Simulation and parametric study on a Beta-type Stirling refrigerating machine

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Abstract - In this paper, a parametric investigation of domestic regenerative Stirling refrigerating machines is conducted using a modified simple model. The model is adapted from an ideal adiabatic engine model and different losses have been included at different stage based on their effect. The model is validated experimentally using the Beta type FEMTO 60 Stirling engine as a case study. The simulation and experimental results show that the coefficient of performance of the Stirling cycle refrigerator at a charging pressure of 17.5 bar, frequency of 12.1 Hz, cold end temperature of -22^{o}C and using nitrogen as a working fluid are found as 38.2% and 35% respectively. Furthermore, the effects of phase angle, working fluid type, operating pressure, and operating frequency of a machine on the performance of a regenerating Stirling cycle refrigerator are investigated.

Keywords: Stirling cycle refrigerator; Modified adiabatic model; Phase angle; Working fluid; Parametric study.

Nomenclature

C_p	isobaric specific heat capacity, J.kg^{-1}.K^{-1}
C_v	isochoric specific heat capacity, J.kg^{-1}.K^{-1}
D	diameter of piston, m
L	length of piston, m
m	mass, kg
$\dot{m}_{i,in}$	inlet mass flow rate, kg.s^{-1}
$\dot{m}_{i,o}$	outlet mass flow rate, kg.s^{-1}
P	pressure, bar
Q	heat, J
R	gas constant, J.kg^{-1}.K^{-1}
T	temperature, K
$T_{i,in}$	inlet fluid temperature, K
$T_{i,o}$	outlet fluid temperature, K
W	work, J
Greek symbols	
μ	viscosity, Pa.s
Index and exponent	
c	compression space
cr	chiller
e	expansion space
g	gas
h	hot heat exchanger

1. Introduction

Nowadays, vapor compression refrigerating machines are used commonly as refrigerating equipment and for air conditioning. They are highly efficient and simple in design. However, the emission of high Chlorofluorocarbon (HCFC) refrigerant has aggravated such ecological problems created by human activities. As the environment is getting polluted by human activities, concerns about global warming and environmental pollution have been increased worldwide. The reduction of greenhouse gases is taken as the most important measure in managing global warming.

Stirling cycle machine is one of the alternatives that could run with environmental friendly fluid. Stirling cycle machine is a type of closed thermodynamic cycle machine. The first Stirling

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cycle was invented in 1816 by Robert Stirling as a heat engine to convert thermal energy to mechanical energy. The air was used as a working fluid to replace the steam engine since they were prone to life-threatening explosions. The Stirling cycle refrigerating machine, which is the counterpart of the Stirling engine, was first recognized in 1832 [1]. The system was practically realized in 1862 when Alexander Kirk built and patented a closed cycle Stirling refrigerator. In the year 1971, Beale stated that by reversing the cycle, Stirling cycle machines could be used for both work producing or refrigerating purpose [2].

According to the classical theory of thermodynamics, the performance of a Stirling cycle machine is a function of pressure, ratio of temperature, speed, phase angle, type of fluid, effectiveness of heat exchanger and volumes [3]. The parameters phase angle, ratio of temperature, ratio of swept volume and volume of the heat exchangers must be selected at the design stage of the Stirling cycle machine.

An analytical model was developed for displacer gap losses which are the sum of enthalpy pumping and shuttle heat losses for Beta and Gamma engine [4]. The pressure difference between compression and expansion spaces was taken into consideration.Theoretical and experimental evaluation of Gamma-type Stirling refrigerator was conducted [5]. The optimum theoretical and experimental analysis coefficients of performances from the research were reported as 0.28 and 0.27 respectively.

In the present study, the developed numerical model is validated using experiment and the effect of different working fluids and phase angle on domestic refrigerating performance is analyzed. The analysis is conducted for air, nitrogen, helium and hydrogen at different operating frequencies.

2. Theory

In this paper, a numerical model which was described in the researchers' previous work for Beta type Stirling cycle domestic refrigerating machine is simulated [6] and the effects of different parameters are evaluated. In developing the model, in addition to other common design features, the authors consider the exact geometrical features for Beta type configuration. This includes the effect of the overlap volume and give a deeper insight on the nature effect of losses i.e. losses may have effect on working conditions (pressure and temperature) have been included directly to the differential equation, those that have effect on the temperature of working fluid are included in the modified simple analysis, and others that could only affect the cooling performance are analyzed separately. The final cooling performance is evaluated by considering ideal performance and all these losses.

2.1. Modified ideal adiabatic modeling

The modified ideal adiabatic model is adapted from Urieli's ideal adiabatic model [3]. In this model, the authors' major adaption is the identification of losses that could have a direct effect on the working condition of the Stirling cycle machine and systematically introducing these losses into the differential equations of the ideal adiabatic model. Therefore, shuttle heat loss, that is transfer of heat from hot space to cold space with the movement of displacer, and gas leakage to the crank case through a gap between piston and cylinder are identified to have an effect on the working condition (pressure and temperature) of the working gas and hence on overall performance of the machine. Hence, differential equations of mass and energy conservation of the original ideal adiabatic analysis of the Stirling refrigeration machine have been modified to include the mass leakage and shuttle heat loss effects respectively. The working fluid used in this research work is assumed as ideal gas which is valid in most Stirling machine analysis. The mass conservation equation is modified as equation (1):

$$m_c + m_h + m_r + m_{cr} + m_e - m_{leak} = M_t \tag{1}$$

where m_{leak} denotes mass leakage to crank case due to the clearance. The mass leakage to the crank case was calculated based on [3, 7, 9] using equation (2):

$$m_{leak} = D\pi \frac{P + P_{buffer}}{4RT_g}(U_p J - \frac{J^3}{6\mu}(\frac{P - P_{buffer}}{L})) \tag{2}$$

The energy conservation equation for each control volume shown in Figure 1 is written as equation (3):

$$\delta\dot{Q}_i - \delta\dot{Q}_{shut} - \delta\dot{W}_i = C_p(T_{i,o}\dot{m}_{i,o} - T_{i,in}\dot{m}_{i,in}) + C_v d(m_i T_i)/dt \tag{3}$$

where, $\delta\dot{Q}_{shut}$ is the shuttle heat loss, $\dot{m}_{i,o}, \dot{m}_{i,in}$ outlet and inlet mass flow rates, $T_{i,o}$, and $T_{i,in}$ outlet and inlet temperatures of fluid.

The shuttle heat loss as given in [9, 10, 11] is calculated by using equation (4):

$$\delta\dot{Q}_{shut} = \frac{\pi s^2 K_g D_d}{8JL_d}(T_c - T_e) \tag{4}$$

where, $s, K_g, D_d, J,$ and L_d are stroke, thermal conductivity of gas, diameter of displacer, gap between displacer and cylinder, and displacer length. T_c and T_e are gas temperature in compression and expansion spaces respectively.

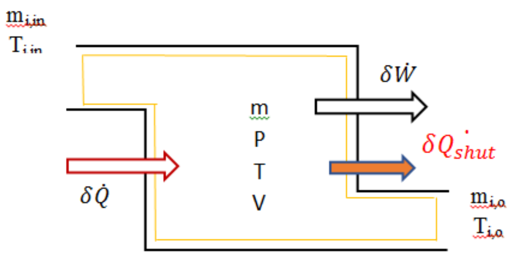

Figure 1 : *Open system representing a control volume ($\delta\dot{Q}_{shut}$ applicable only for compression and expansion spaces)*

Generally gas leakage has a direct effect on pressure and pressure changes. Shuttle heat loss has a direct effect on change in pressure and mass accumulations on working spaces.

2.2. Modified simple analysis

This research follows a second order decoupled numerical analysis method in which the analysis considers the refrigeration system as a number of separate but interrelated factors. The factors are decoupled, the basic mass flow, heat, and power transfers are determined using ideal isothermal analysis and modified ideal adiabatic analysis (considering shuttle and gas leakage losses). The energy transfers obtained are then corrected using losses determined from modified simple analysis. The heat transfer losses, in this modified simple analysis include losses due to internal conduction in the regenerator, regenerator ineffectiveness/external conduction loss, heat conduction loss and pumping losses. The losses included in modified simple analysis associated with power losses are losses due to pressure drop in heat exchangers (friction losses), losses due to finite speed of piston and mechanical friction losses. The detailed equations for each type of losses have been presented in authors' previous research [6]. The various losses are indicated in Figure 2.

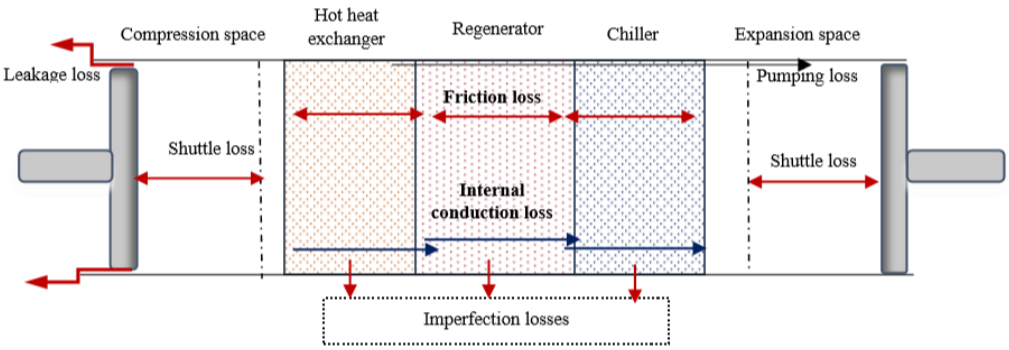

Figure 2 : *Mapping of various heat and power losses (adapted from [9]).*

3. Experimental work and Validation

The simulation result of the model is evaluated by considering the FEMTO 60 Stirling engine as a case study. The considered experimental device is a reversible thermal machine (motor and/or receiver) that operates between two heat sources at constant temperatures. It works according to the Stirling cycle. The working fluid used for the experiment is nitrogen gas. The hot heat exchanger and the chiller both have slot geometric arrangement and the configuration of the regenerator is an annular configuration with a stainless steel woven screens matrix. The detail experimental setup is presented in [6] and [8]. The main parameters and dimensions of the experimental device are shown in Table 1.

No	Parameters	value
1	Hot heat temperature (K)	305
2	Cooling temperature (K)	270
3	Piston diameter (mm)	60
4	displacer diameter (mm)	59
5	Piston stroke (mm)	40
6	Compression space swept volume (cm^3)	103
7	Expansion space swept volume (cm^3)	113
8	Working gas	Nitrogen
9	Frequency (Hz)	7.5
10	Charging pressure (bar)	20

Table 1 : *Specifications of Stirling cooling machine.*

The refrigerator input power could be found using equation (5) from the electrical power consumed as follows:

$$\dot{W}_{in} = \eta_m \eta_e \dot{W}_e \quad (5)$$

Where, η_m is the mechanical efficiency, η_e is electrical efficiency and $\dot{W}_e$ is the electrical power consumed by the motor.

The refrigerator cooling capacity is obtained from equation (6) as :

$$\dot{Q}_{cr} = \dot{m}_{cr} C_p \Delta T \tag{6}$$

Where, $\dot{m}_{cr}$, C_p, and ΔT are the water flow rate through chiller heat exchanger, the specific heat capacity of water, and the temperature difference between water inlet and outlet of the coil in the chiller respectively. Thermocouples applied to measure the temperature of water inlet and outlet.

The cooling coefficient of performance of the regenerative refrigerating machine is determined from input power and cooling capacity using equation (7).

$$COP = \frac{\dot{Q}_{cr}}{\dot{W}_{in}} \tag{7}$$

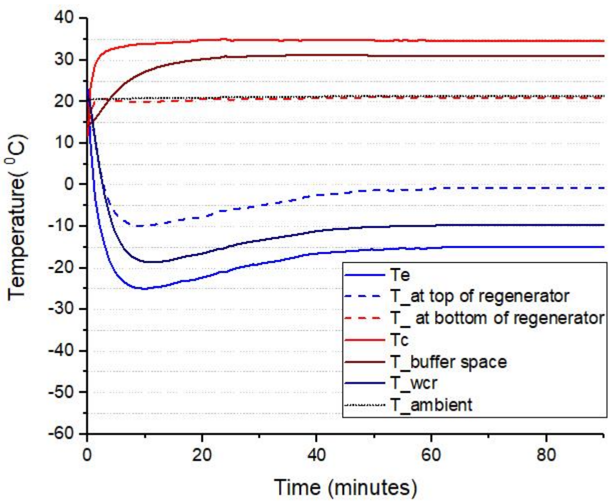

Figure 3 : *Experimental temperature distribution (P=17.5 bar, operating frequency =12.1 Hz and cooling load =451 W)*

Figure 3 is a plot of experimental temperature variation in different parts of the refrigerating machine. The experiment was run for 90 minutes to be sure of the trend of the cooling process for a long period of time. The stabilization temperature at the cold end is -15^oC(258 K) at a cooling load of 451 W, charging pressure of 17.5 bar and frequency of 12.1 Hz. Such Stirling refrigerator needs 3 minutes only to reach such a low temperature and the stabilization temperature is achieved after 40 minutes. The minimum temperature found is -24.9^oC and achieved 10 minutes after starting the operation. The buffer space temperature rises approximately by (10^oC) from the ambient temperature. Furthermore, the stabilized temperature difference between the compression space (warm section) and the buffer space is less than (4^oC). This result shows that there seems more gas leakage towards the buffer space.

The experimental COP of refrigerating machine for a cold end temperature of -22^oC and -33^oC are found respectively as 35% and 29% respectively. For the same cold end temperature the COP from simulation is found as 38.2% and 32%. This shows that the simulation result approaches the experimental result with an error of +9.2%.

4. Simulation results and discussion

The refrigeration rotational speed varied between 435 to 725 rpm during the tests when nitrogen is used as the working fluid. The charging pressure is varied between 15-25 bar and a phase angle of 90^o. The simulation results of the developed model are presented in this section.

Figure 4 demonstrates a simulated plot of pressure versus volume for modified simple analysis in expansion and compression spaces. In this case the pressure drop in three heat exchangers (chiller, regenerator and hot heat exchanger) is considered.

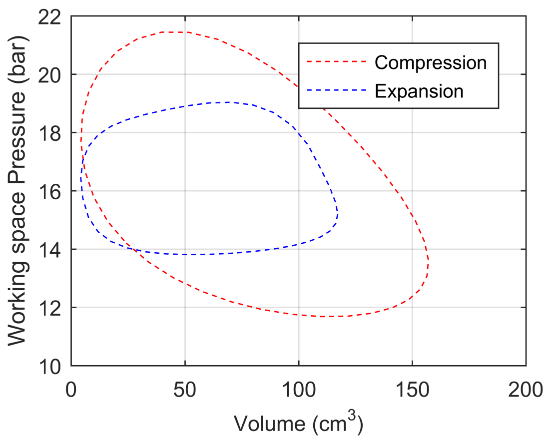

Figure 4 : *PV-diagram from modified simple analysis simulated ($T_h = 305K$,$T_{cr} = 295$ K and P = 17.5 bar)*

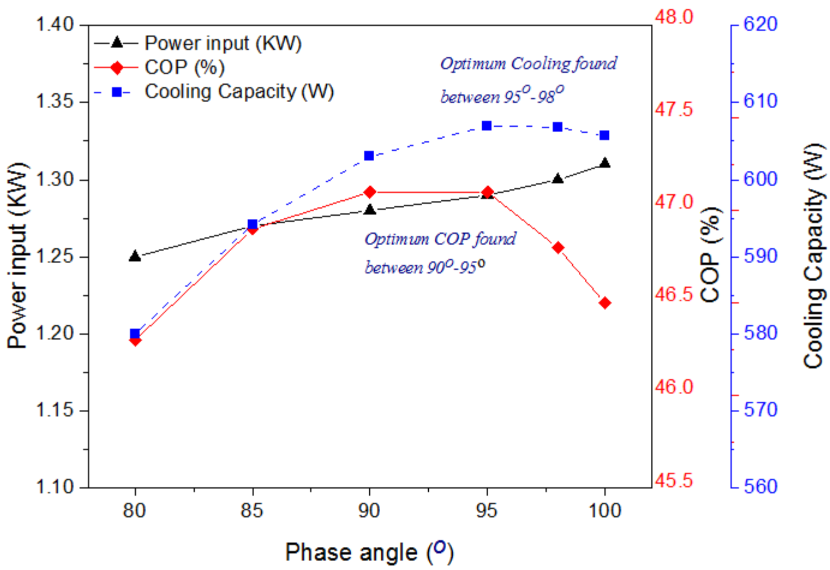

Figure 5 : *Effect of phase angle on the performance of Stirling refrigerator*

The effect of phase angle between the piston and displacer positions on the performance of regenerative Stirling cycle refrigerating machine is evaluated using the model at different phase angles. Figure 5 illustrates the effect of phase angle on the required power input, COP and cooling production of the refrigerating machine. As could be seen from this figure, required input power increases with phase angle. On the other hand, the COP and cooling production

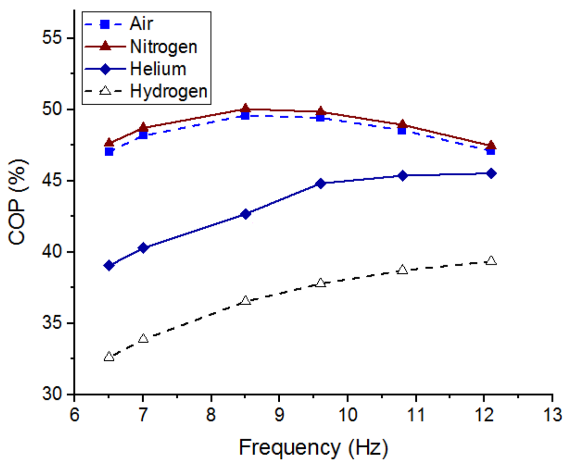

Figure 6 : *Effect of working fluid types on COP of Stirling cycle refrigerator with frequency at (T_h = 305 K, T_{cr} = 295 K and P = 17.5 bar)*

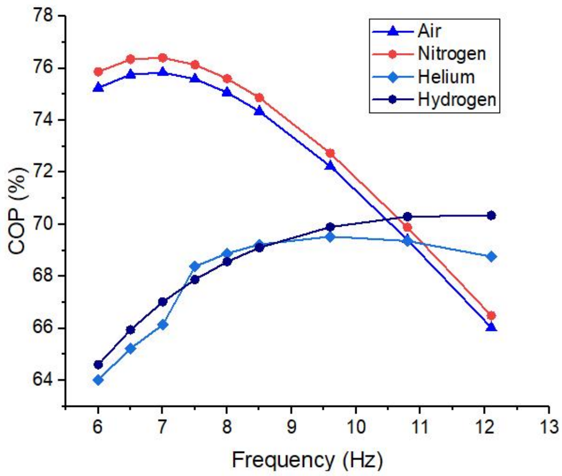

Figure 7 : *Effect of working fluid types on COP of Stirling cycle refrigerator with frequency at (T_h = 305 K, T_{cr} = 295 K and P = 25 bar)*

of a refrigeration machine has maximum value with different phase angles. The optimum COP exists between phase angles of $90^o - 95^o$ and the optimum cooling production could be found between phase angles of $95^o - 98^o$.

Figure 6 displays the influence of working fluids on the coefficient of performance of the refrigerating machine with operating frequency at $T_h = 305$ K, $T_{cr} = 295$ K and charging pressure of 17.5 bar. It can be observed that air and nitrogen have better COP, with nitrogen slightly outperforms and the optimum COP is 50% at a frequency of 8.5 Hz. The COP for helium and hydrogen relatively increase with operating frequency or the optimum COP found at relatively higher frequency.

Figure 7 displays the influence of working fluids on the coefficient of performance of the refrigerating machine with operating frequency at $T_h = 305$ K, $T_{cr} = 295$ K and charging pressure of 25 bar. It can be observed that air and nitrogen have better COP than helium and hydrogen. The optimum COP for air and nitrogen is found at a frequency of 7 Hz and their respective values are 75.8% and 76.41% respectively. The optimum COP for helium is found at a frequency of 9.6 Hz and for hydrogen is at 12.1 Hz. From Figure 5 and figure 6, it could be seen that as pressure increases, the cooling performance increases and the optimum value is found at relatively lower operating frequency/speed. The pressure drop increase with frequency and the COP drops at higher frequency. The lower the molar mass of a gas, the less it is affected by frequency. As a result the COP drops faster for air and nitrogen than for helium and hydrogen.

Figure 6 and 7 confirm that as pressure increases the refrigeration performance seems to be more sensitive to the speed of machine.

5. Conclusion

In this paper, simulation of a developed modified simple model for domestic cooling application is conducted. The effect of phase angle, working fluid type, operating pressure and operating frequency on the performance of the the Stirling refrigerator is investigated. Based on the simulation and experimental investigations, the following results are found:

- The optimum refrigerating performance both for cooling production and COP is found at a phase angle of 95^o.

- The optimum performances is found at an operating frequency of 7 Hz for nitrogen and air, at 9.6 Hz for helium and at 12.1 Hz for hydrogen gas.

- The simulation and experimental results show that the coefficient of performance of the Stirling cycle refrigerator at a charging pressure of 17.5 bar, frequency of 12.1 Hz, cold end temperature of -22^oC and using nitrogen as a working fluid are found as 38.2 % and 35 % respectively. This shows that the simulation result approaches the experimental result with an error of 9.1 %.

- The coefficient of performance of a Stirling cycle refrigerator increases with charging pressure.Nitrogen is the most efficient working fluid and still air is very comparable gas. These two working gases are more efficient especially at relatively lower operating speed/frequency.

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