Linear stability analysis of a ferrofluid in a radially heated concentric cylindrical annulus with an applied magnetic field furnished by stack of magnets in the inner cylinder.

Anupam M HIREMATH[1], Antoine MEYER[1], Harunori N YOSHIKAWA[2], Innocent MUTABAZI[1*]

[1]LOMC UMR CNRS 6294, UNIVHAVRE, Normandie Université
53 rue de Prony, 76600-Le Havre (France)
[2] INPHYNI UMR CNRS 7010, Université de Côte d'Azur, Institut de Physique de Nice
Avenue Joseph Vallot, 06108-Nice (France)
*(Corresponding author: innocent.mutabazi@univ-lehavre.fr)

Abstract - The linear stability analysis (LSA) was conducted on a ferrofluid confined in an infinitely long cylindrical annulus, with differential heating between the inner and the outer cylinder. A stack of magnets inside the inner cylinder providing a magnetic field. In addition, the cylinders can rotate rigidly with an angular frequency Ω. The ferrofluid is subject to the Archimedean buoyancy due to the terrestrial gravity $\boldsymbol{g}$, magnetic buoyancy due to magnetic gravity $\boldsymbol{g_m}$, and centrifugal buoyancy due to the centrifugal acceleration. In this study we identify the different types of instalibity modes that can develop in such a fluid system.

Nomenclature

A, C Coefficients for base state axial velocity.
F_B Magnetic buoyancy due to the Kelvin force.
$\boldsymbol{g}$ Terrestrial gravity, $\mathrm{m.s^{-2}}$
$\boldsymbol{g_m}$ Magnetic gravity, $\mathrm{m.s^{-2}}$
$\boldsymbol{g_c}$ Centrifugal gravity, $\mathrm{m.s^{-2}}$
$\boldsymbol{v}$ Velocity,$\mathrm{m.s^{-1}}$
Ra_m Magnetic Rayleigh number
Ta Taylor number
Pr Prandtl number
Gr Grashoff number
K_0, K_1 Modified Bessel functions of second kind
$\boldsymbol{M}$ Magnetization,$\mathrm{A.m^{-1}}$
$\boldsymbol{B}$ Applied magnetic field,T
R_1 Inner radius, m
R_2 Outer radius, m
T_1 Inner cylinder temperature, K
T_2 Outer cylinder temperature, K
$d = R_2 - R_1$ Width of the gap, m
n Number of azimuthal modes

Greek symbols

Ω Rotation angular frequency $\mathrm{rad.s^{-1}}$
ω Mode angular frequency $\mathrm{rad.s^{-1}}$
$\eta = R_1/R_2$ Radius Ratio
λ_b Axial spacing of the magnets, m
Θ Dimensionless temperature of the base flow
π Dimensionless modified pressure
ν Kinematic viscosity, $\mathrm{m^2.s^{-1}}$
α Thermal expansion coefficient, $\mathrm{K^{-1}}$
α_m Thermal variation of magnetization, $\mathrm{K^{-1}}$
$\gamma_a = \alpha\Delta T$ Dimensionless thermal expansion coefficient
κ_b Axial wavelength of the magnetic field, $\mathrm{m^{-1}}$
ρ_{ref} Density of fluid at reference temperature, $\mathrm{kg/m^3}$

1. Introduction

Ferrofluids are stable colloids of nano particles nearly $10nm$ in diameter of ferro or ferri-magnetic particles in a carrier fluid. The stability of these fluids is governed by the Brownian

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motion in the fluid. A wide range of carrier fluids having oil or aqueous base [1] could be used to fabricate, and many ferrofluids are commercially available to satisfy particular applications.

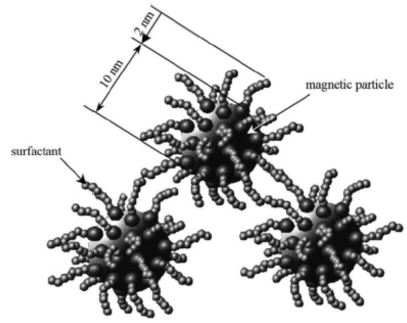

Figure 1 : *Description of a ferromagnetic particle.[2]*

Fig.1 shows the magnetic particles coated with a surfactant to avoid any coagulation between the particles. These particles need to remain suspended in the fluid.

Under the influence of an external magnetic field these fluids tend to orient themselves in the direction of the applied field. Ferrofluids, due to their thermal capacity, are used in several heat transfer applications. The external applied field can have an influence on the fluid dynamics. Hence, a linear stability analysis (LSA) has been performed to understand the stability of the ferrofluids in micro-gravity as well as in terrestrial conditions.

2. Flow equations and system under study

In this section we will describe the system flow and the governing equations.

2.1. Flow configuration

In the Fig.2 we have two coaxial cylinders which are infinitely long along the vertical direction and the gap between the two cylinders is filled with Newtonian ferrofluid. The inner cylinder is at radius R_1 and it is maintained at temperature T_1, the outer cylinder has a radius R_2 at temperature T_2, where $T_1 > T_2$.

The magnetic field is provided by the stack of permanent magnets placed inside of the inner cylinders and these magnets are evenly placed at a distance of λ_b, hence the axial wave number of the applied field is given by $\kappa_b = 2\pi/\lambda_b$.

2.2. Body force

The flow is subject to three body forces : the Archimedean buoyancy $\rho\theta\boldsymbol{g}$, the centrifugal buoyancy $\rho\theta\boldsymbol{g_c}$ and the magnetic buoyancy $\rho\theta\boldsymbol{g_m}$ where $\theta = T - T_{ref}$, the reference temperature T_{ref} can be chosen either at the inner cylinder or the outer cylinder or at the central cylindrical flow surface. The magnetic buoyancy stems from the Kelvin force acting on a ferrofluid in a magnetic field $\boldsymbol{B}$[1].

$$\boldsymbol{F_B} = M_0 \boldsymbol{\nabla}|\boldsymbol{B}| + |\boldsymbol{B}|\boldsymbol{\nabla} M_0 \quad (1)$$

In the Boussinesq approximation, the second term in Eqn.1 is of second order and hence it can be neglected. The magnetization density M_0 is a function that decreases with the temperature and is defined as

$$M_0 = M_{ref}[1 - \alpha_m \theta] \quad (2)$$

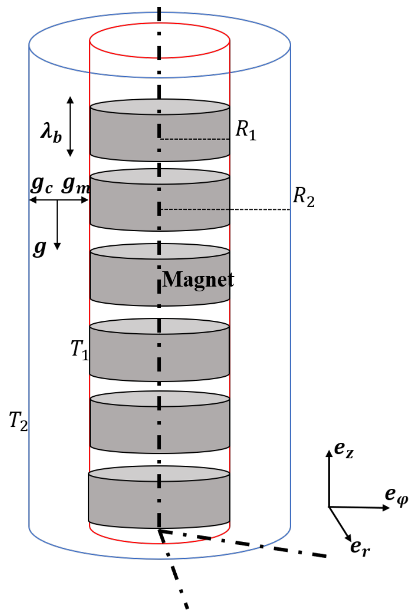

Figure 2 : *Flow configuration : Cylindrical annulus with the inner cylinder filled of magnets stacked along the vertical axis.*

where $\alpha_m = K/M_{ref}$, $K = -\partial M/\partial T$ is the pyromagnetic coefficient. Hence the magnetic buoyancy becomes,

$$\boldsymbol{F_B} = -\alpha_m M_{ref} B_0 \kappa K_1(\kappa r)\theta \boldsymbol{e_r} + \boldsymbol{\nabla}\left(M_{ref} B_0 K_0(\kappa r)\right) \tag{3}$$

The gradient term in the Eqn.(3) is integrated into the pressure term and the non conservative part is defined as the magnetic buoyancy $\rho\alpha\theta g_m$ where we have introduced the centripetal $magnetic\ gravity$ defined as,

$$\boldsymbol{g_m} = \frac{\alpha_m M_{ref} B_0 \kappa_b K_1(\kappa_b r)}{\alpha \rho_{ref}} \boldsymbol{e_r} \tag{4}$$

2.3. Governing equations

We obtain the following set of dimensionless flow equations,

$$\boldsymbol{\nabla}.\boldsymbol{v} = 0 \tag{5}$$

$$\frac{d\boldsymbol{v}}{dt} + (\boldsymbol{v}.\boldsymbol{\nabla})\boldsymbol{v} = -\boldsymbol{\nabla}\pi + \Delta\boldsymbol{v} - Gr\theta\boldsymbol{e_z} - \frac{Ra_m}{Pr}\theta\boldsymbol{e_r} - \gamma_a \frac{v^2}{r}\theta\boldsymbol{e_r} \tag{6}$$

$$\frac{d\theta}{dt} + (\boldsymbol{v}.\boldsymbol{\nabla})\theta = \frac{1}{Pr}\Delta\theta \tag{7}$$

where the control parameters are defined as, Prandtl number $Pr = \nu/\kappa$, magnetic Rayleigh number $Ra_m = \alpha\Delta T g_m(\overline{R})d^3/\nu\kappa$ defined at the mid gap and the Grashof number $Gr = -\alpha\Delta T g d^3/\nu^2$.

2.4. Base state flow

The base state flow is considered to be stationary, axisymmetric and axially invariant and can only depend on the radial coordinate.

2.4.1. Temperature and velocity profile

The temperature in the base state is given by,

$$\Theta = \frac{\ln[r(1-\eta)]}{\ln(\eta)} \tag{8}$$

and the axial velocity is given by [3],

$$W_b = A\left\{C\left[(1-\eta)^2 r^2 - 1 + (1-\eta)^2\Theta\right] - 4\left[r^2(1-\eta)^2 - \eta^2\right]\Theta\right\} \tag{9}$$

where the coefficients A and C are obtained from the no-slip conditions at the cylindrical walls

$$A = \frac{1}{16(1-\eta)^2} \tag{10}$$

$$C = \frac{(1-\eta^2)(1-3\eta^2) - 4\eta^4\ln(\eta)}{(1-\eta^2)^2 + (1-\eta^4)\ln(\eta)} \tag{11}$$

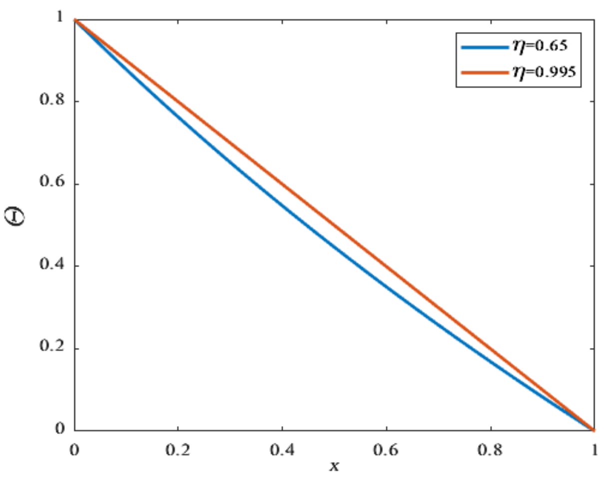

Figure 3 : *Variation of the base state temperature with the dimensionless radial coordinate*

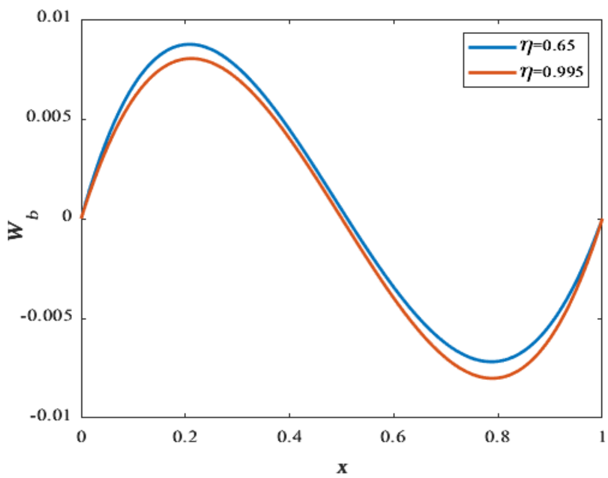

Figure 4 : *Variation of the axial velocity with the dimensionless radial coordinate*

Fig.3 and Fig.4 show the curvature dependence of the temperature and axial velocity of the base flow. The flow is ascending near the inner cylinder and it is a descending near the cold outer cylinder.

2.4.2. Magnetic gravity in the base state

We can estimate the magnitude of the magnetic gravity in the cylindrical annulus from the Eqn.(4). For, we choose the value $\lambda_b = 3.54d$ as suggested by Tagg and Weidman [1]. We use data of the water-based ferrofluid Fe_3O_4 from [4] [5] to calculate the values of the magnetic averaged gravity $\langle g_m \rangle = \frac{1}{R_2 - R_1}\int_{R_1}^{R_2} \boldsymbol{g_m} dr$ given in Table(1)

3. Results

The study was conducted to understand the thermo-magnetic convection in a cylindrical annulus filled with a water-based ferrofluid whose Prandtl number is $Pr = 15$.

η	$\langle g_m \rangle / g$
0.65	15.5371
0.7	15.4513
0.95	15.1262

Table 1 : *Ratio of the average magnetic gravity to the terrestrial gravity.*

We have superimposed infinitesimal perturbations of the base flow and linearized resulting equations.The perturbations have been expanded into normal modes of the form $\exp(st+in\varphi+ikz)$, where $s=\sigma+i\omega$ is the complex temporal growth rate, k_z is the axial wavenumber, n is the number of modes in the azimuthal direction. The resulting eigenvalue problem is solved with Chebyshev collocation method.

3.1. Thermo-magnetic convection in microgravity

In the space where the gravity is negligible the Archimedean buoyancy vanishes and has no effect on the flow destabilization. The thermal convection can thus be induced by the magnetic buoyancy. We have investigated the thermomagnetic convection in microgravity conditions and the effect of solid body rotation on the thermomagnetic convection.

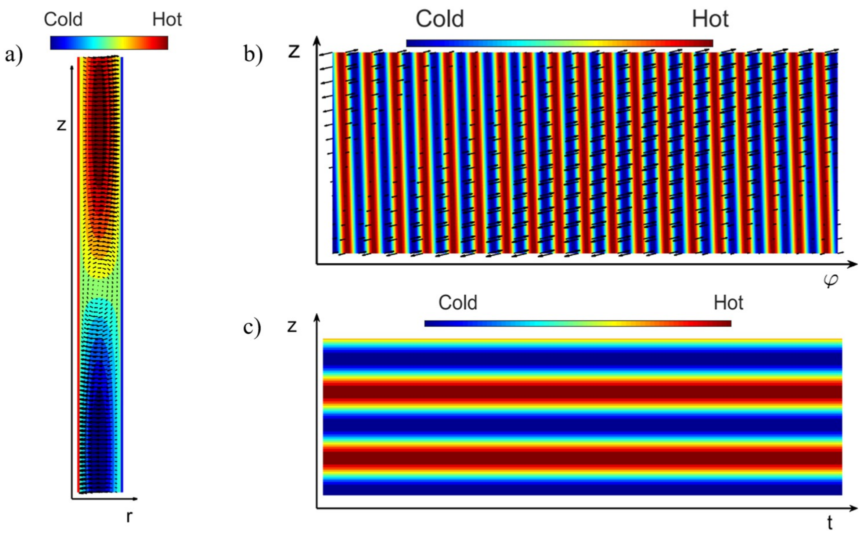

Figure 5 : *Eigenfunctions of velocity and temperature perturbations for stationary cylinders in microgravity for $\eta=0.85$, $Ta=0$*

In Fig.5 we have plotted the eigenfunctions of velocity and temperature perturbations for zero rotation i.e. $Ta=0$. Fig.5a shows the cross section in the r-z plane of the critical modes. These critical modes have helical structure Fig.5b ($k_c=0.42, n_c=19$) and they are stationary Fig.5c ($\omega=0$).

When a small body rotation is added, the threshold of the thermomagnetic convection is delayed and the critical modes become columnar oscillatory modes Fig.6c ($\omega\neq 0$).

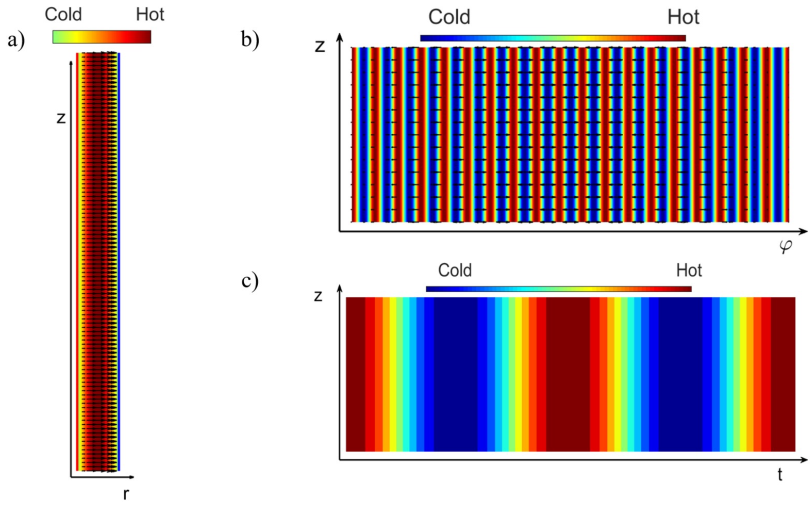

Figure 6 : *Eigenfunctions of velocity and temperature perturbations in microgravity* $\eta = 0.85$, $Ta = 40$

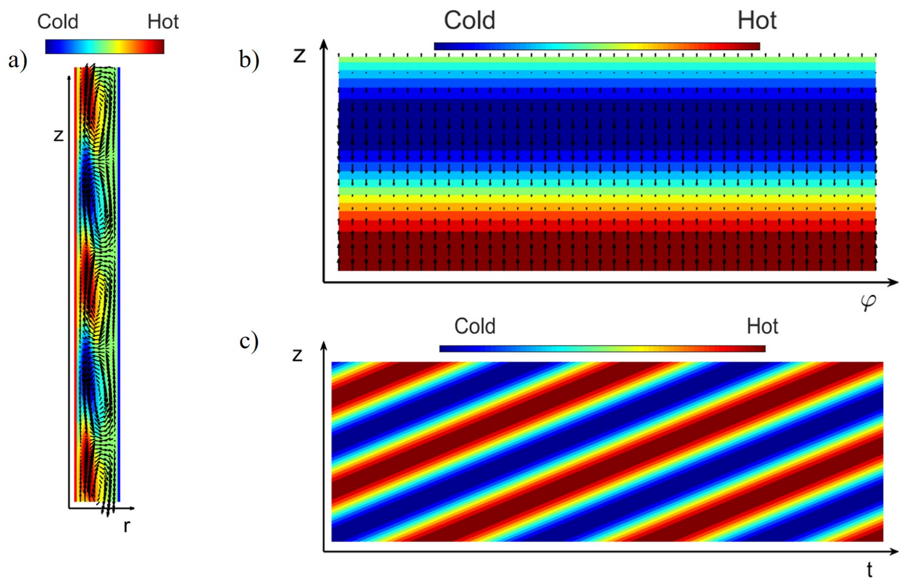

Figure 7 : *Eigenfunctions of velocity and temperature perturbations for thermal modes with no rotation in terrestrial conditions* $\eta = 0.9$.

3.2. Thermo-magnetic convection in terrestrial conditions

In the presence of the Earth's gravity we need to include the Archimedean buoyancy which has an influence on the stability of the flow.

In the terrestrial conditions for $Pr = 15$, there exist two types of modes: the thermal modes and magnetic modes. Thermal modes exist for $Pr > 11.5$ [3] for small values of Ra_m. For greater values of Ra_m we find that the modes are magnetic.

Similar analogy can be made with dielectrophoretic fluids where for higher values of electric field and Pr, critical modes are thermal modes and electric modes[6].

For the case of no rotation the thermal modes are oscillatory and magnetic modes are sta-

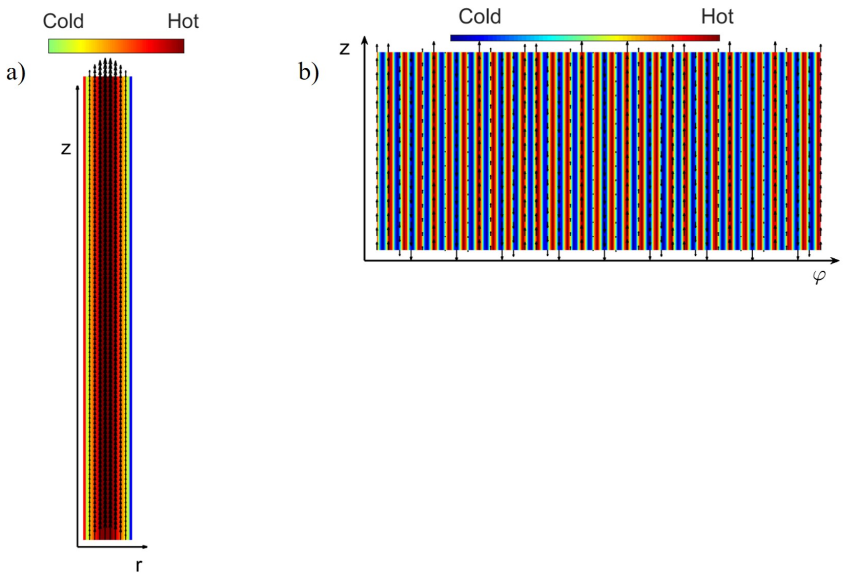

Figure 8 : *Eigenfunctions of velocity and temperature perturbations for magnetic modes with no rotation in terrestrial conditions* $\eta = 0.9$.

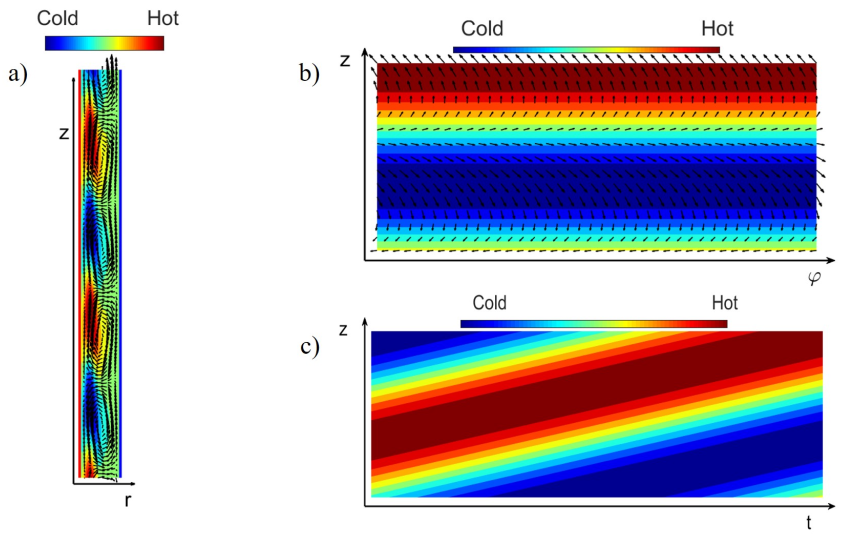

Figure 9 : *Eigenfunctions of velocity and temperature perturbations for thermal modes with small rotation in terrestrial conditions* $Ta = 20$ $\eta = 0.9$.

tionary as shown in Fig.7 and Fig.8 and in the case of $Ta = 20$ both the thermal and magnetic modes are oscillatory Fig.9 and Fig.10.

4. Conclusion

For a small amount of applied magnetic field the magnetic gravity g_m generated is on an average 15 times grater than the terrestrial gravity. In microgravity situation it is the centripetal gravity g_m which destabilizes the flow and the centrifugal gravity g_c is the stabilizing force showing that the rotation of cylinders delays the onset of the instabilities.

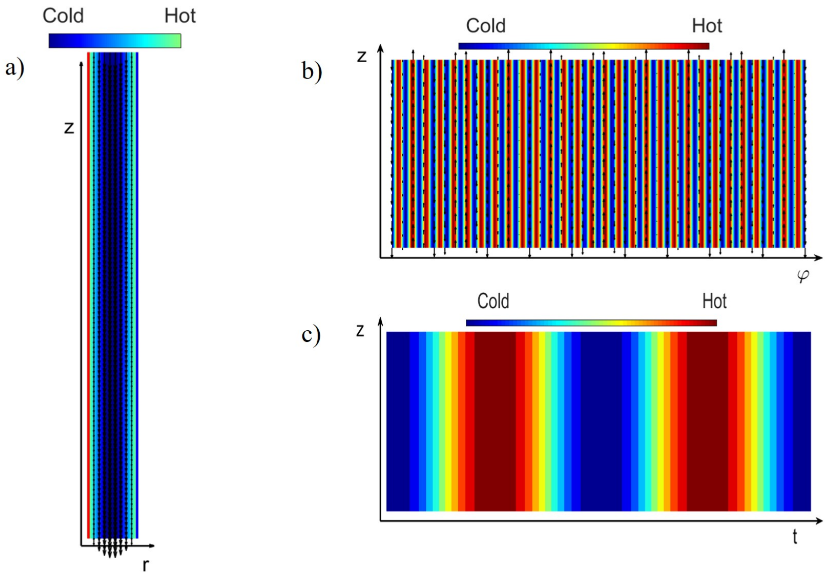

Figure 10 : *Eigenfunctions of velocity and temperature perturbations for magnetic modes with small rotation in terrestrial conditions* $Ta = 20$ $\eta = 0.9$.

The nature of modes in microgravity are always magnetic and can be stationary or oscillatory depending on the rotation rate of the cylinders. For stationary cylinders they are non-axisymmetric and for rotating cylinders they are columnar. In the terrestrial conditions there is a vertical velocity $\boldsymbol{W_b}$ induced by the torque of the Archimedean buoyancy. For small number of Ra_m the critical modes are thermal and for large number of Ra_m they are magnetic. Thermal modes are oscillatory and magnetic modes are stationary only in zero rotation, otherwise they are oscillatory.

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