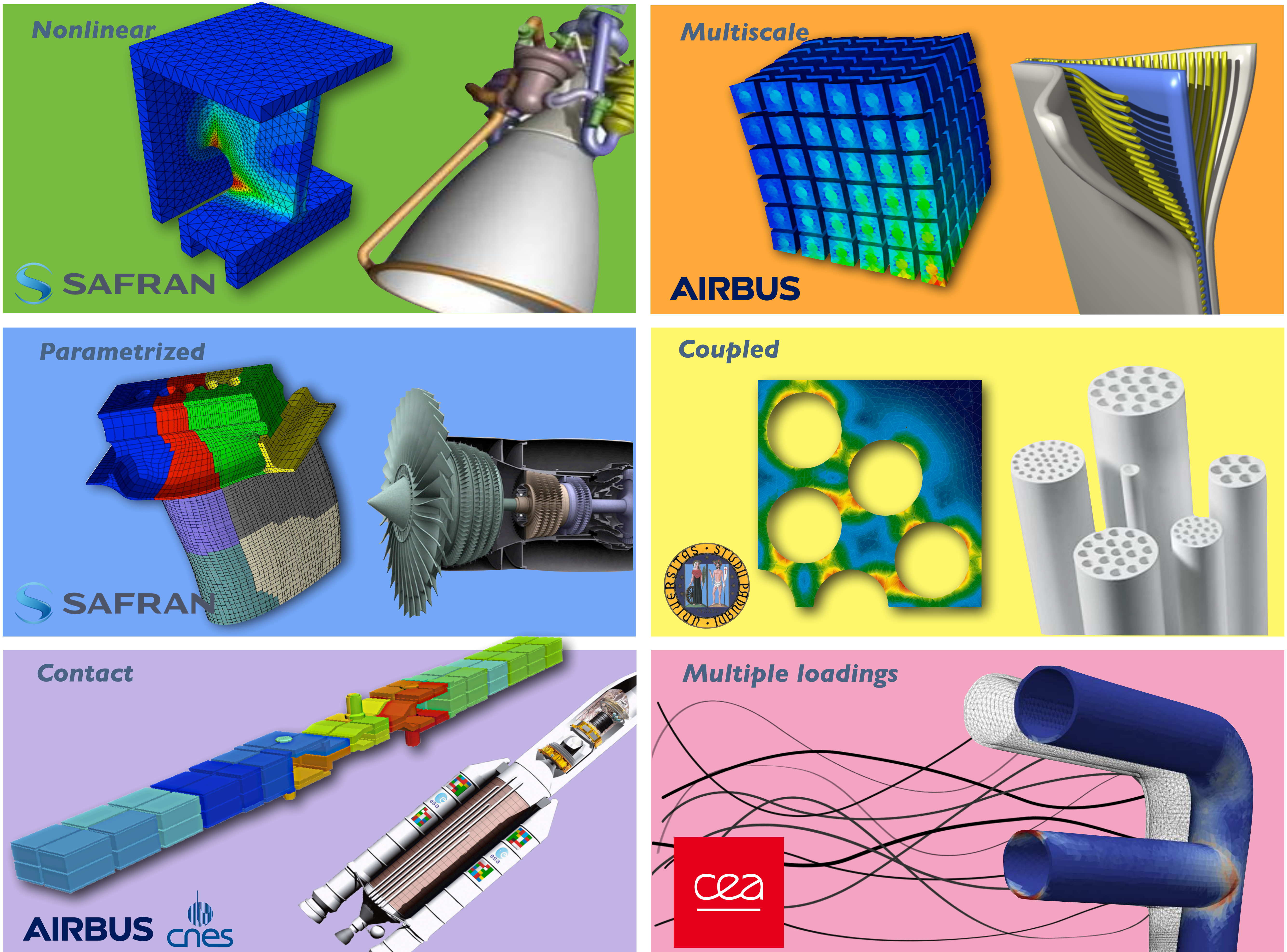
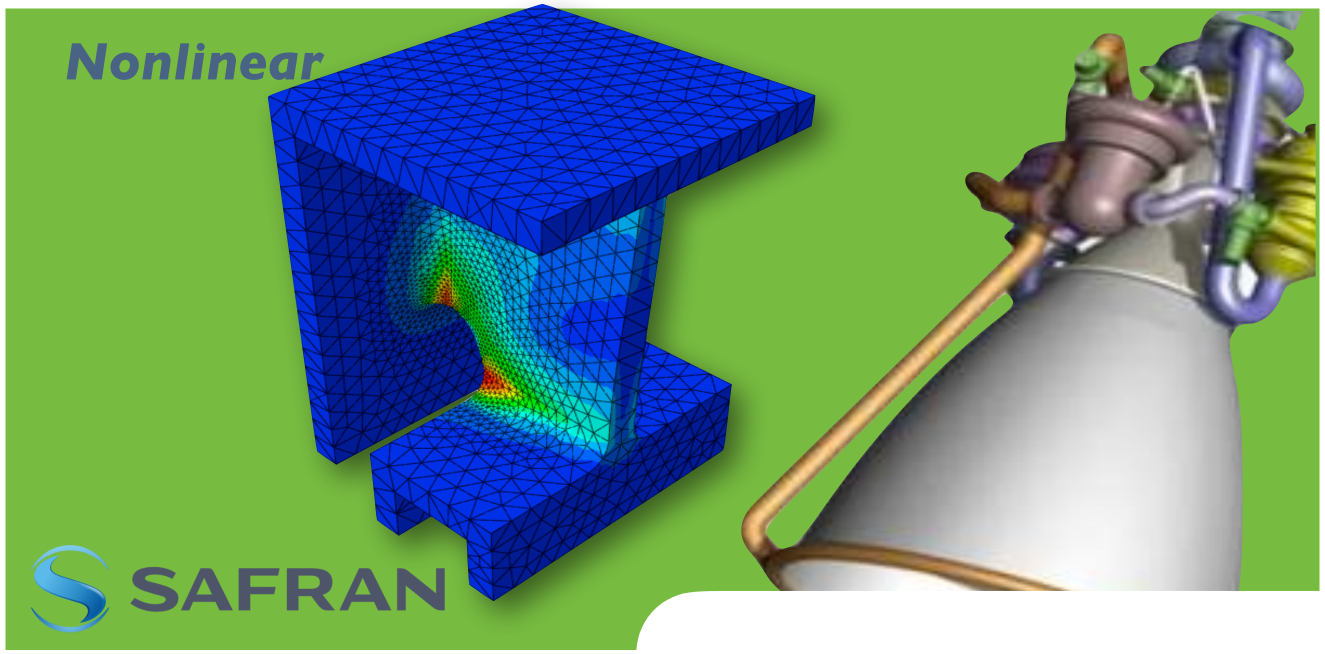
Réduction de modèles en mécanique non linéaire pour la construction d'abaques virtuels

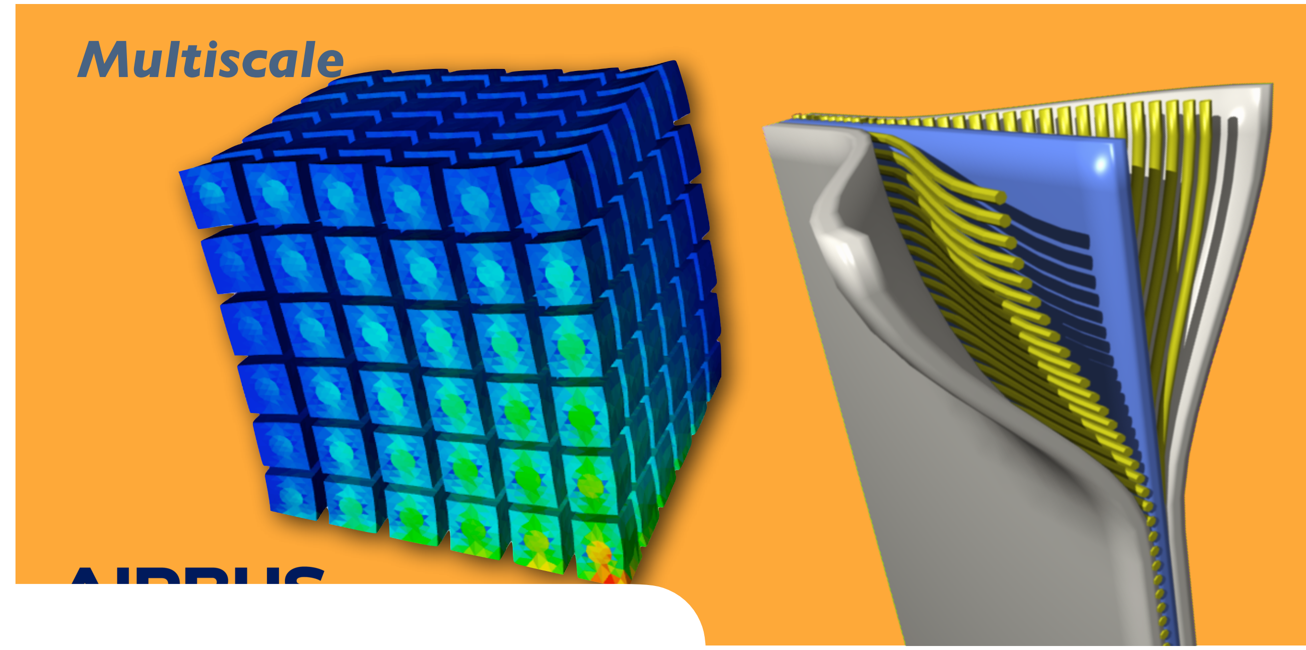
pour les problèmes paramétrés, dynamiques, multiphysiques, multiéchelles…

David Néron

P.-A. Boucard, A. Fau, P.-A. Guidault, P. Ladevèze F. Louf (LMPS)
Pierre-Etienne Charbonnel (CEA), F. Feyel (SAFRAN), R. Scanff (SIEMENS)
A. Daby-Seesaram, E. Foulatier, P.-E. Malleval, V. Matray, N. Relun, F. Wurtzer…


Nonlinear
SAFRAN
Multiscale
AIRBUS
Parametrized
SAFRAN
Coupled
Contact
AIRBUS
cnes
Multiple loadings
cea



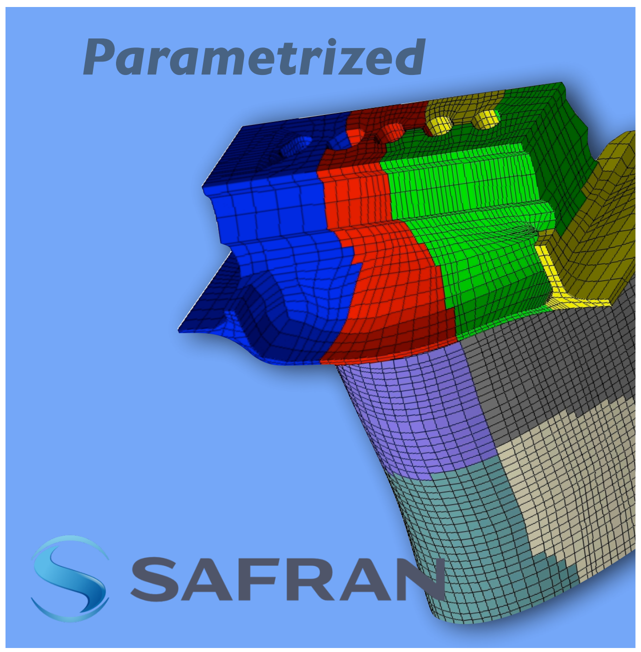


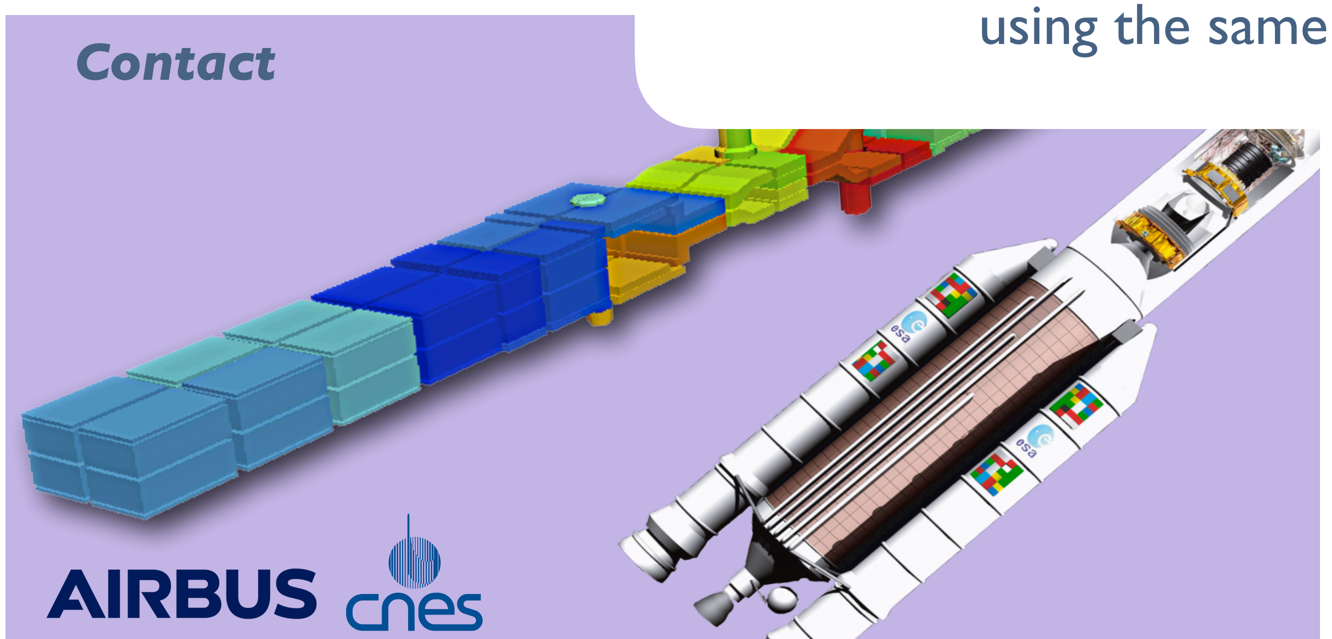
In our team, model reduction by;

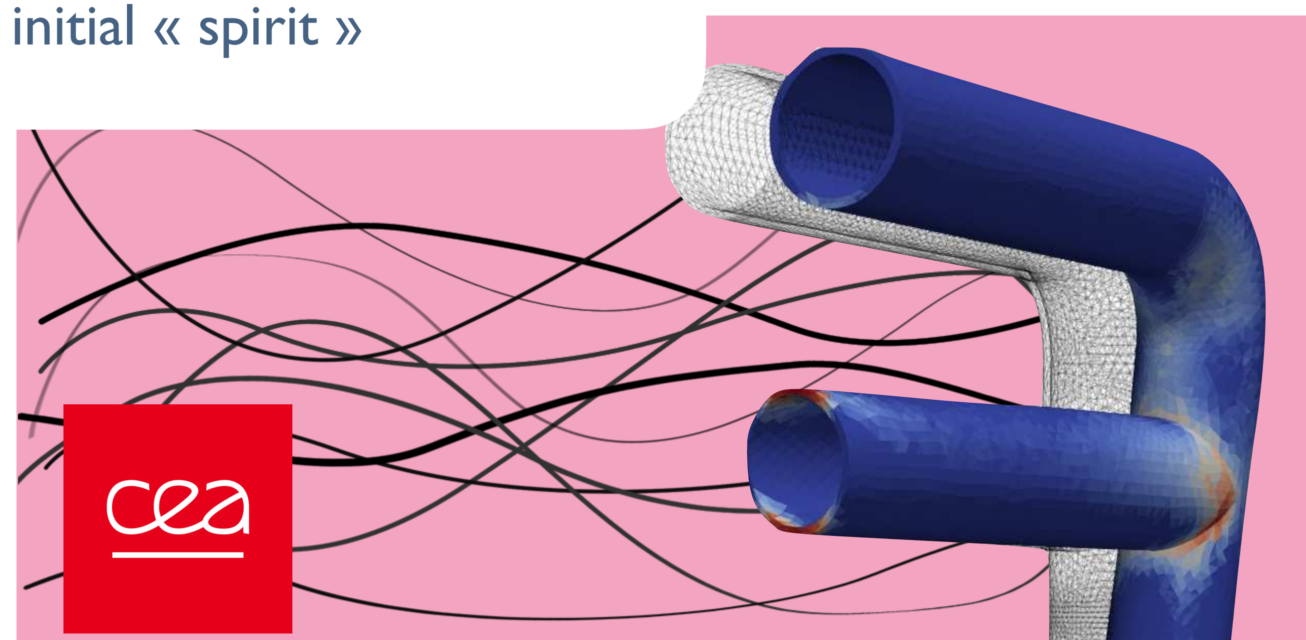
LATIN
+
Proper Generalized Decomposition (PGD)

Initially introduced for the nonlinear problems
many works for more than 30 years

Since adapted to many other situations
using the same initial « spirit »







Outline

1. The LATIN method and Proper Generalized Decomposition
2. Solving parametrized problems to build virtual charts
3. Many queries in multiphysics problems
4. Conclusion

LMPS

Outline

1. The LATIN method and Proper Generalized Decomposition
2. Solving parametrized problems to build virtual charts
3. Many queries in multiphysics problems
4. Conclusion

LMPS

Reduced-Order Modeling (ROM)

Tentative definition

- capture **main features** of the behavior, retaining the **accuracy** of the approximation
- use the **redundancy** of information

➡ possibility of approximating a **complex system** using only a **handful of DOFs**

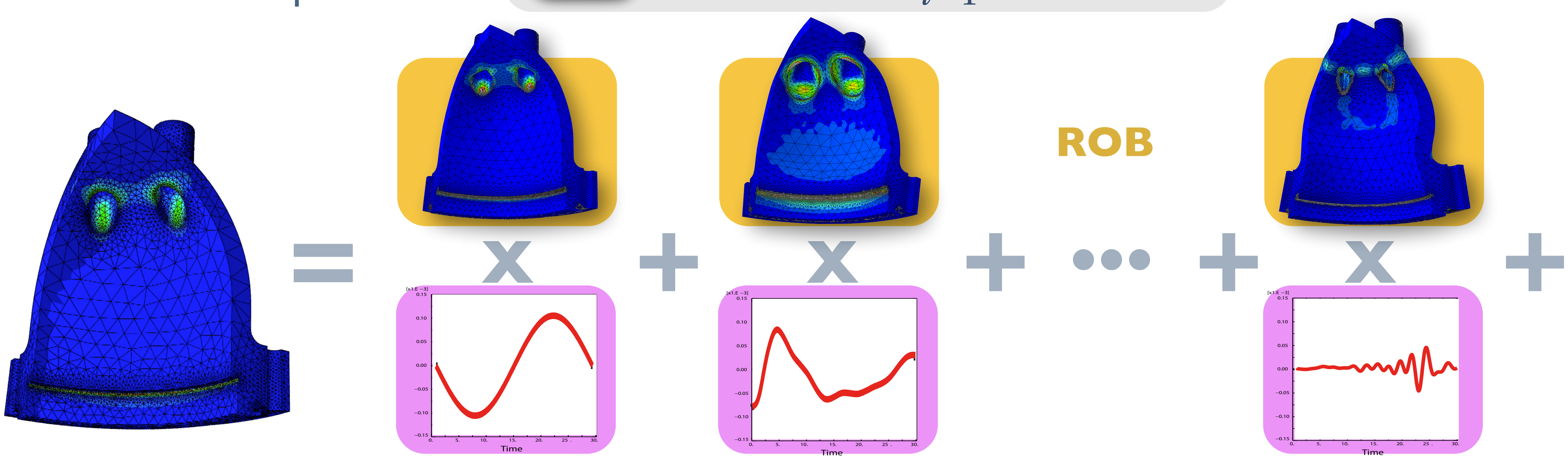
The behavior can be defined

- explicitly **Given:** $u(t,M)$ or implicitly **PDE:** $\mathscr{L}(u(t,M)) = 0$

Separation of variables

- best finite sum decomposition

$$u(t,M) \approx \sum_{i=1}^{m} \lambda_i(t)\Lambda_i(M)$$



Reduced-Order Modeling (ROM)

Tentative definition

- capture **main features** of the behavior, retaining the **accuracy** of the approximation
- use the **redundancy** of information

➡ possibility of approximating a **complex system** using only a **handful of DOFs**

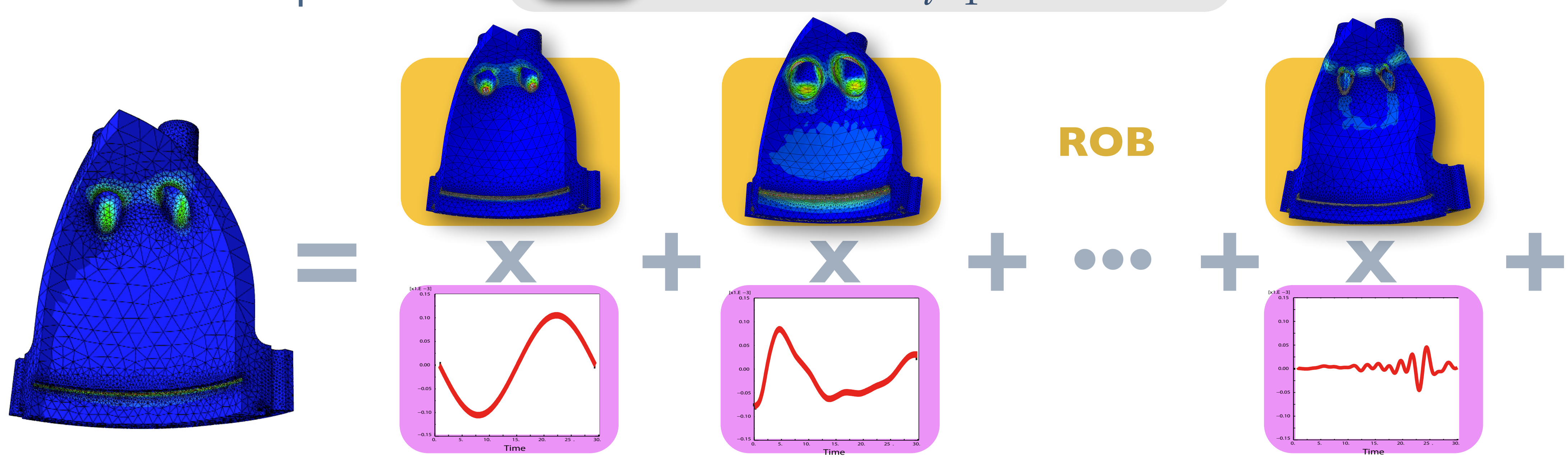
The behavior can be defined

- explicitly **Given:** $u(t,M)$ or implicitly **PDE:** $\mathscr{L}(u(t,M)) = 0$

Separation of variables

- best finite sum decomposition

$$u(t,M) \approx \sum_{i=1}^{m} \lambda_i(t)\Lambda_i(M)$$



Particular cases

- if one imposes to one of the families λ_i or Λ_i to be orthogonal

  POD *(Proper Orthogonal Decomposition)*

  also known (depending on the community) as

  KLD *[Karhunen 43] [Loeve 55],* ***PCA*** *[Pearson 1901] [Hotteling 33]*

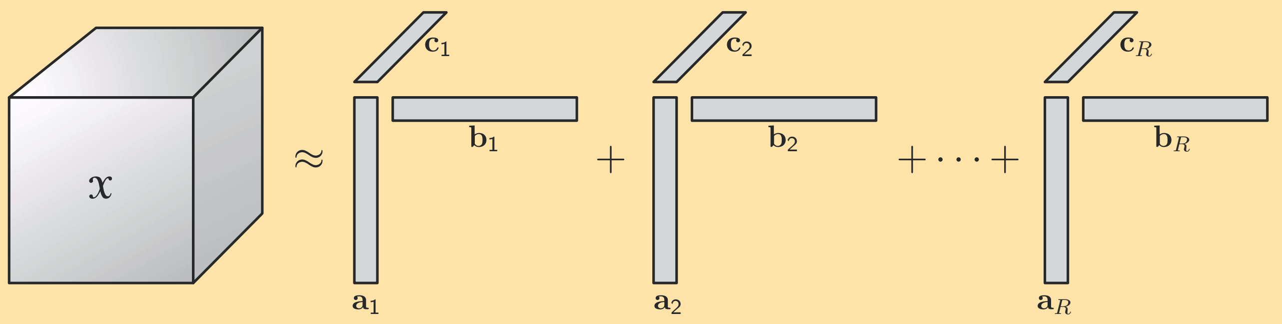
- in finite dimension (our case after discretization)

  SVD *[Ekardt & Young 39]*

```
matlab
[U,S,V] = svd(X)
X = U*S*V'
```

- in finite dimension but with more than two variables

  HOSVD *[Baranyi et al. 06]*



More generally

- linear algebra of tensor decomposition
  - *1st-order = vector, 2nd-order = matrix, >2-order = high-order tensors*

LMPS

How to chose/build the ROB?

- **Building a given ROB before solving the PDE** (a posteriori method)
  - principle: **offline/online computations, learning phase**
  - **Proper Orthogonal Decomposition** (POD)

    > *[Sirovich 87] [Holmes et al. 93] [Krysl et al 00] [Kunisch and Wolkwein 02]*
    > *[Willcox et al. 02] [Picinbono et al. 03] [Bergmann et al. 05] [Lieu et al. 06]*
    > *[Gunzburger et al. 06] [Niroomandi et al. 08] [Farhat et al. 08] [Matthies et al. 10] ...*

  - **Reduced-Basis** (RB) especially for parametrized problems

    > *[Maday et al. 02] [Patera et al. 02] [Rozza et al. 07] [Haasdonk et al. 08] [Boyaval et al. 09] ...*

- **Without assuming any ROB before solving the PDE** (a priori method)
  - principle: **automatic generation of the most relevant ROB**
  - **Proper Generalized Decomposition** (PGD)

    > *[Ladevèze 85, 99] ... [Ladevèze et al. 99-11] [Nouy and Ladevèze 03, 04]*
    > *[Ladevèze et al. 08, 09, 10] [DN and Dureisseix 08] [Boucard, DN 11-13] ...*
    > *[Chinesta 06-] [Nouy et al. 07-] [Ammar and Chinesta 06] [Leygue et al 10-]*
    > *[Ryckelynck 06] [Beringhier et al. 10] ...*

LMPS

Idea

- minimization of a residual, Galerkin formulation, Petrov-Galerkin formulation

PDE: $\mathscr{L}(u(t, M)) = 0$

ex: linear elasticity $\mathbf{u}^{\star T}[\mathbf{K}\mathbf{u}(t) - \mathbf{f}(t)] = 0$

Reformulation in the separated-variable framework

PGD in a nutshell

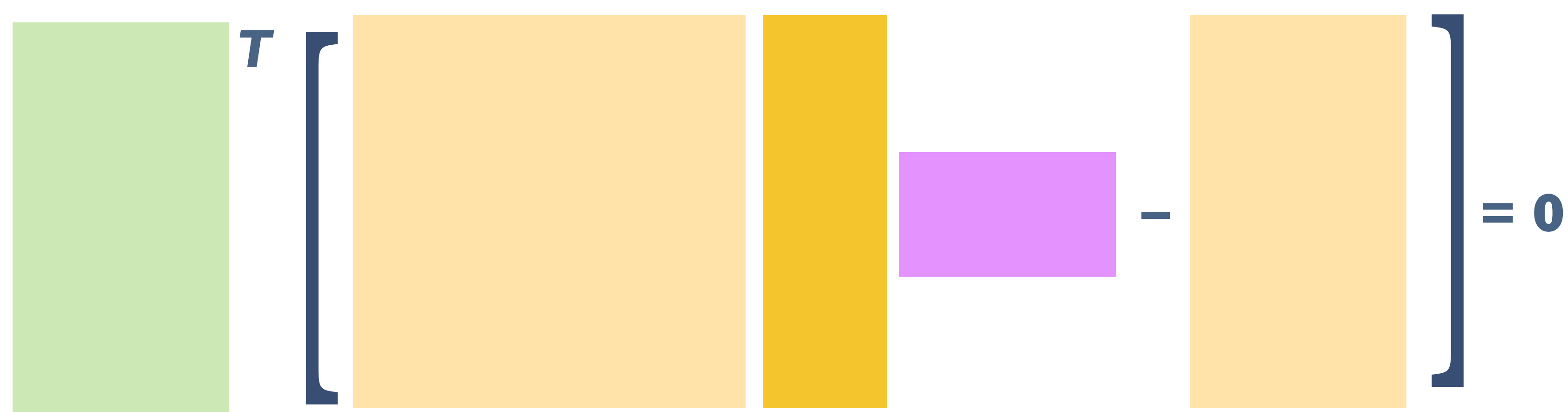
Idea

- minimization of a residual, Galerkin formulation, Petrov-Galerkin formulation

PDE: $\mathscr{L}(u(t,M)) = 0$

ex: linear elasticity $\mathbf{u}^{\star T}[\mathbf{K}\mathbf{u}(t) - \mathbf{f}(t)] = 0$

Reformulation in the separated-variable framework



PGD in a nutshell

Idea

- minimization of a residual, Galerkin formulation, Petrov-Galerkin formulation

PDE: $\mathscr{L}(u(t, M)) = 0$ ex: linear elasticity $\mathbf{u}^{\star T}[\mathbf{K}\mathbf{u}(t) - \mathbf{f}(t)] = 0$

Reformulation in the separated-variable framework

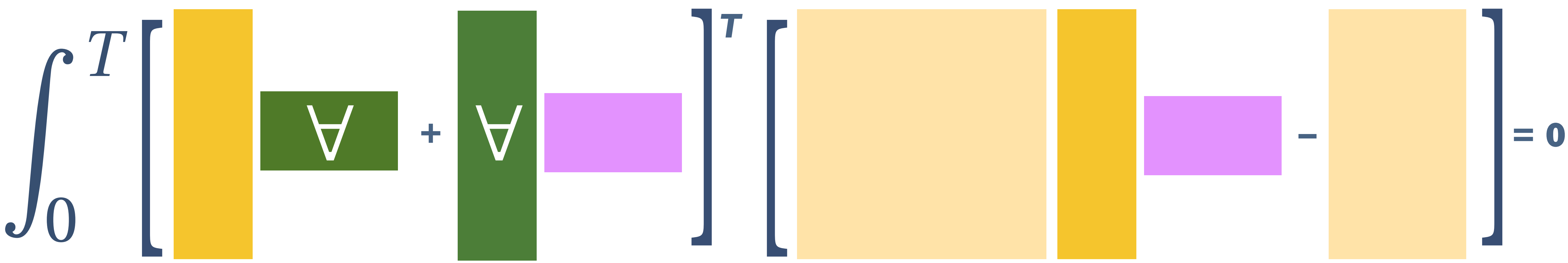
Idea

- minimization of a residual, Galerkin formulation, Petrov-Galerkin formulation

PDE: $\mathscr{L}(u(t, M)) = 0$

ex: linear elasticity $\mathbf{u}^{\star T}[\mathbf{K}\,\mathbf{u}(t) - \mathbf{f}(t)] = 0$

Reformulation in the separated-variable framework



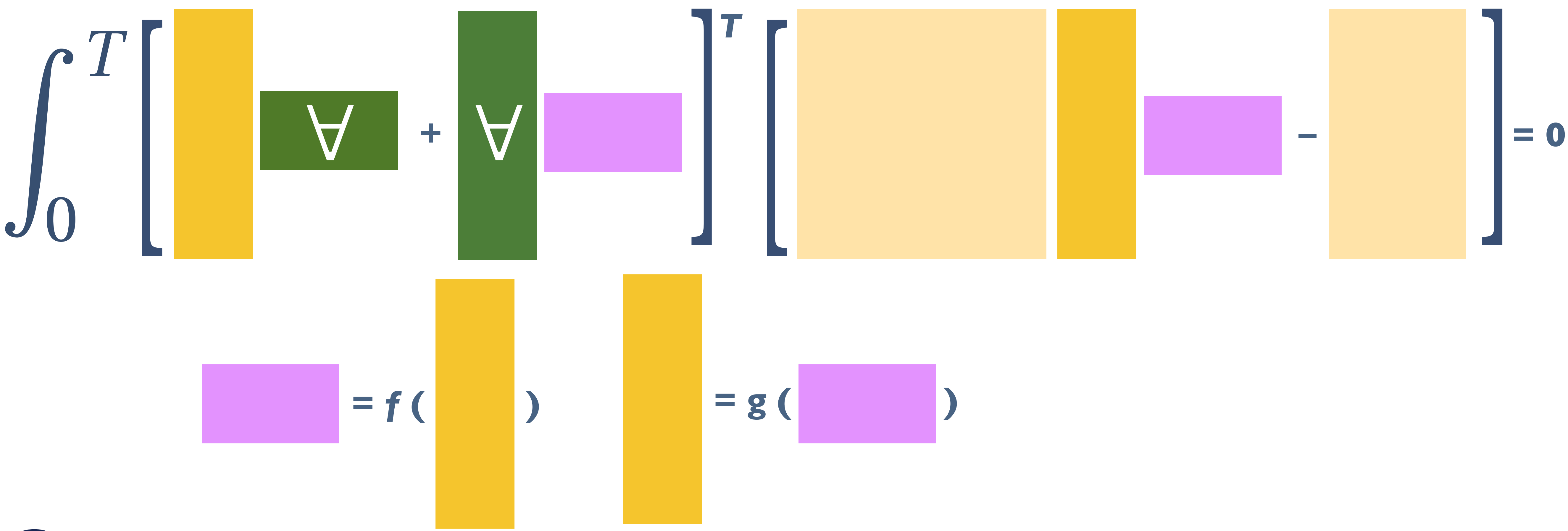
Idea

- minimization of a residual, Galerkin formulation, Petrov-Galerkin formulation

PDE: $\mathscr{L}(u(t, M)) = 0$

ex: linear elasticity $\mathbf{u}^{\star T}[\mathbf{K}\mathbf{u}(t) - \mathbf{f}(t)] = 0$

Reformulation in the separated-variable framework



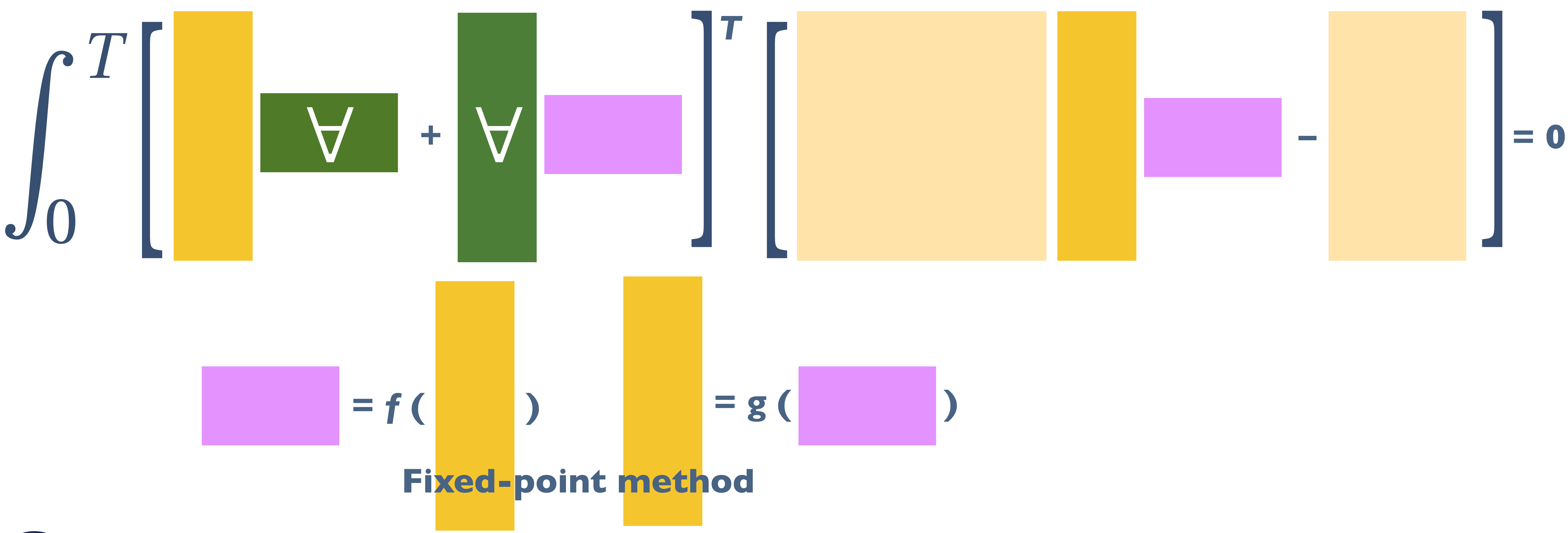
PGD in a nutshell

Idea

- minimization of a residual, Galerkin formulation, Petrov-Galerkin formulation

PDE: $\mathscr{L}(u(t, M)) = 0$ ex: linear elasticity $\mathbf{u}^{\star T}[\mathbf{K}\mathbf{u}(t) - \mathbf{f}(t)] = 0$

Reformulation in the separated-variable framework



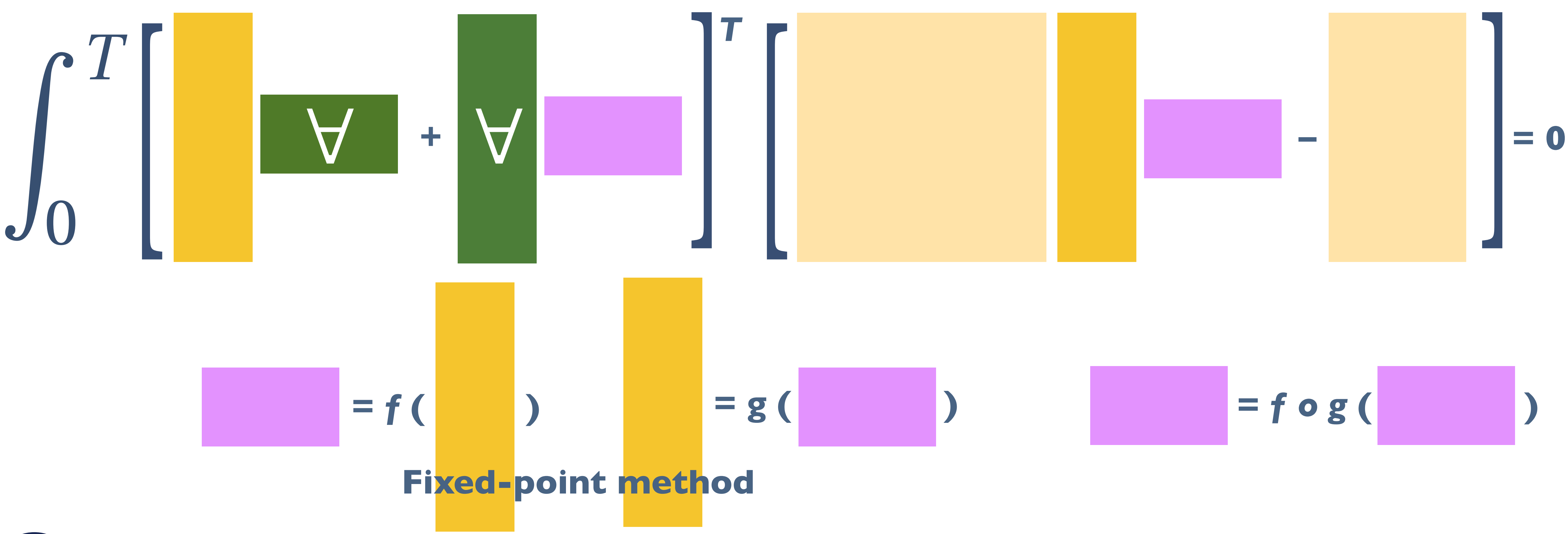
PGD in a nutshell

Idea

- minimization of a residual, Galerkin formulation, Petrov-Galerkin formulation

PDE: $\mathscr{L}(u(t,M)) = 0$ ex: linear elasticity $\mathbf{u}^{\star T}[\mathbf{K}\mathbf{u}(t) - \mathbf{f}(t)] = 0$

Reformulation in the separated-variable framework



PGD in a nutshell

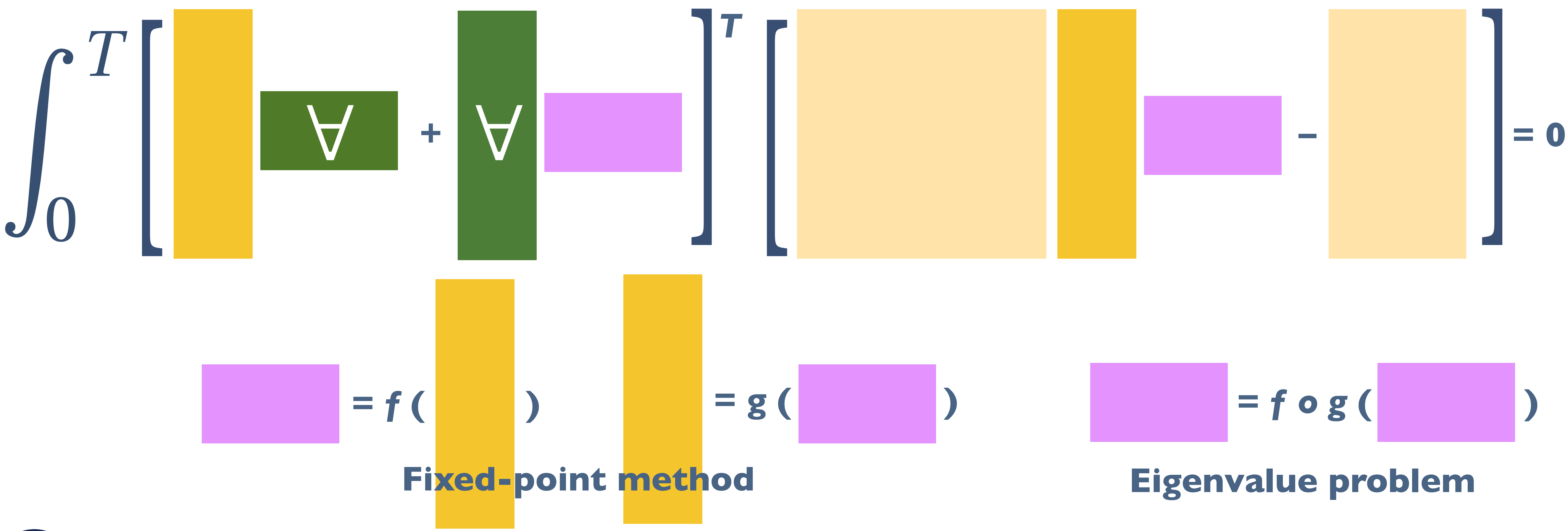
Idea

- minimization of a residual, Galerkin formulation, Petrov-Galerkin formulation

PDE: $\mathscr{L}(u(t, M)) = 0$

ex: linear elasticity $\mathbf{u}^{\star T} [\mathbf{K}\mathbf{u}(t) - \mathbf{f}(t)] = 0$

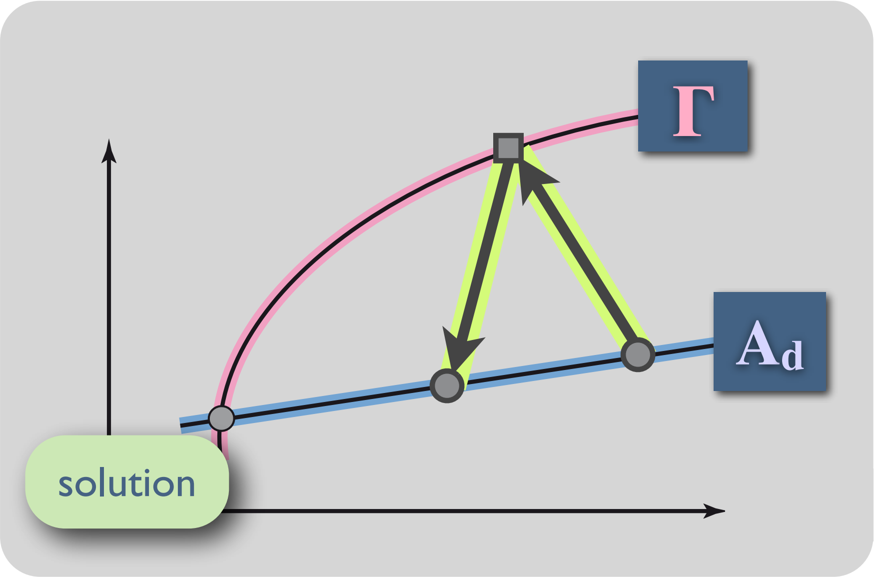
Reformulation in the separated-variable framework



PGD in the LATIN framework

LATIN

- non incremental computational strategy
- books [Ladevèze 85] [Ladevèze 99]
- originally designed for nonlinear problems
  > *separation between nonlinear and linear equations*
- since used for coupled problems
  > *separation between coupled and uncoupled equations*
- or for multiscale problems
  > *separation between equations defined at the scale of subdomains and equation which link subdomains…*



Model reduction method PGD

- formerly « radial loading approximation »
- renamed in 2010 by P. Ladevèze and F. Chinesta
- Proper Generalized Decomposition (PGD) to show the link with POD

$$u(t,M) \approx \sum_{i=1}^{m} \lambda_i(t)\Lambda_i(M)$$

Mechanical problem

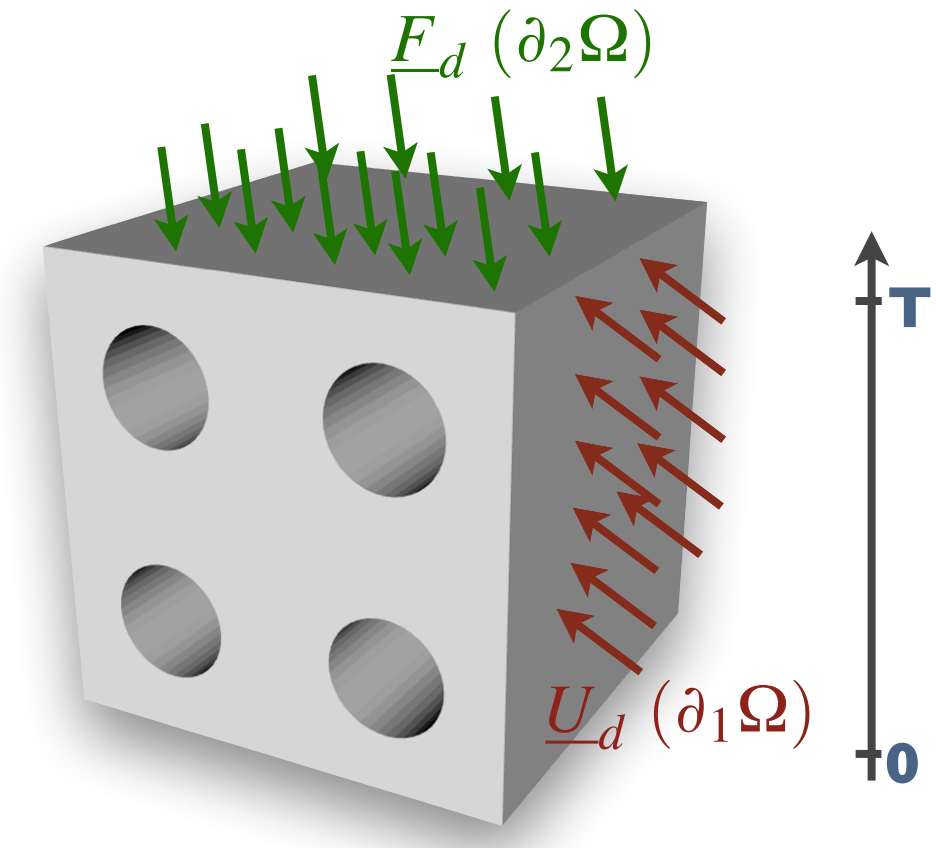
Framework

- small perturbation, quasi-static evolution, isothermal

State of the structure

- defined by $\mathbf{s} = (\varepsilon_p, X, \sigma, Y)$

  - ε_p — *inelastic part of strain field*
  - X — *remaining internal variables*
  - σ — *stress field*
  - Y — *variables conjugate of X*



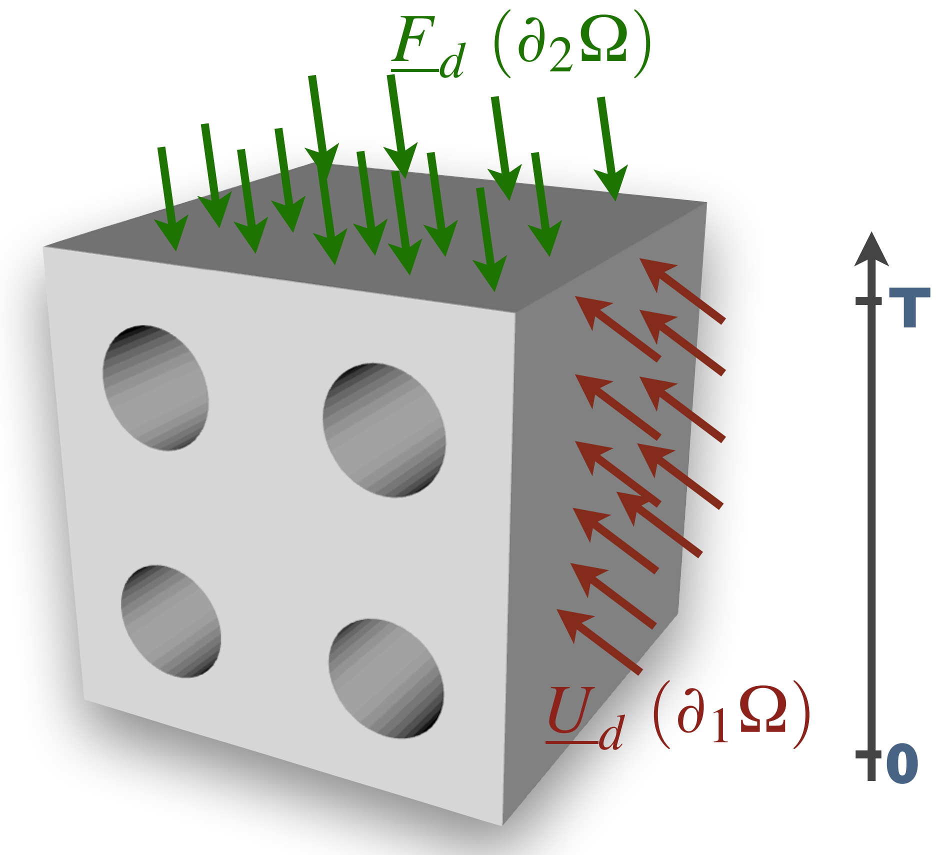
Mechanical problem

Framework

- small perturbation, quasi-static evolution, isothermal

State of the structure

- defined by **s** = $(\varepsilon_p, X, \sigma, Y)$



Governing equations

- **kinematic admissibility**

  compatibility of strain

  prescribed displacement

- **static admissibility**

  equilibrium equation

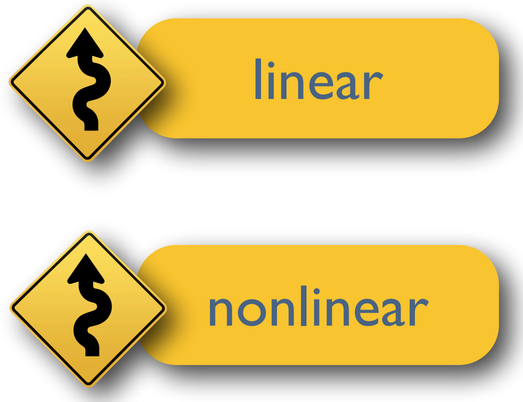
- **nonlinear material behavior** (Marquis-Chaboche elastic-viscoplastic material)

  state equations

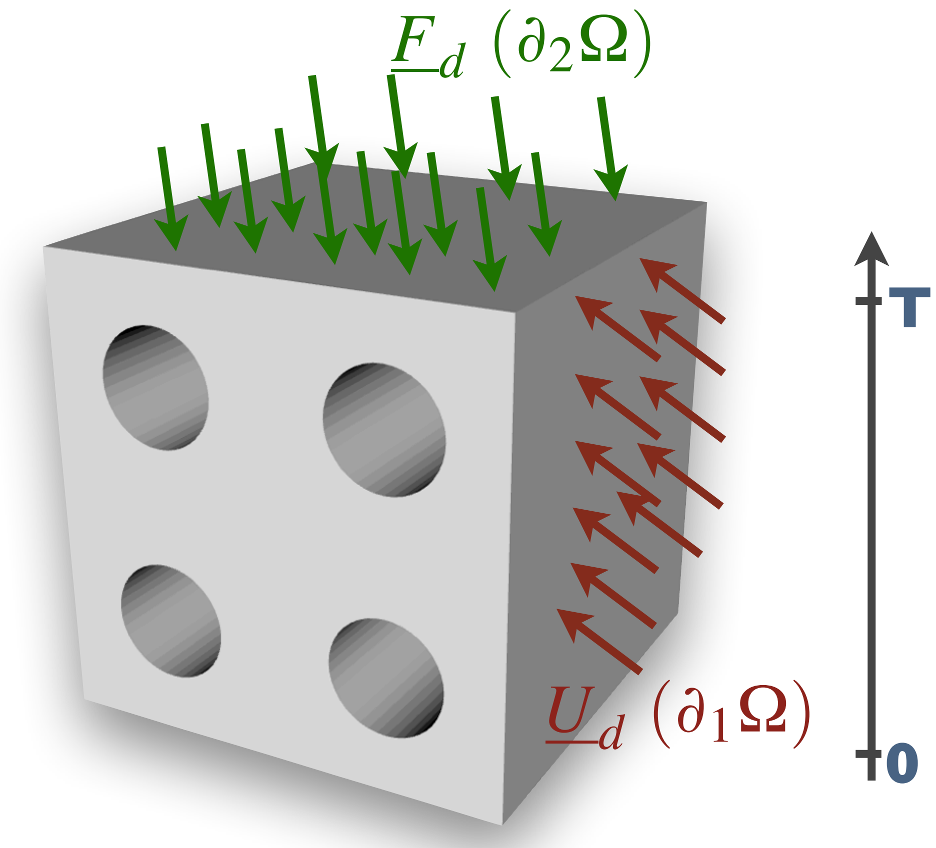
$$\sigma = K\varepsilon_e \qquad Y = \frac{2}{3}CX$$

  evolution laws

$$\dot{\varepsilon}_p = \frac{3}{2}\left\langle \frac{\phi}{K} \right\rangle_+^n \frac{\sigma^D - Y}{(\sigma - Y)_{eq}} \qquad \dot{X} = \dot{\varepsilon}_p - \frac{3}{2}\gamma C^{-1} Y$$



Mechanical problem



Framework

- small perturbation, quasi-static evolution, isothermal

State of the structure

- defined by **s** = $(\varepsilon_p, X, \sigma, Y)$

Governing equations

- **kinematic admissibility**
  - *compatibility of strain*
  - *prescribed displacement*
- **static admissibility**
  - *equilibrium equation*
- **nonlinear material behavior** (Marquis-Chaboche elastic-viscoplastic material)
  - *state equations*

$$\sigma = K\varepsilon_e \qquad Y = \frac{2}{3}CX$$

  - *evolution laws*

$$\dot{\varepsilon}_p = \frac{3}{2}\left\langle \frac{\phi}{K} \right\rangle_+^n \frac{\sigma^D - Y}{(\sigma - Y)_{eq}} \qquad \dot{X} = \dot{\varepsilon}_p - \frac{3}{2}\gamma C^{-1} Y$$

A_d

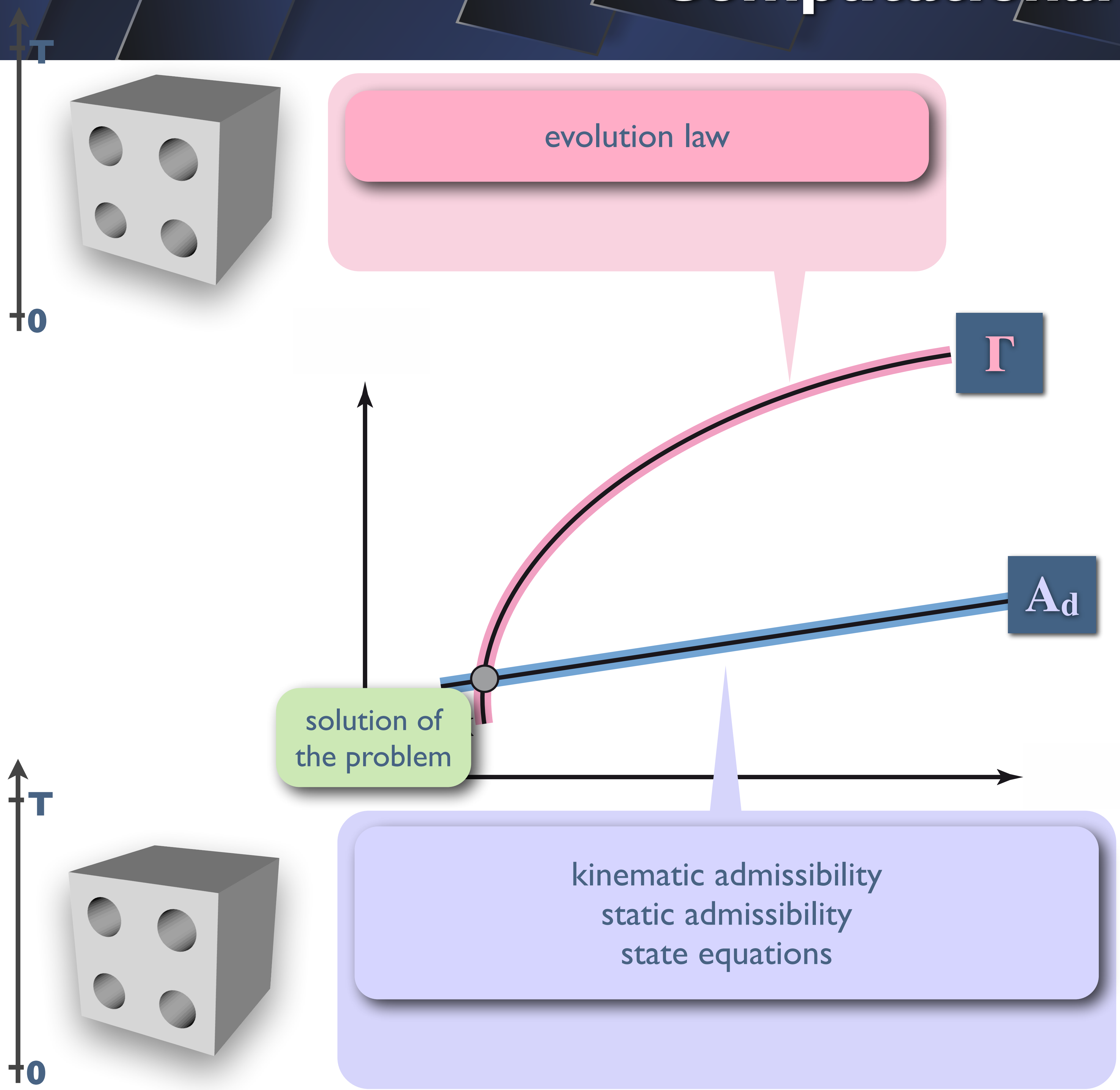
Γ

linear

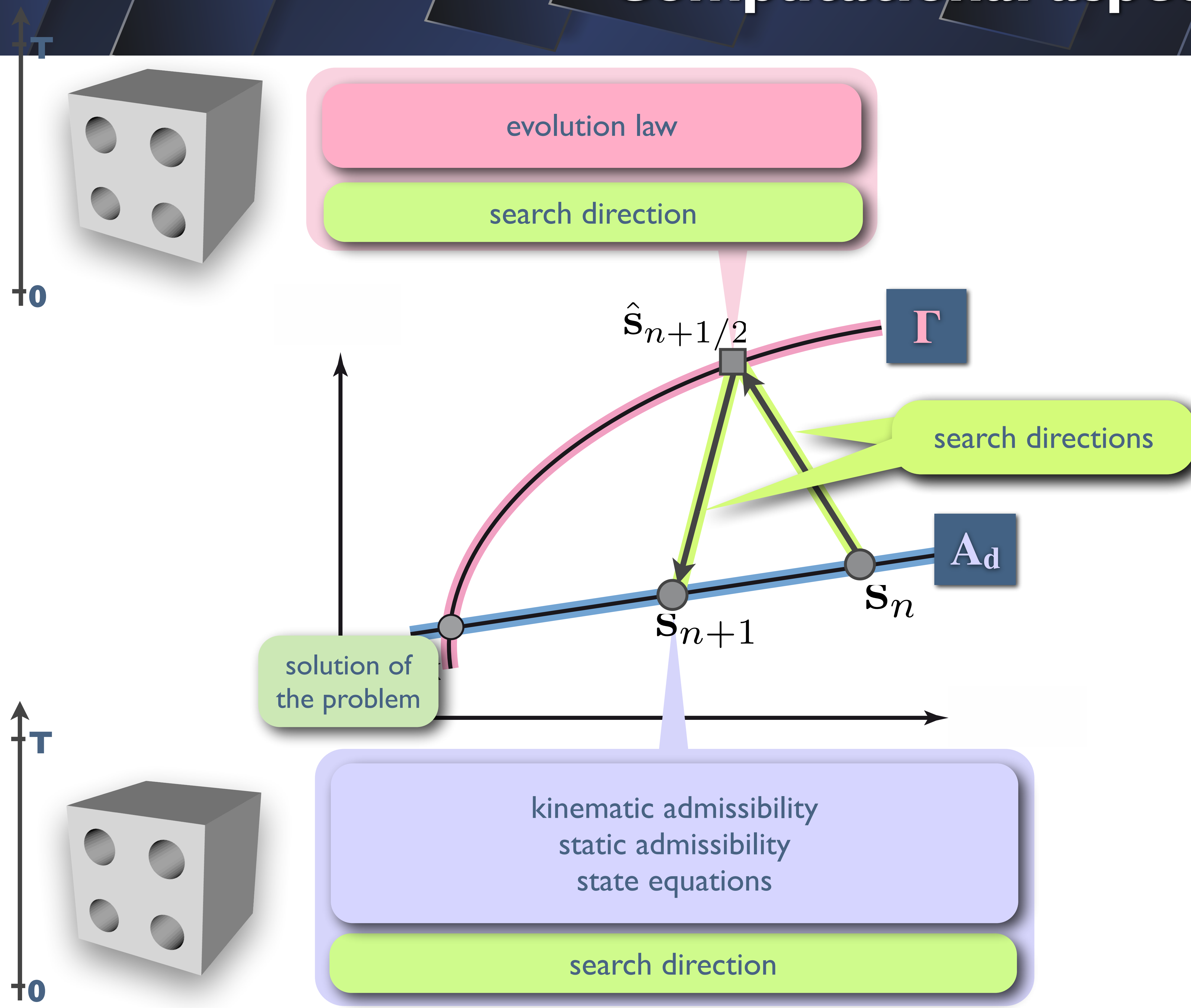
nonlinear

LMPS

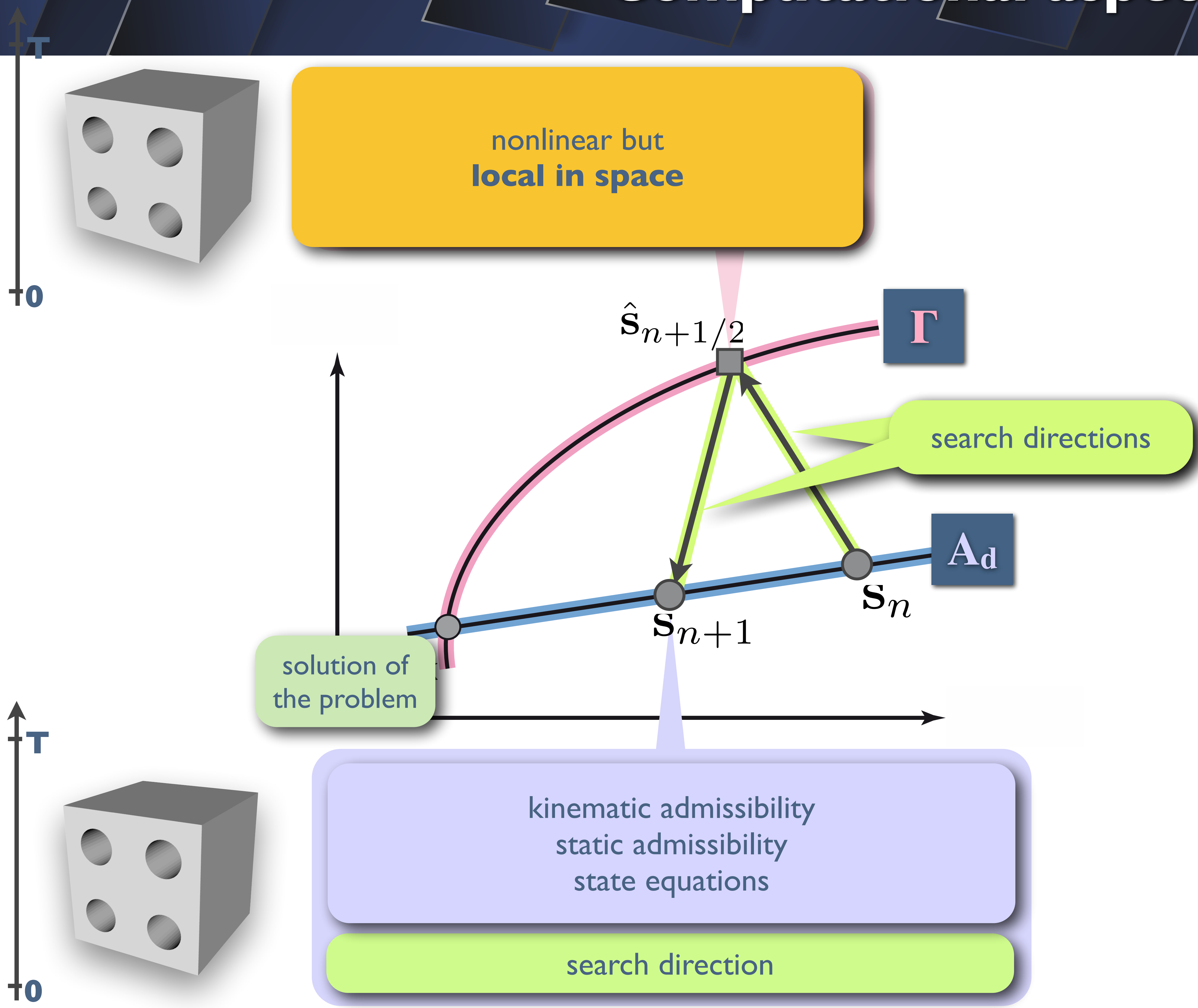
Computational aspects



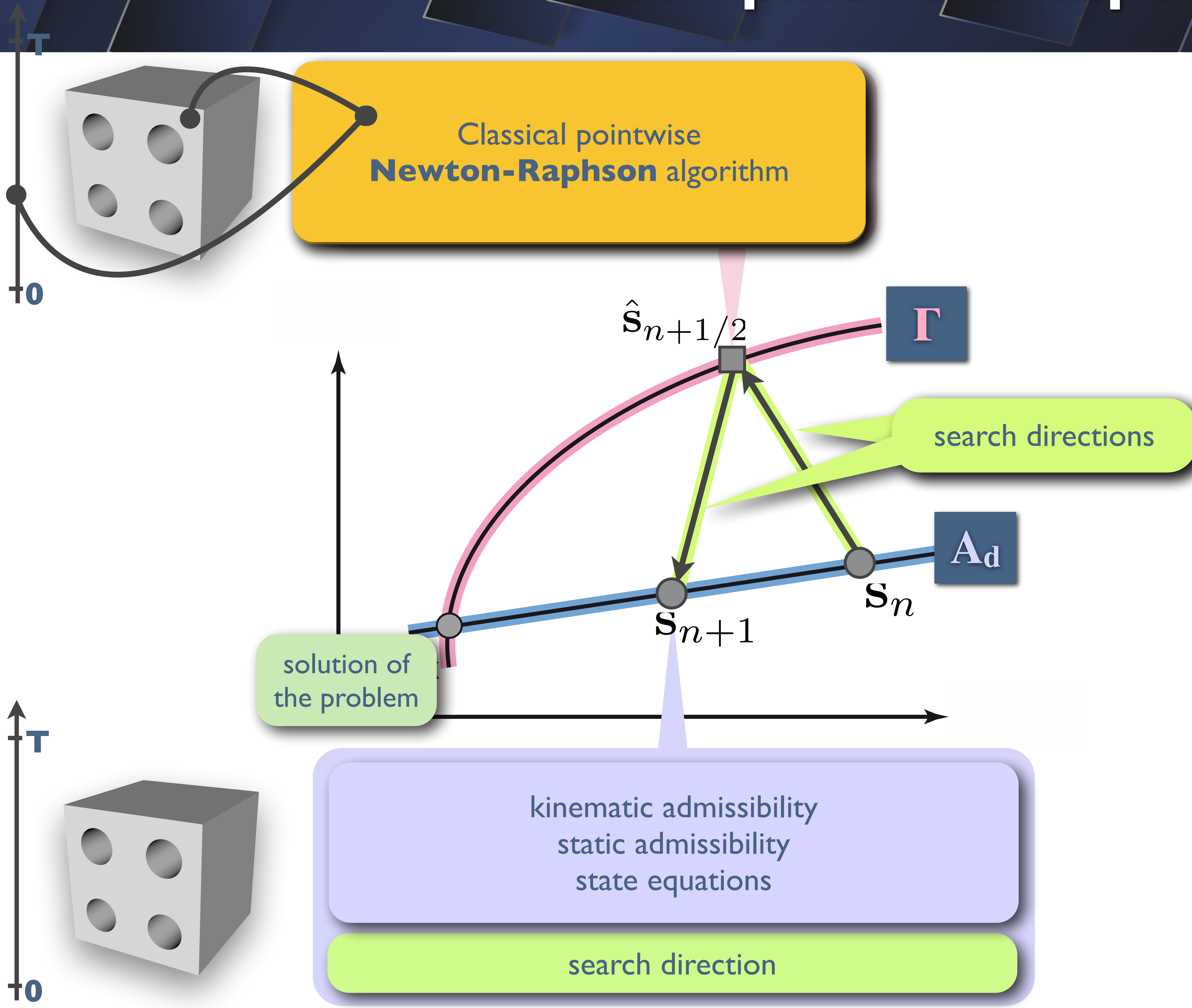
Computational aspects



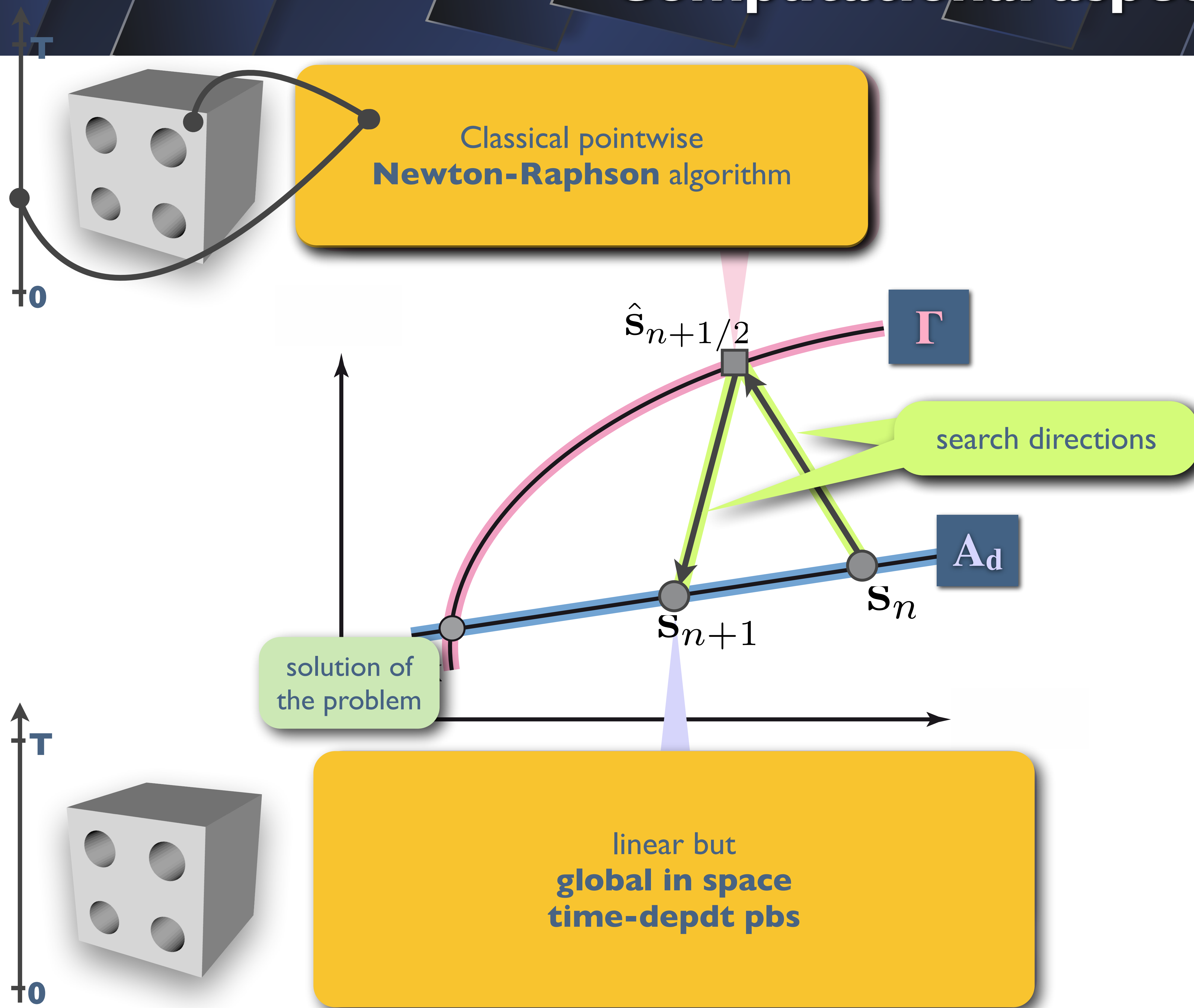
Computational aspects



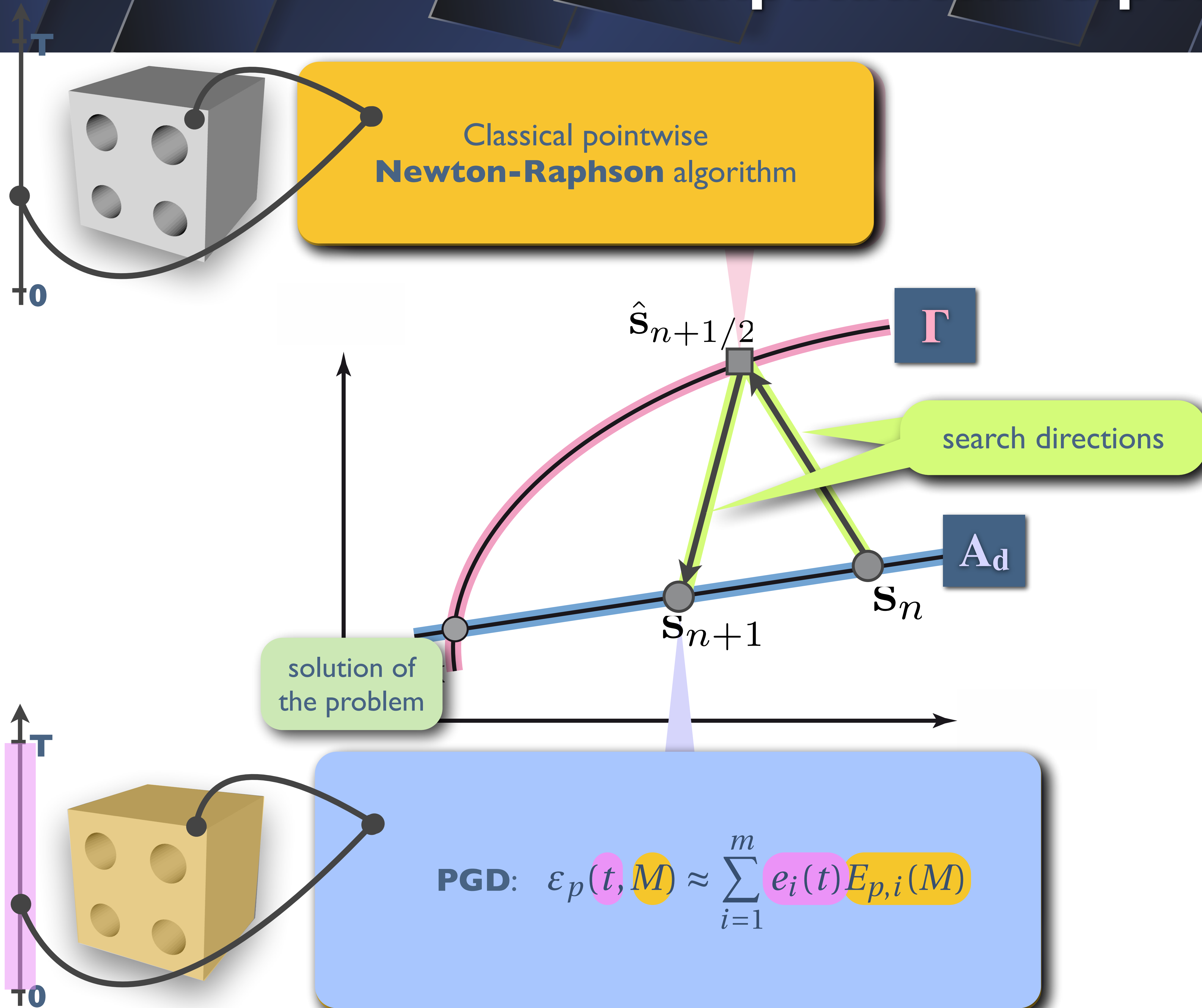
Computational aspects



Computational aspects

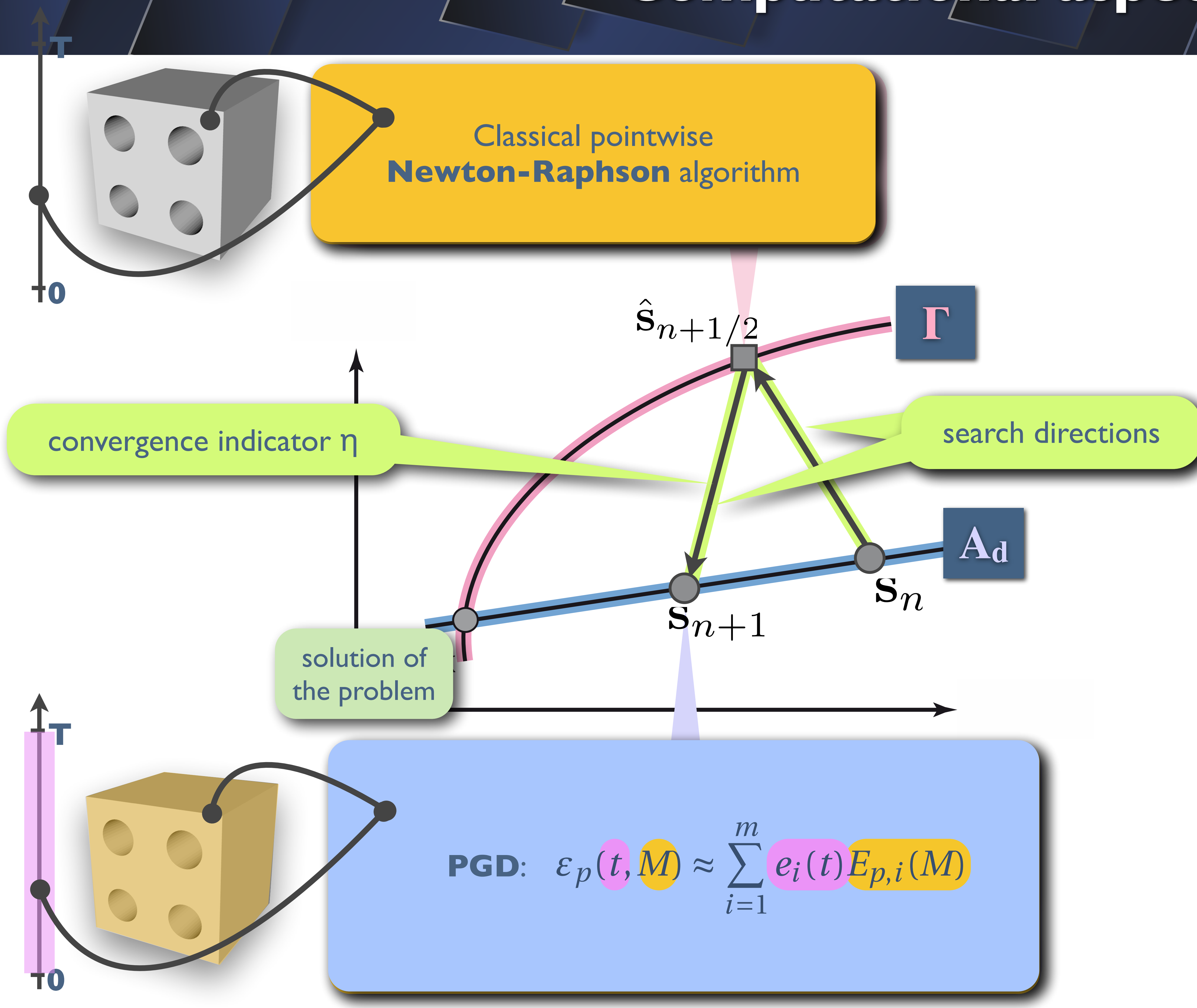


Computational aspects



PGD: $\varepsilon_p(t, M) \approx \sum_{i=1}^{m} e_i(t) E_{p,i}(M)$

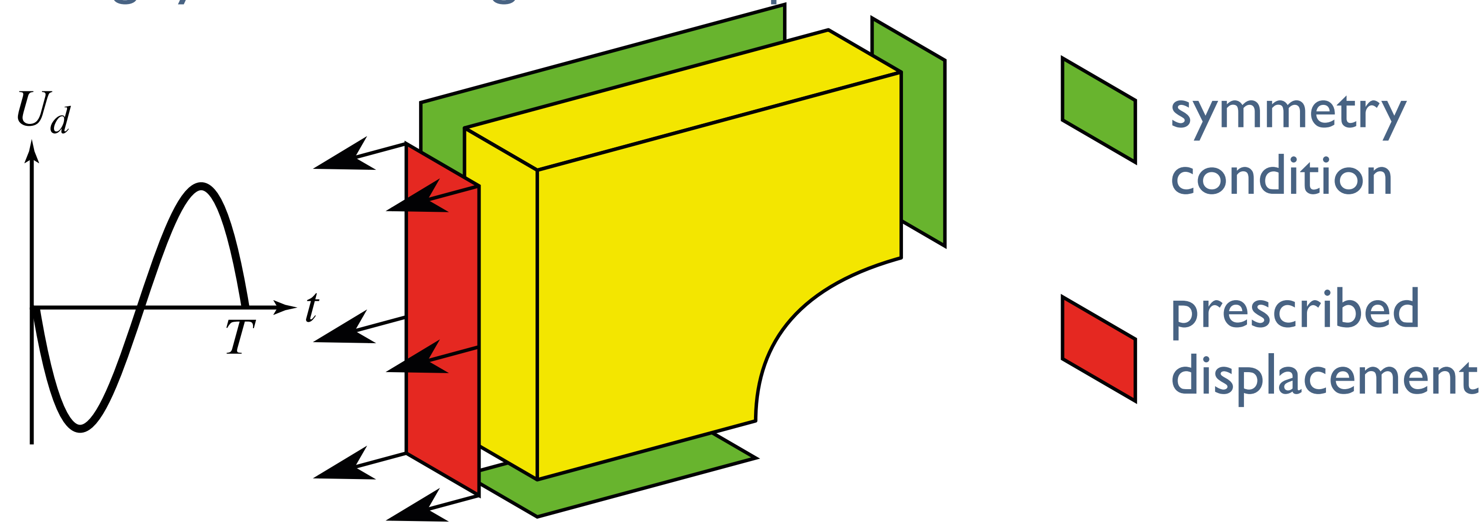
Computational aspects



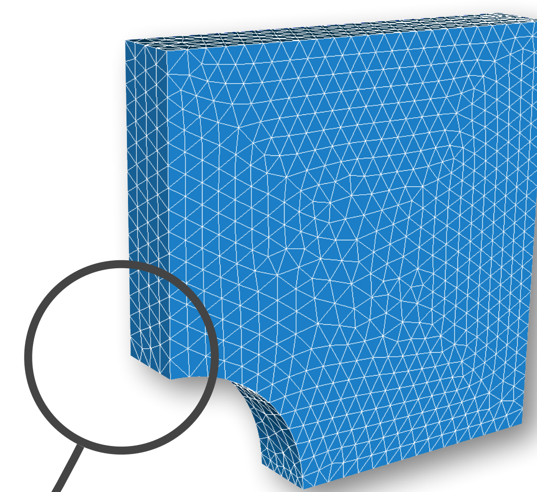
Mini example

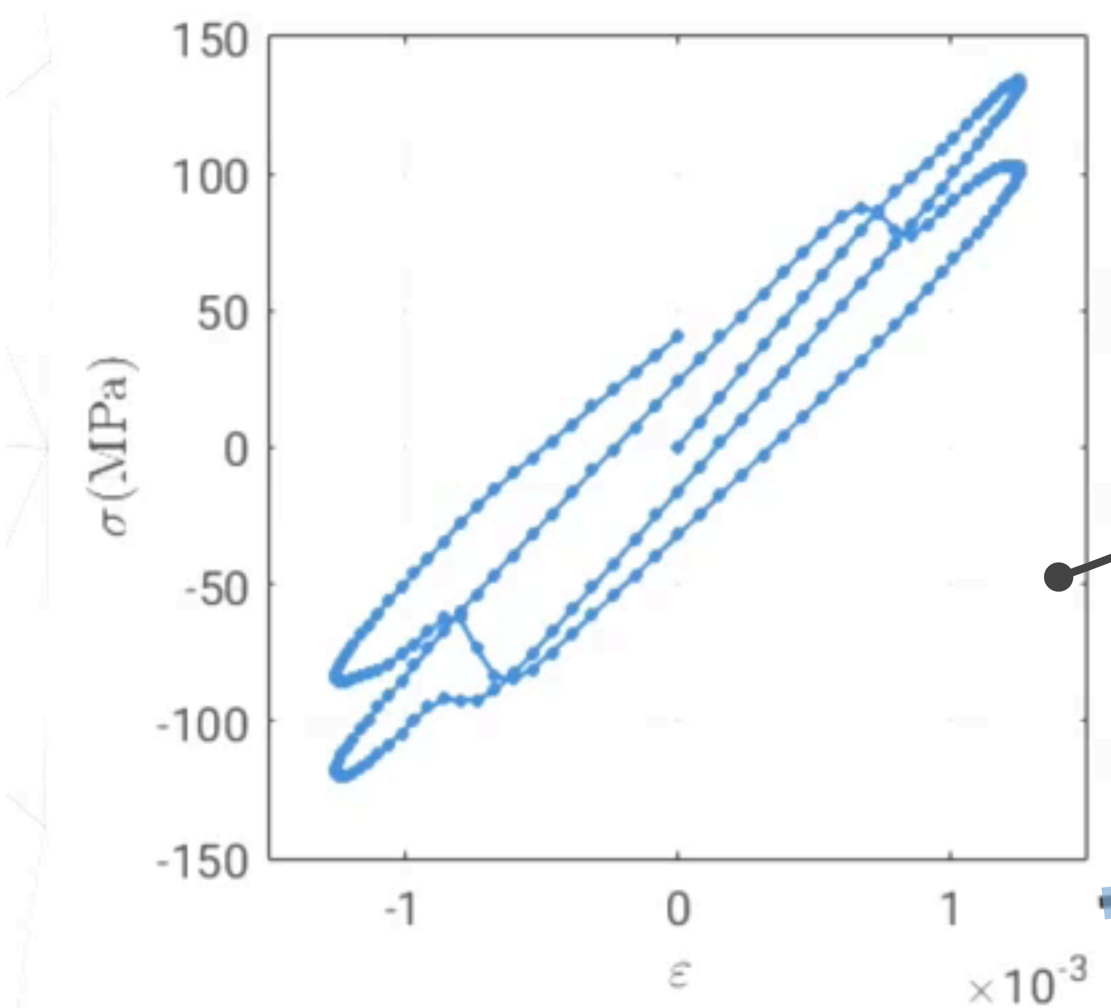
Open hole plate

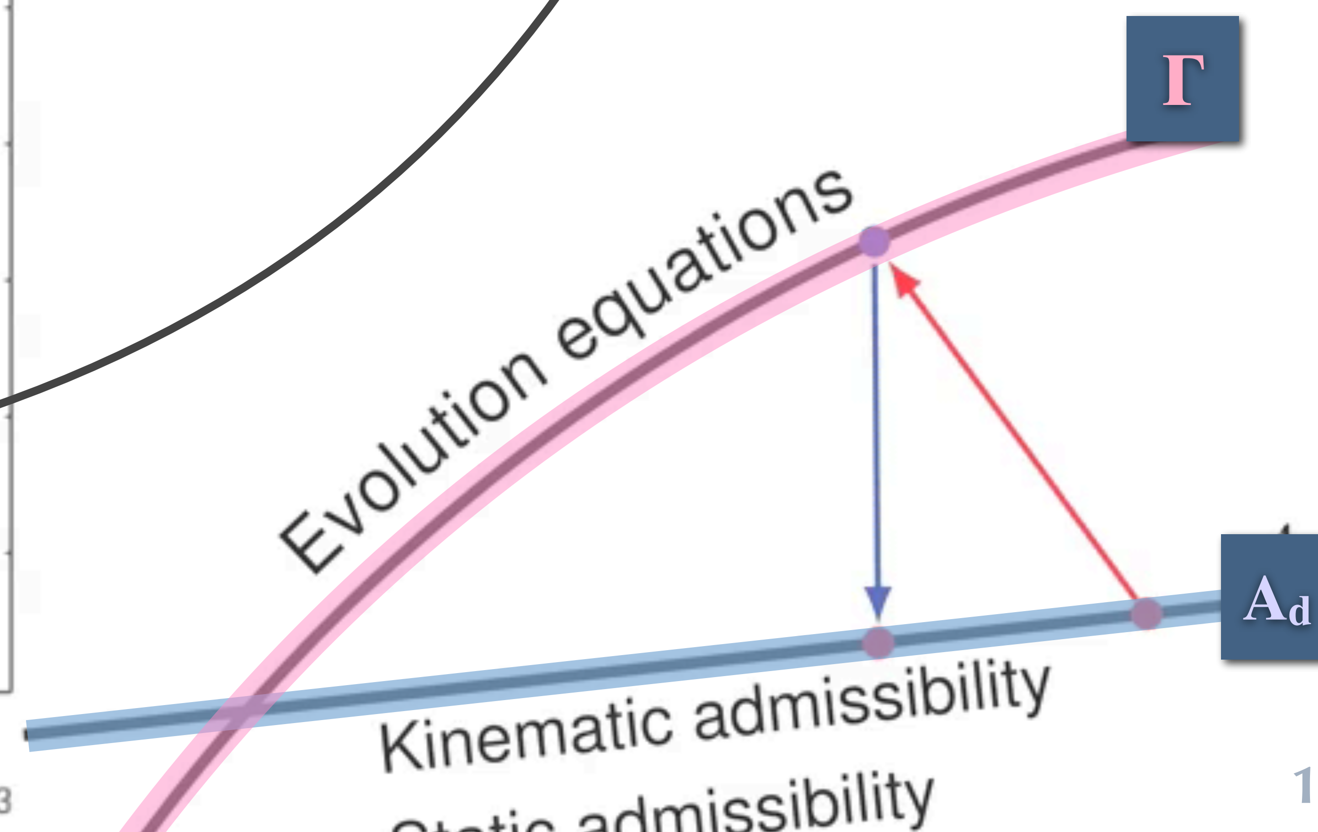
- using symmetries: eighth of the plate



36,954 DOFs
120 time steps

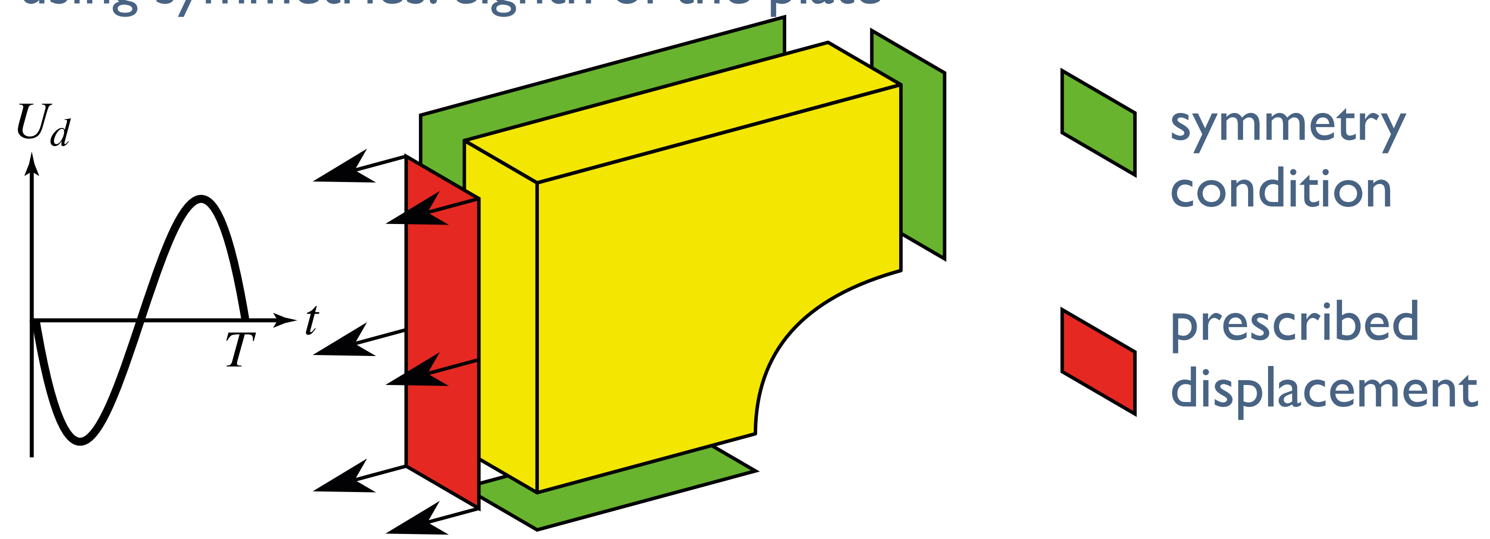




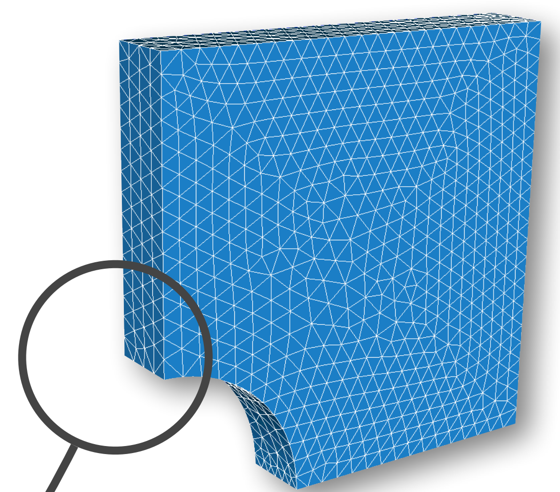


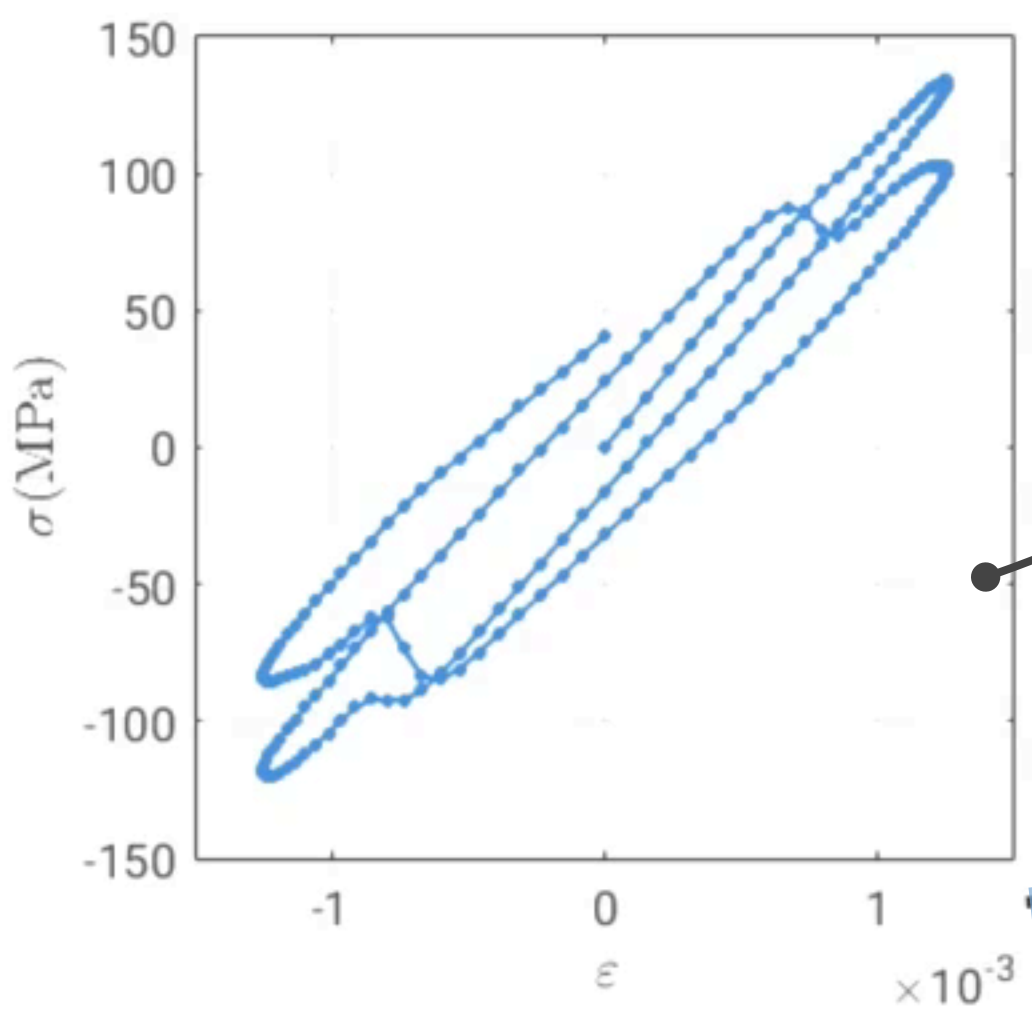
Mini example

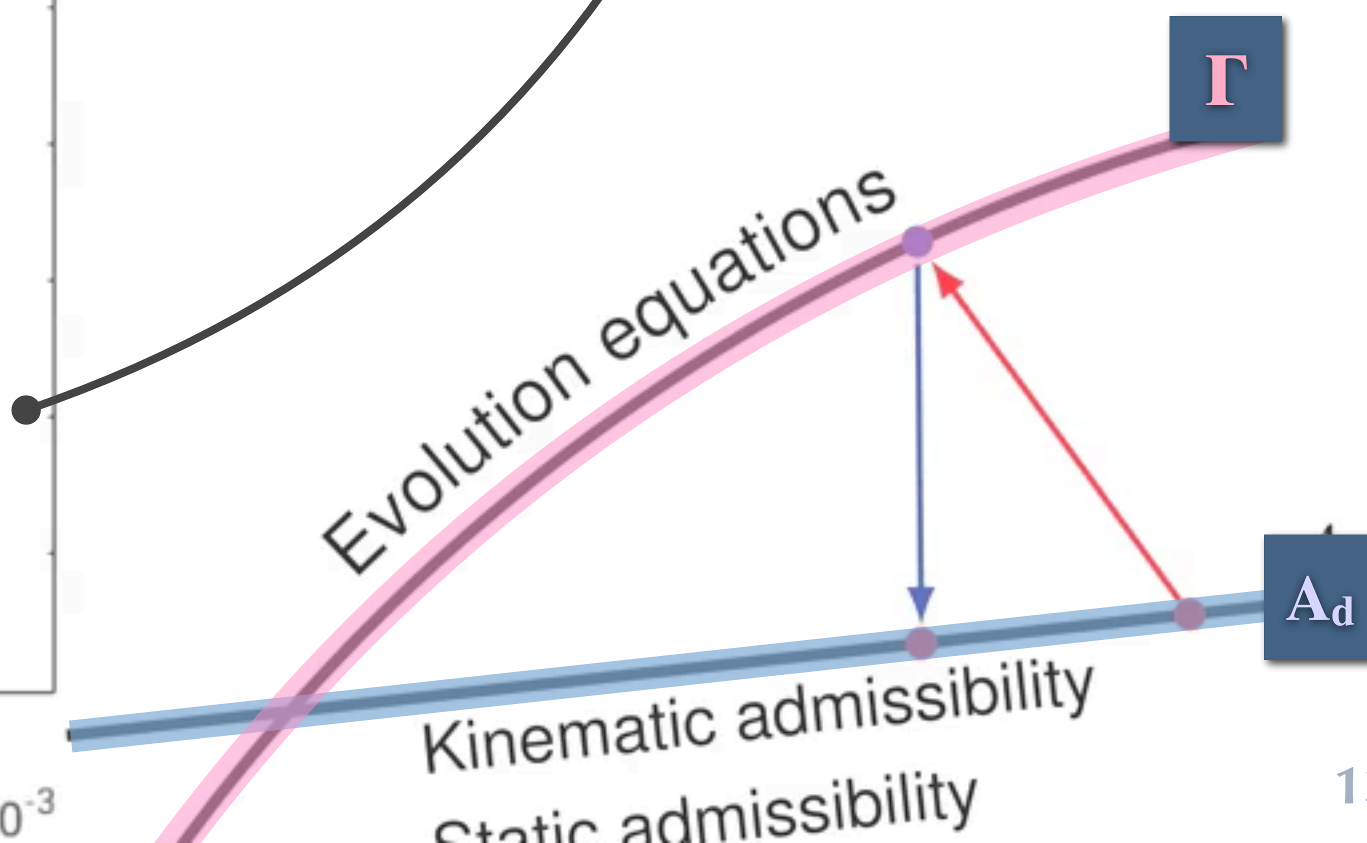
- **Open hole plate**
  - using symmetries: eighth of the plate



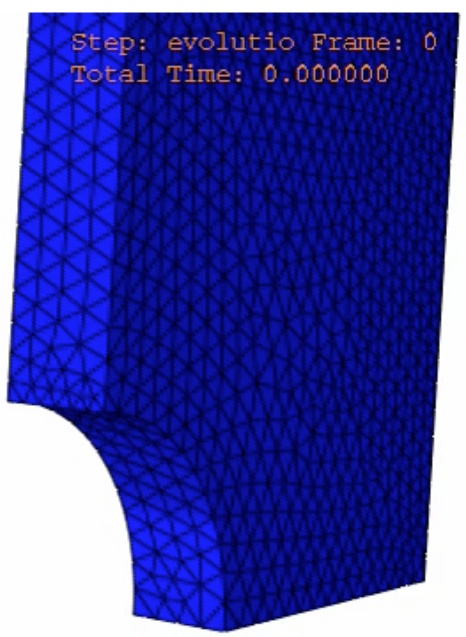
36,954 DOFs
120 time steps



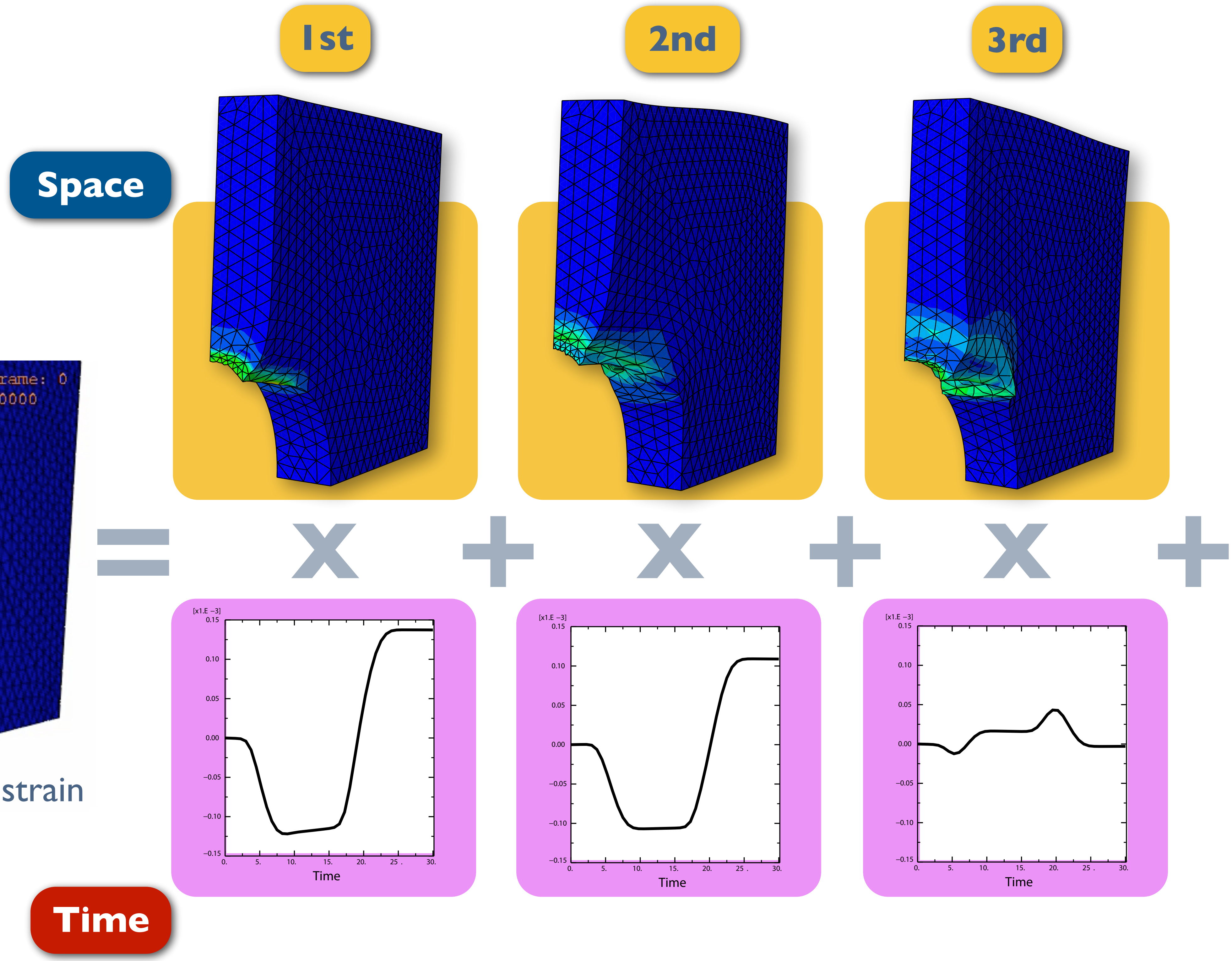




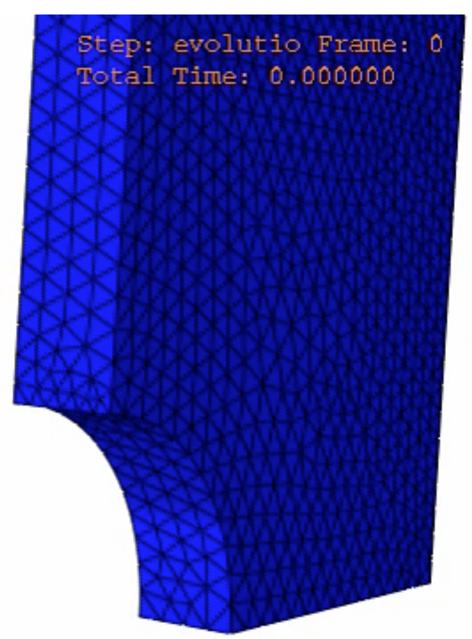
PGD modes



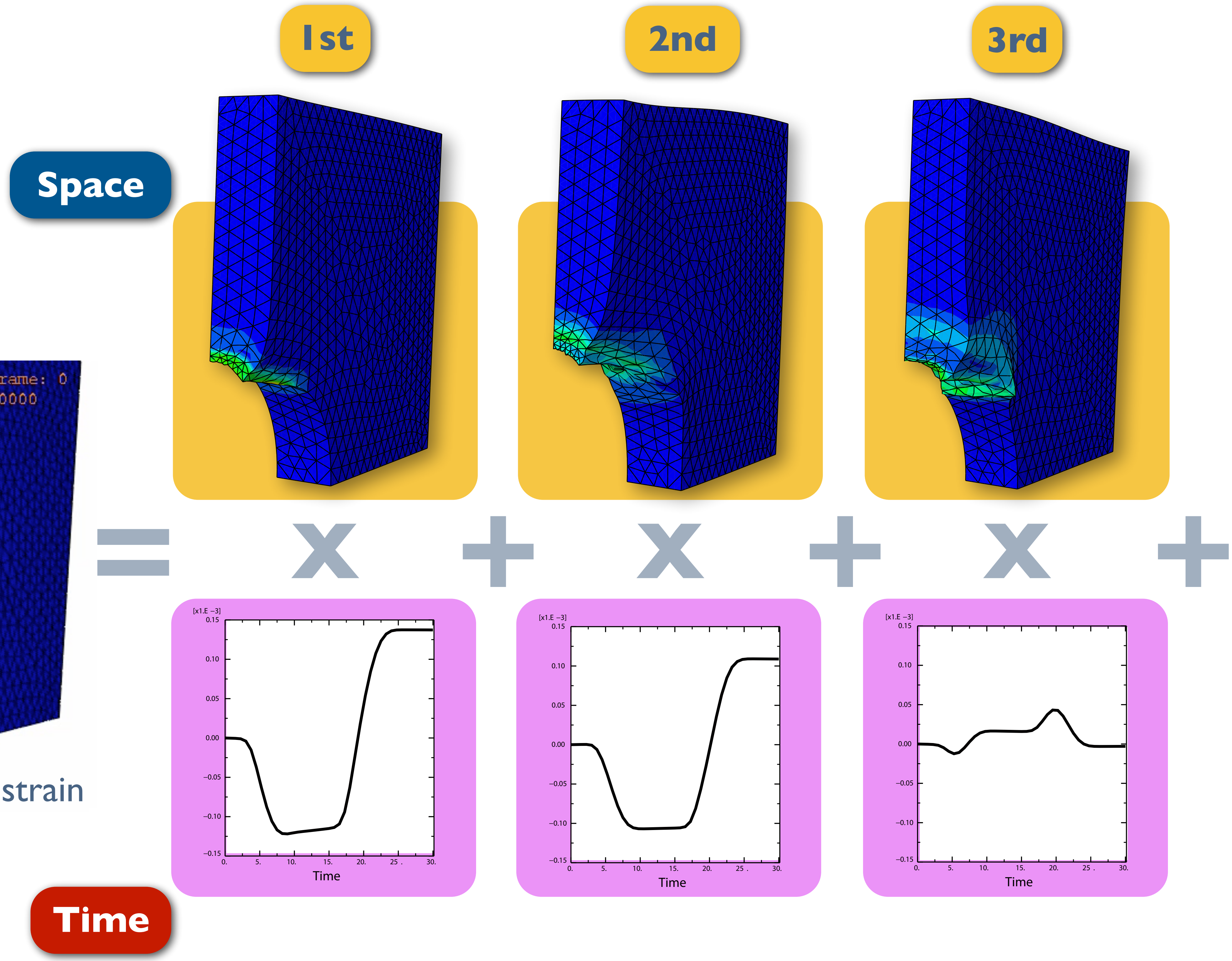
Equiv. plastic strain



PGD modes



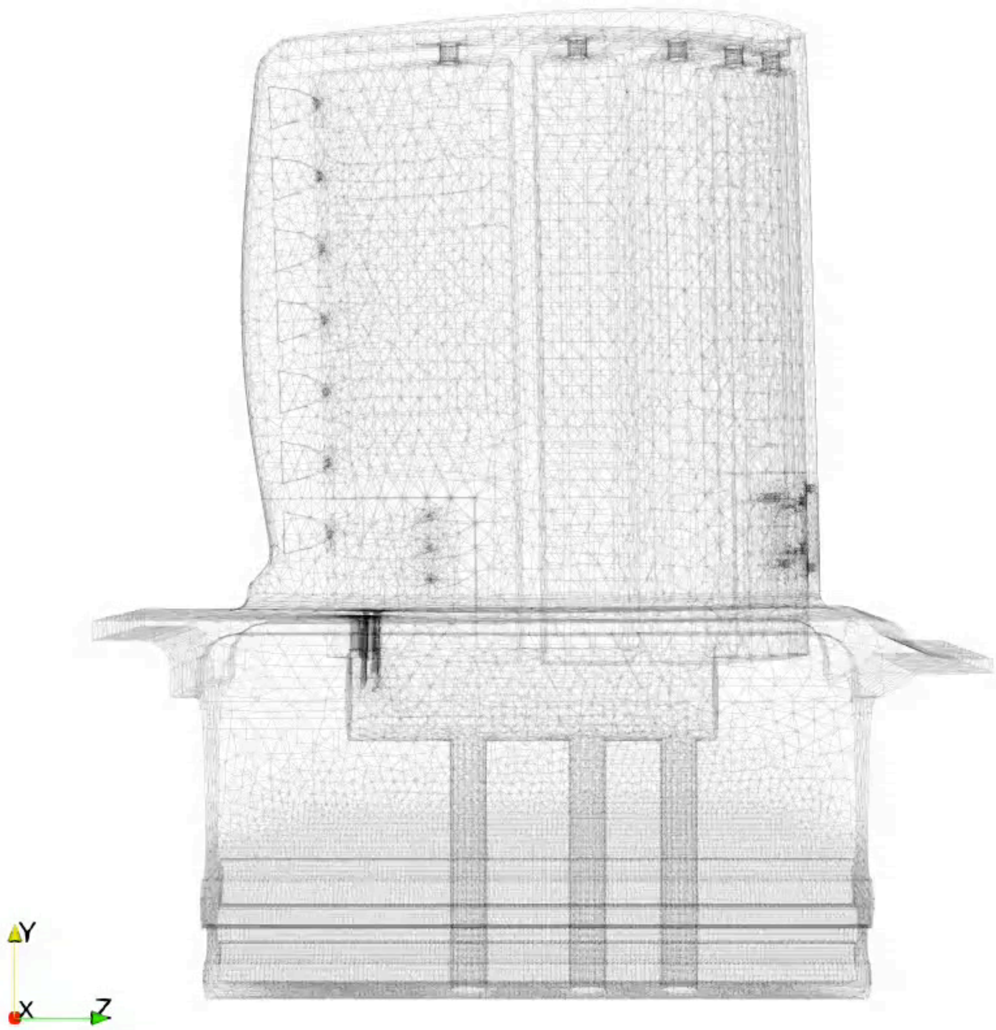
Equiv. plastic strain

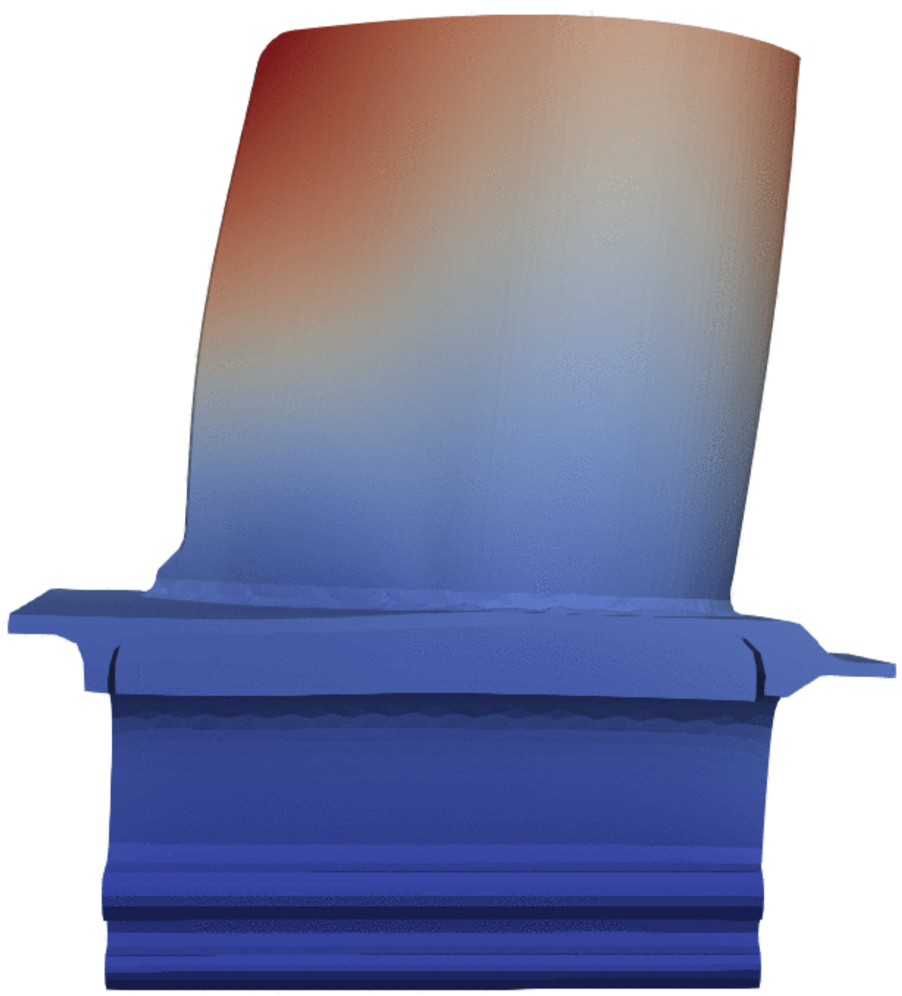


Blade of an aircraft engine

- Chaboche elasto-visco-plastic law with temperature dependence
- 5 MDOFs, 31 time steps, centrifugal inertial forces (rotational speed of 15,000 tr/min)
- [Nachar, Scanff, Ladevèze, Boucard, DN, 2022]

SIEMENS

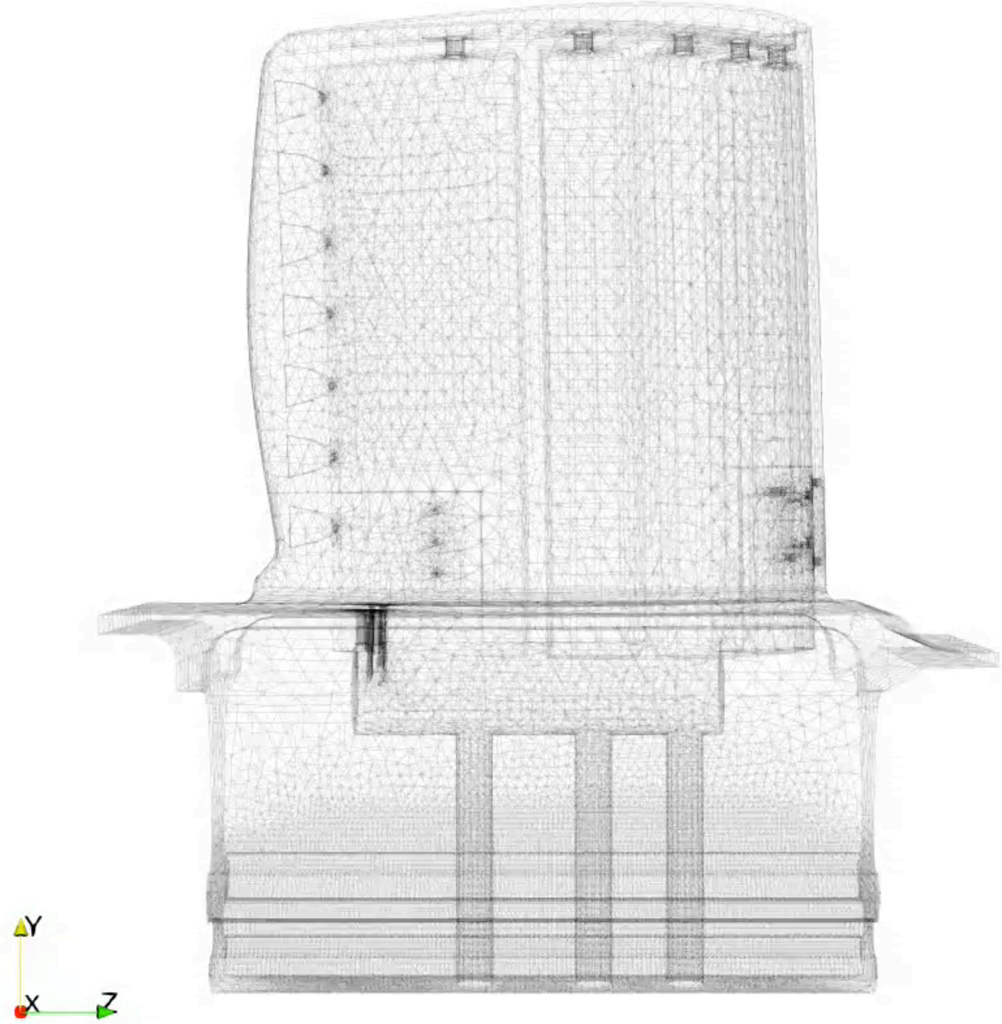


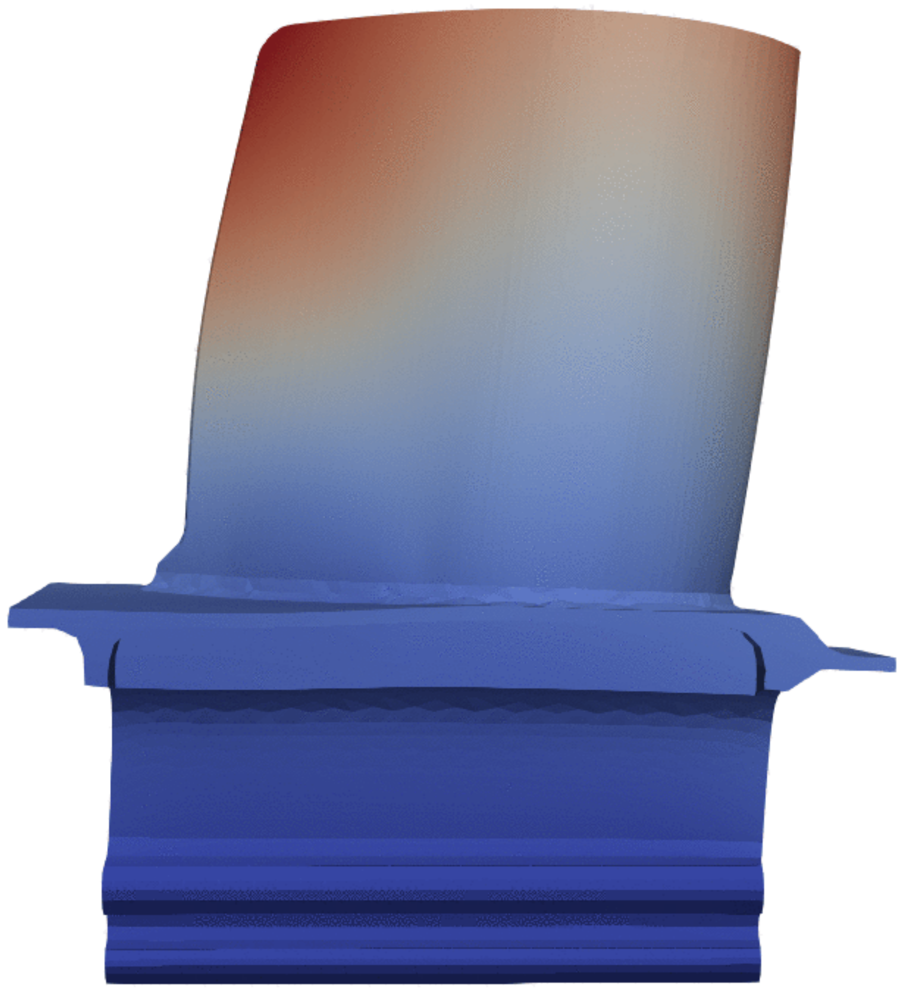


Blade of an aircraft engine

- Chaboche elasto-visco-plastic law with temperature dependence
- 5 MDOFs, 31 time steps, centrifugal inertial forces (rotational speed of 15,000 tr/min)
- [Nachar, Scanff, Ladevèze, Boucard, DN, 2022]

SIEMENS



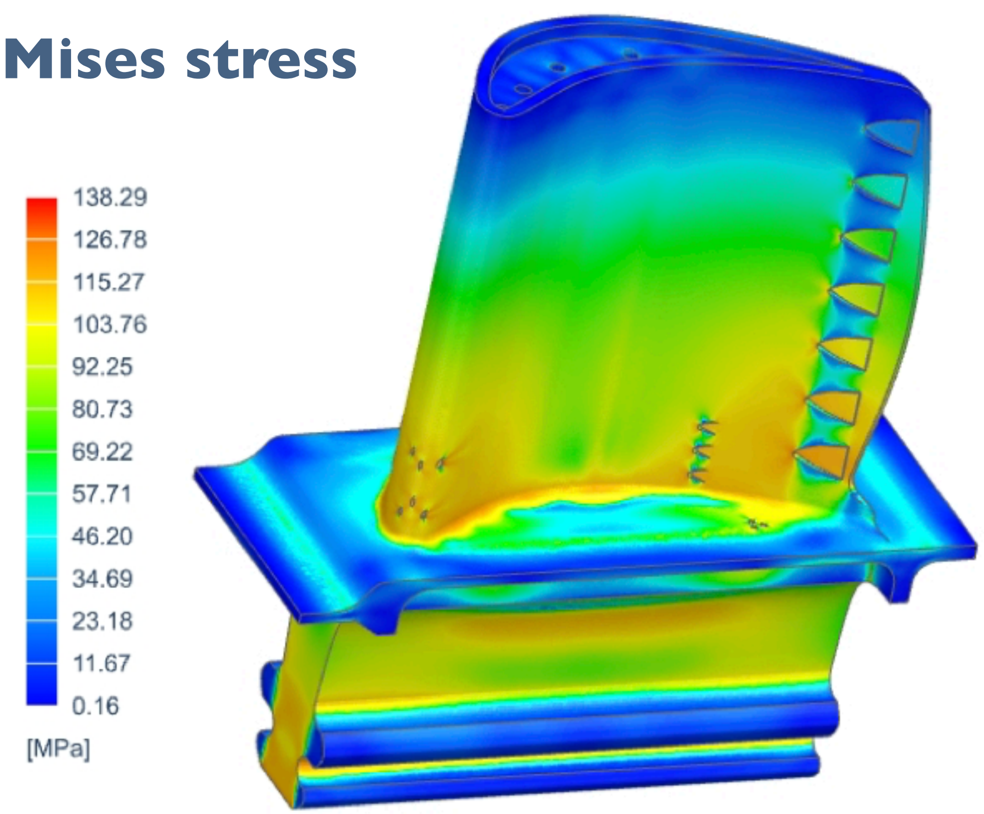


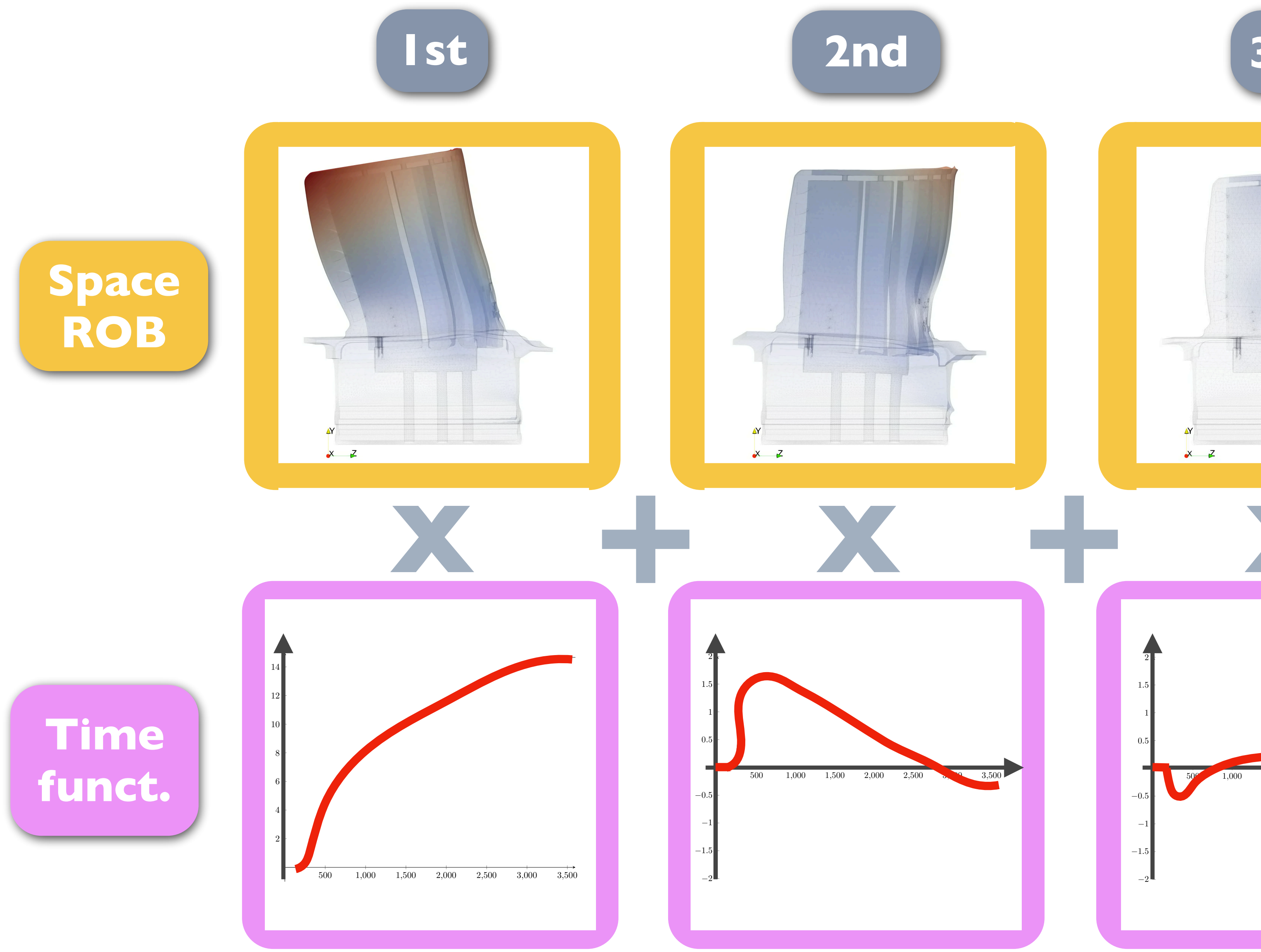
LMPS

Blade of an aircraft engine

- Chaboche elasto-visco-plastic law with temperature dependence
- 5 MDOFs, 31 time steps, centrifugal inertial forces (rotational speed of 15,000 tr/min)
- [Nachar, Scanff, Ladevèze, Boucard, DN, 2022]

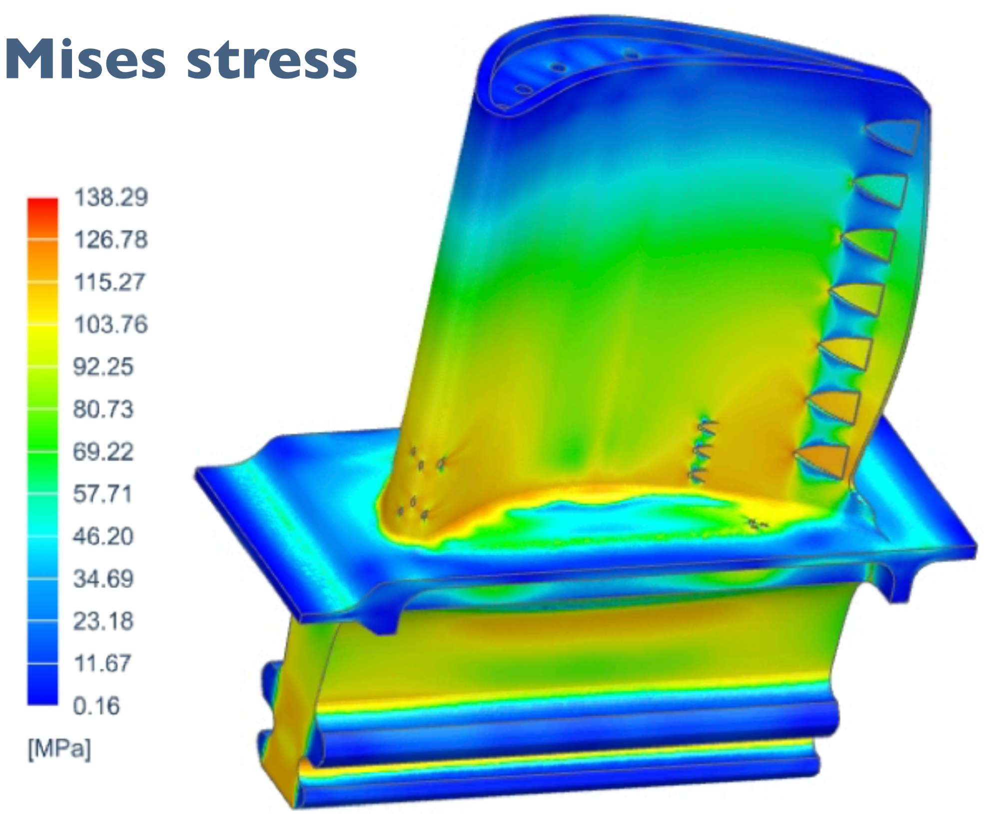
Mises stress

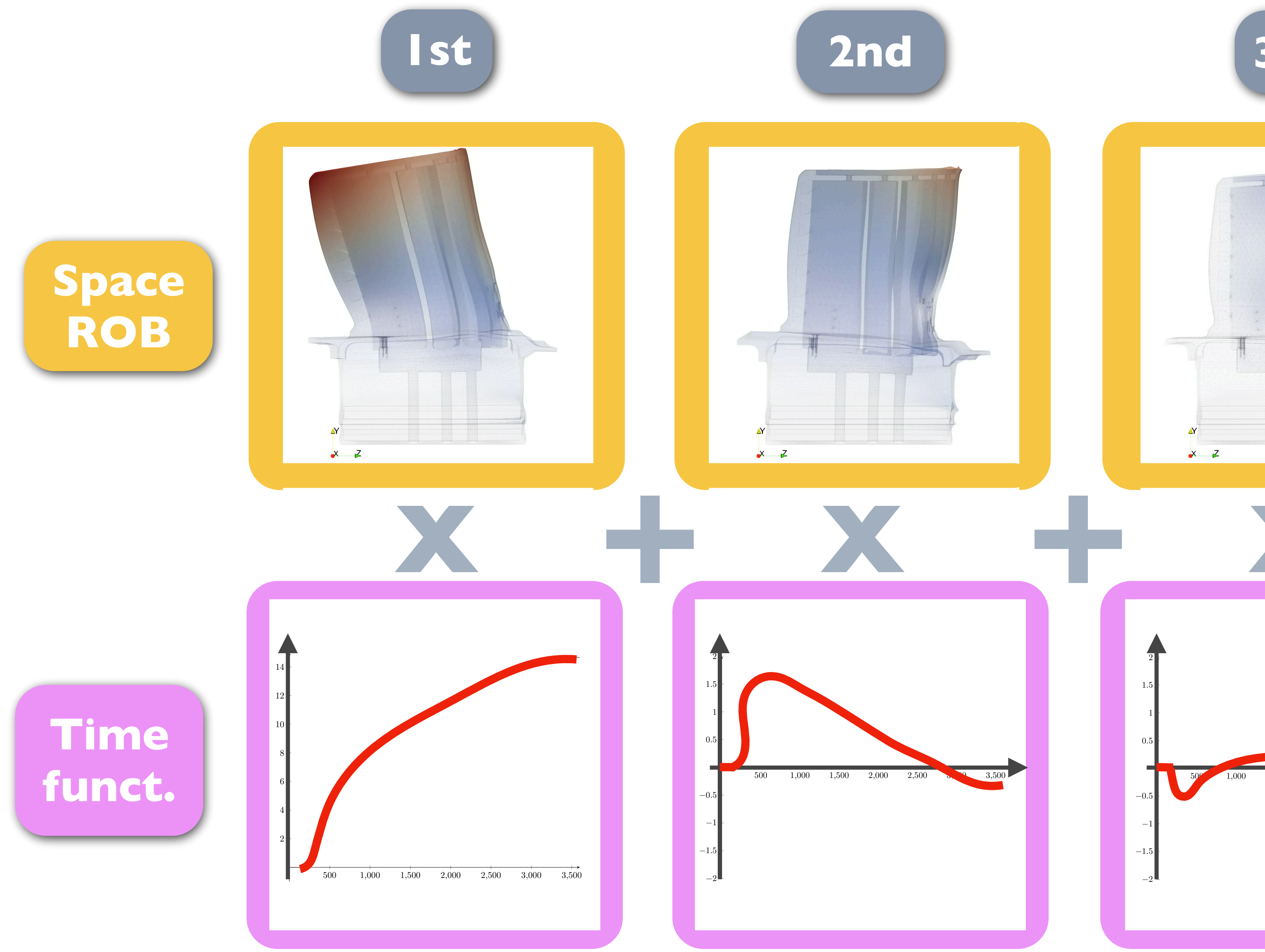




Bigger example

- **Blade of an aircraft engine**
  - Chaboche elasto-visco-plastic law with temperature dependence
  - 5 MDOFs, 31 time steps, centrifugal inertial forces (rotational speed of 15,000 tr/min)
  - [Nachar, Scanff, Ladevèze, Boucard, DN, 2022]



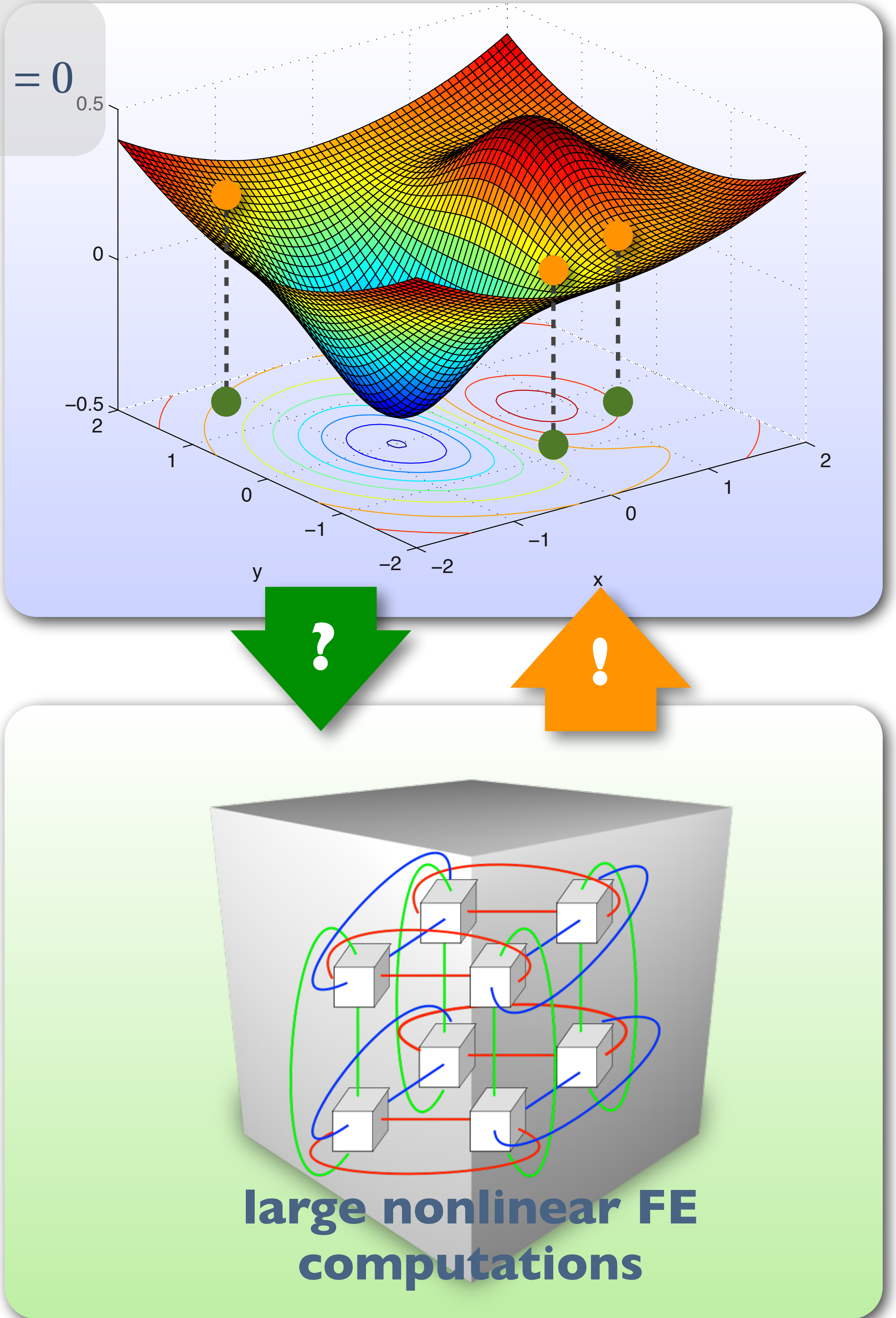


Outline

1. The LATIN method and Proper Generalized Decomposition
2. Solving parametrized problems to build virtual charts
3. Many queries in multiphysics problems
4. Conclusion

LMPS

PDE: $\mathscr{L}(u(t,M)) = 0$ → **µPDE:** $\mathscr{L}(u(t,M),\mu_1,\mu_2) = 0$



External driver algorithm

- reliability method
- optimization algorithm
- construction of a metamodel
- ...

Many queries

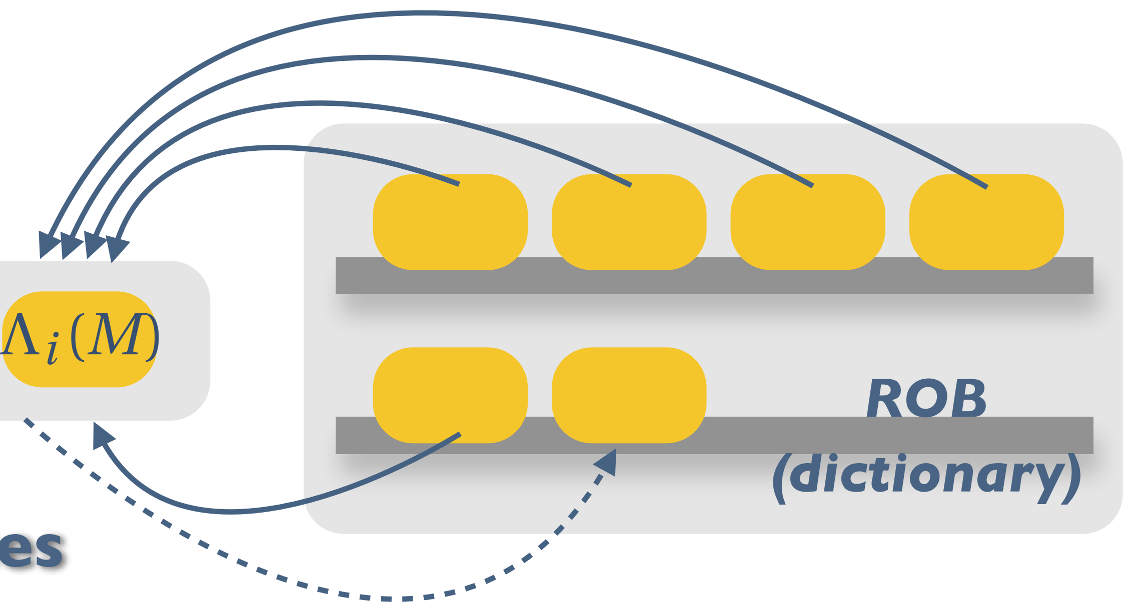
- same large nonlinear problem
- multiple runs for different sets of parameters
- very high CPU cost

PDE: $\mathscr{L}(u(t,M)) = 0$ → **µPDE:** $\mathscr{L}(u(t,M),\mu_1,\mu_2) = 0$

First approach: building a dictionary

- construction of a ROB common to all the sets of parameters
  - *[Boucard, Ladevèze 99] [DN, Boucard et al. 12-14] [Heyberger, Boucard, DN 13]*
  - *[DN, Boucard, Relun 15]*

$$u_{\mu_1,\mu_2}(t,M) \approx \sum_{i=1}^{m} \lambda_i(t)\Lambda_i(M)$$



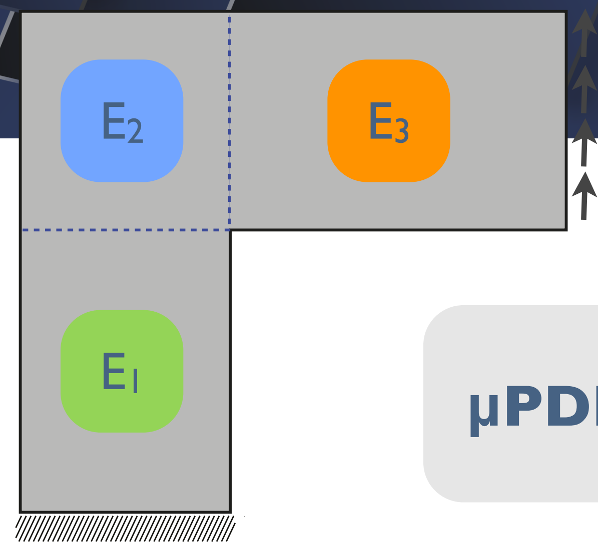
Second approach: extra-coordinates

- introduction of parameters as new coordinates
  - *[Chinesta, Ammar, Cueto, Huerta, Diez, Gonzalez, Leygue, Bordeu ... 12-]*

$$u(t,M,\mu_1,\mu_2) \approx \sum_{i=1}^{m'} \lambda_i(t)\Lambda_i(M)\alpha_i(\mu_1)\beta_i(\mu_2)$$

LMPS

Second approach

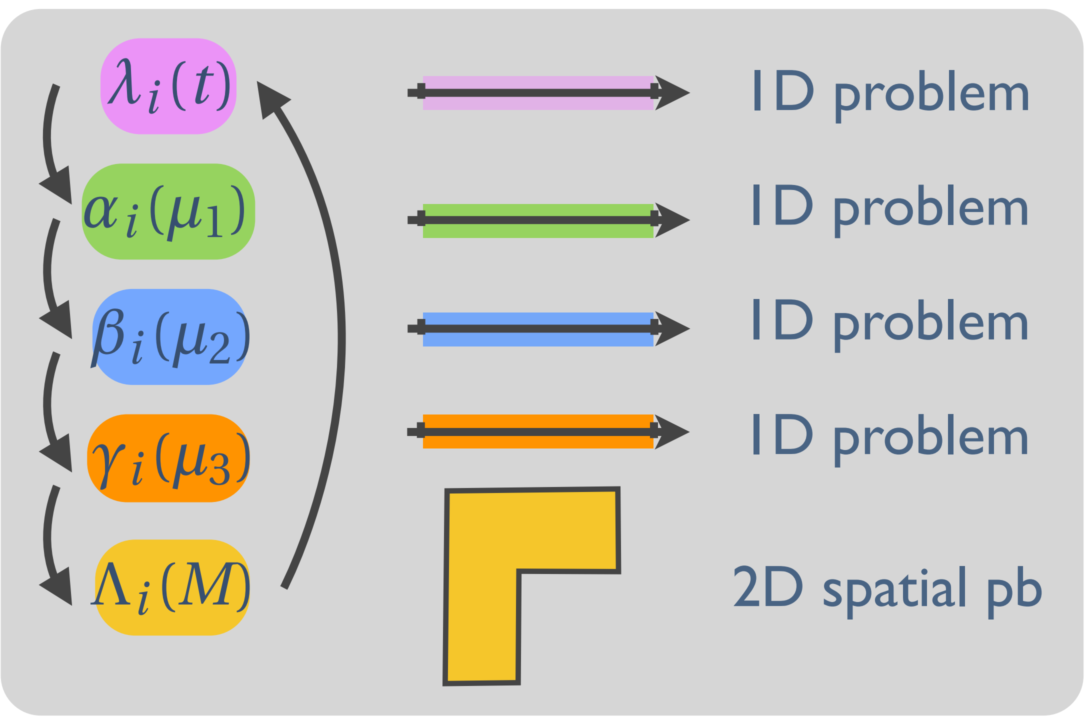


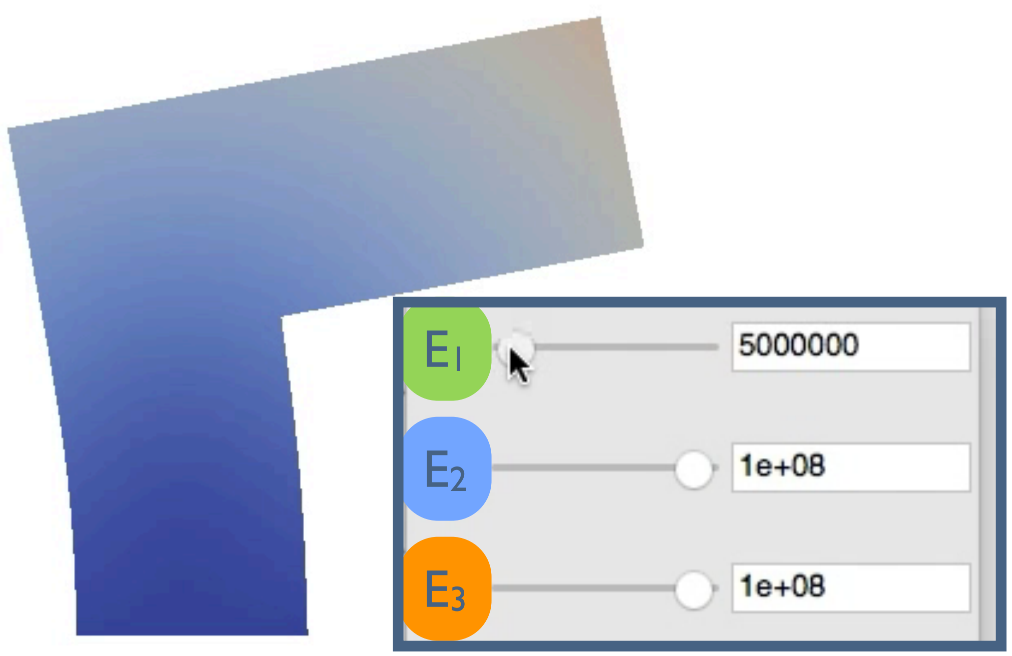
- **Parametrized PDE**
  - with 3 Young moduli as parameters

µPDE: $\mathscr{L}(u(t,M),\mu_1,\mu_2,\mu_3)=0$

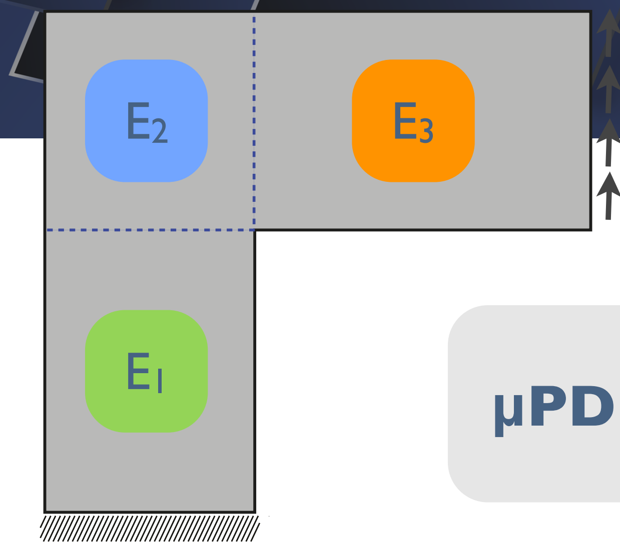
- **Separation of variables**

$$u(t,M,\mu_1,\mu_2,\mu_3) \approx \sum_{i=1}^{m'} \lambda_i(t)\Lambda_i(M)\alpha_i(\mu_1)\beta_i(\mu_2)\gamma_i(\mu_3)$$





Second approach

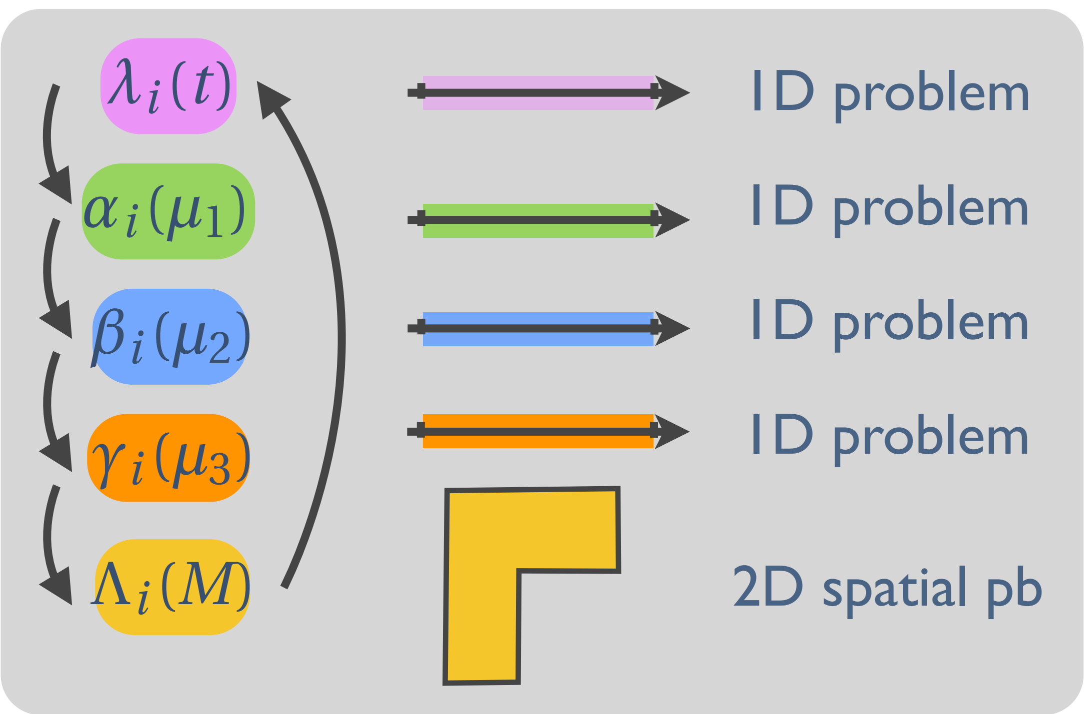


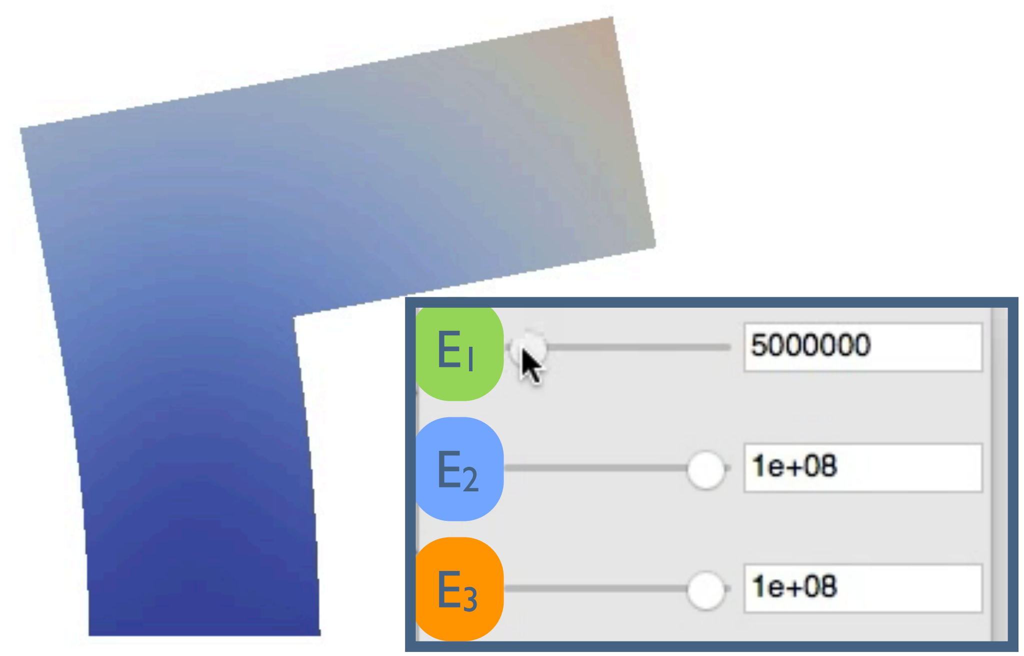
- **Parametrized PDE**
  - with 3 Young moduli as parameters

μPDE: $\mathscr{L}(u(t,M),\mu_1,\mu_2,\mu_3)=0$

- **Separation of variables**

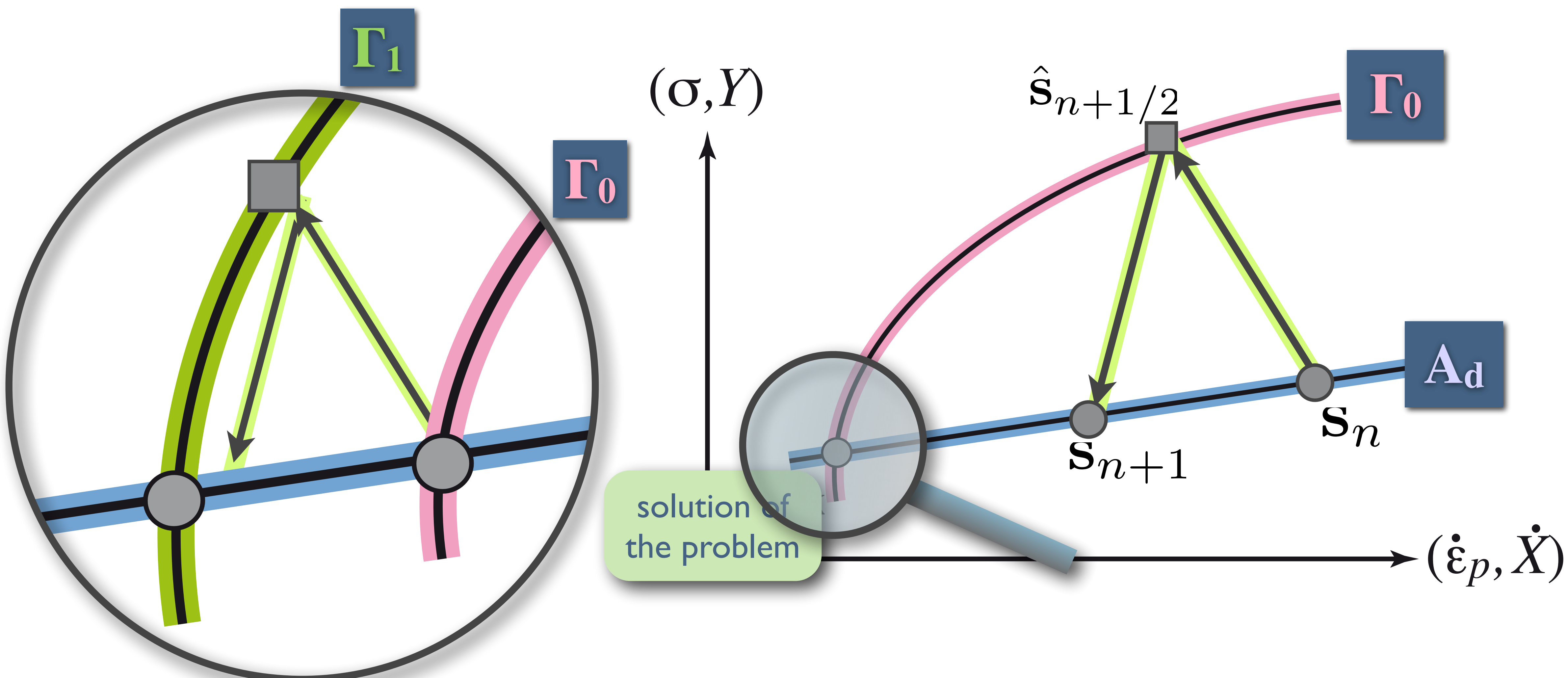
$$u(t,M,\mu_1,\mu_2,\mu_3)\approx\sum_{i=1}^{m'}\lambda_i(t)\Lambda_i(M)\alpha_i(\mu_1)\beta_i(\mu_2)\gamma_i(\mu_3)$$





First approach

- **Taken into account variability of parameters**
  - for example: variation of a material parameter of the nonlinear law
  - first computation for value k1: space Γ_0
  - new computation for value k2: space Γ_1
  - ➡ **reuse of the reduced model obtained from the PGD**
  - ➡ **addition of new pairs only if needed**

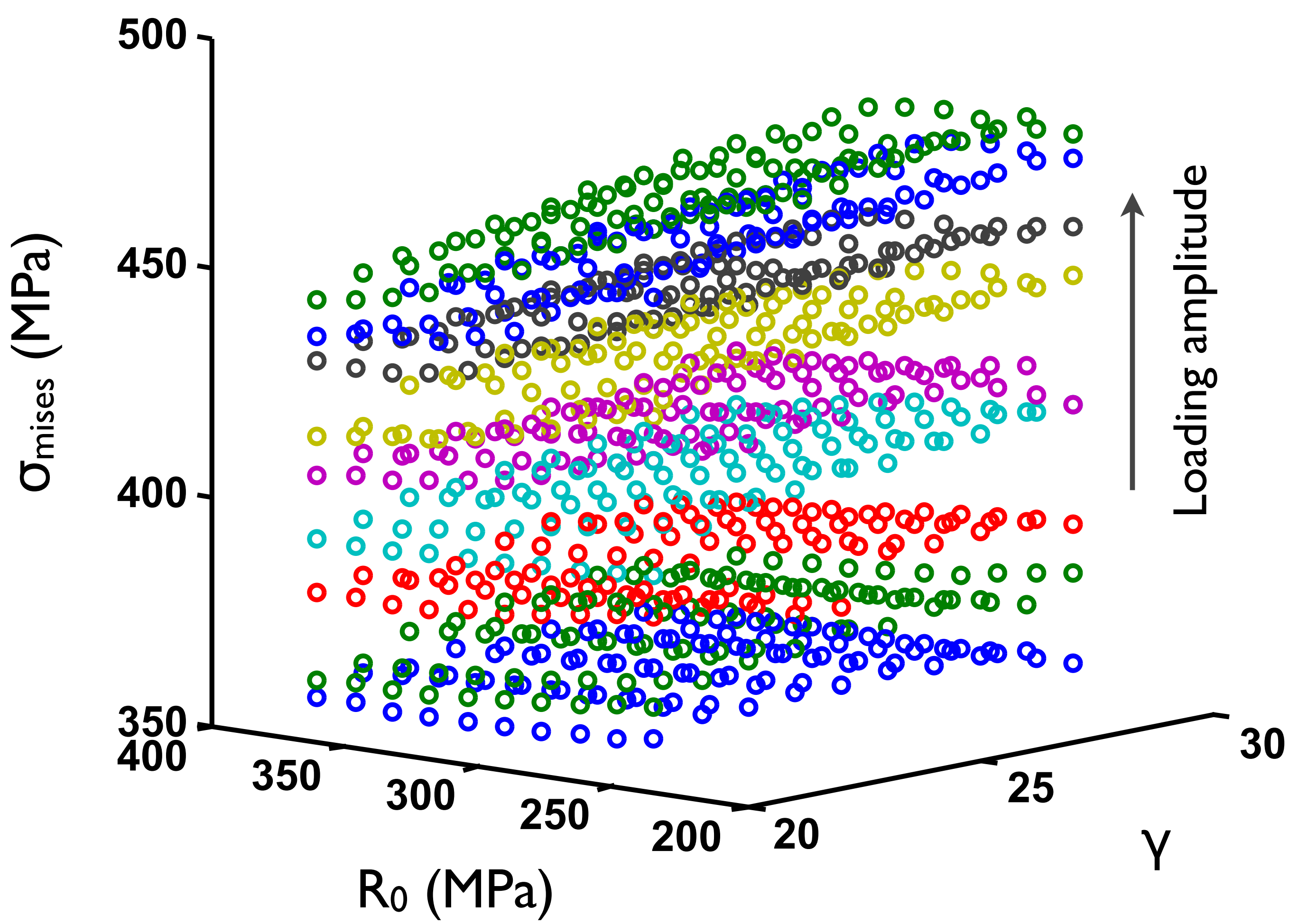


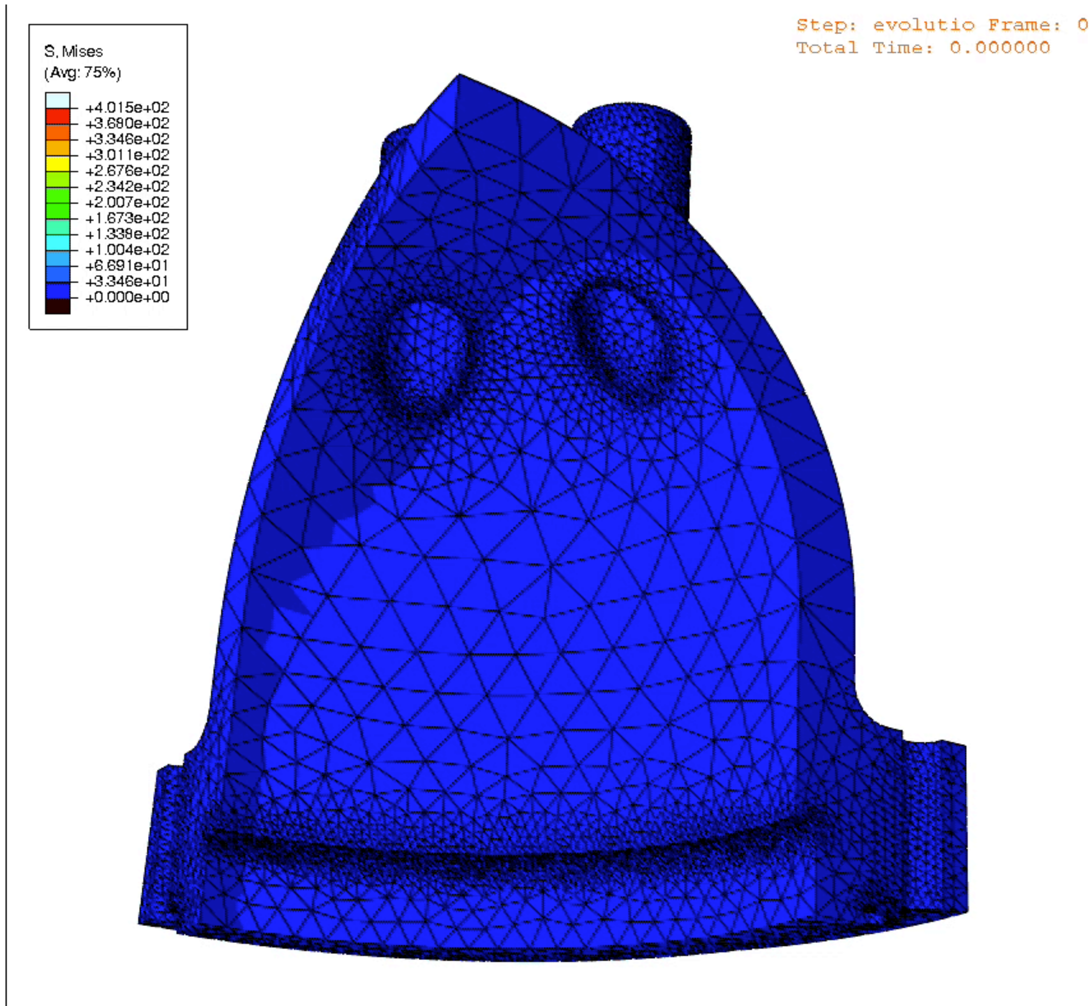
[Boucard and Ladevèze 99], [Allix and Vidal 02], [Boucard and Champaney 03]...

Parametric study

- parameters: loading amplitude and material characteristics (R_0 ,γ)
- 1,000 sets of parameters (**range of variation ±30%**)
- wallclock time for 1 run: **LATIN (2.5 hours)** **ABAQUS (3.5 hours)**
- influence on the maximum value of the σ_{mises}

740,000 DOFs
60 time steps





Mises stress

>35% of variation

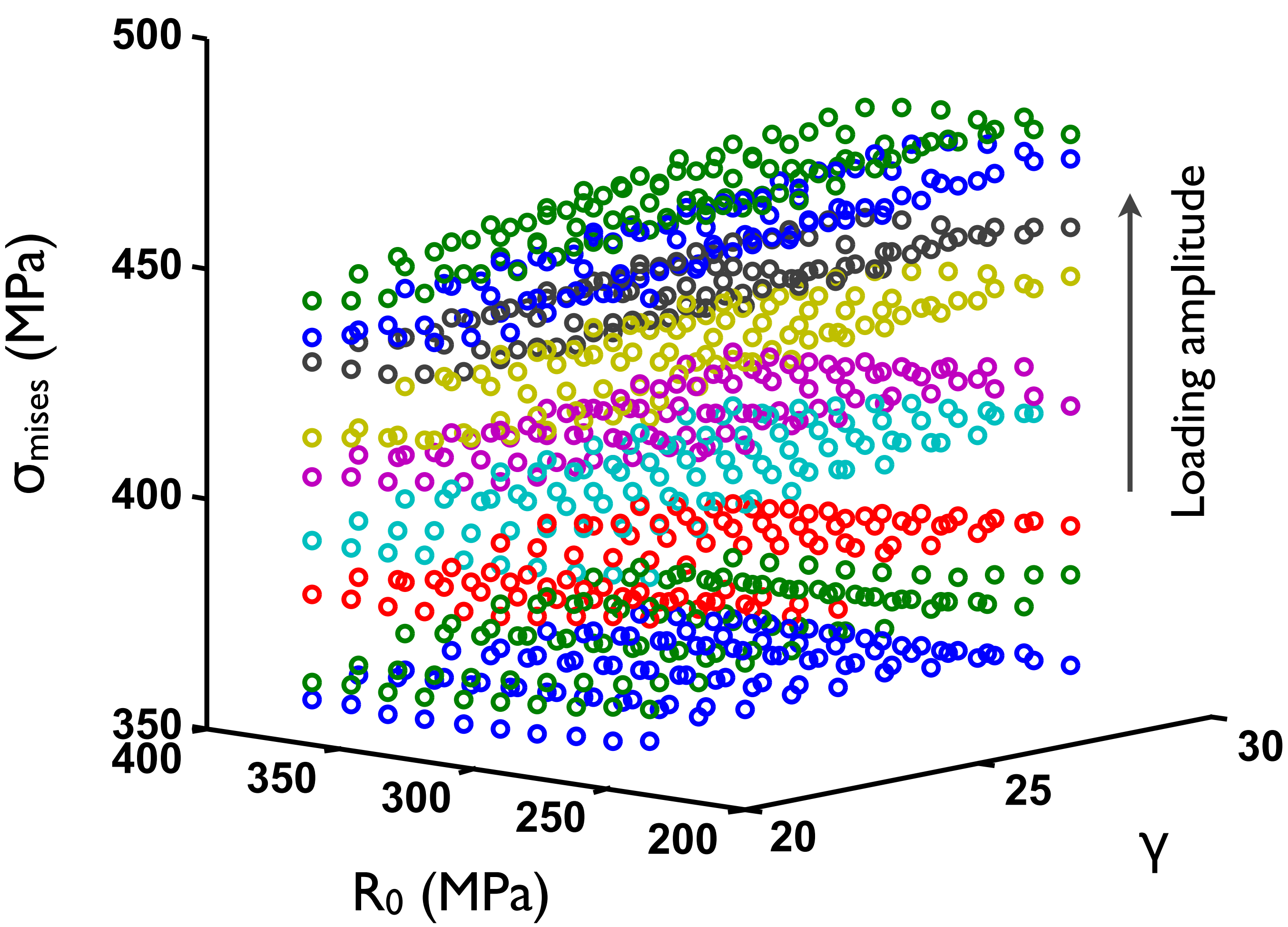
LMPS

Vessel head of nuclear reactor

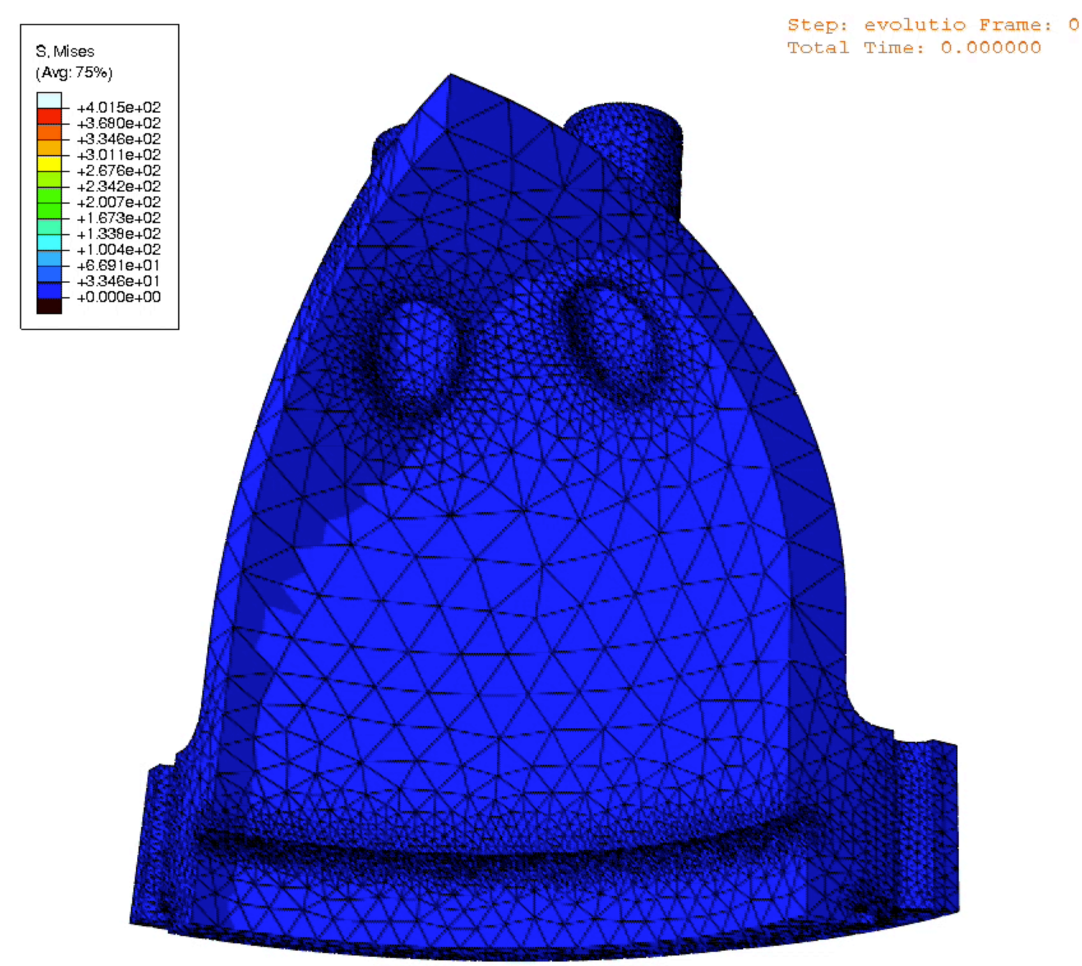
Parametric study

740,000 DOFs
60 time steps

- parameters: loading amplitude and material characteristics (R_0 ,γ)
- 1,000 sets of parameters (**range of variation ±30%**)
- wallclock time for 1 run: **LATIN (2.5 hours)** **ABAQUS (3.5 hours)**
- influence on the maximum value of the σ_{mises}



>35% of variation



Mises stress

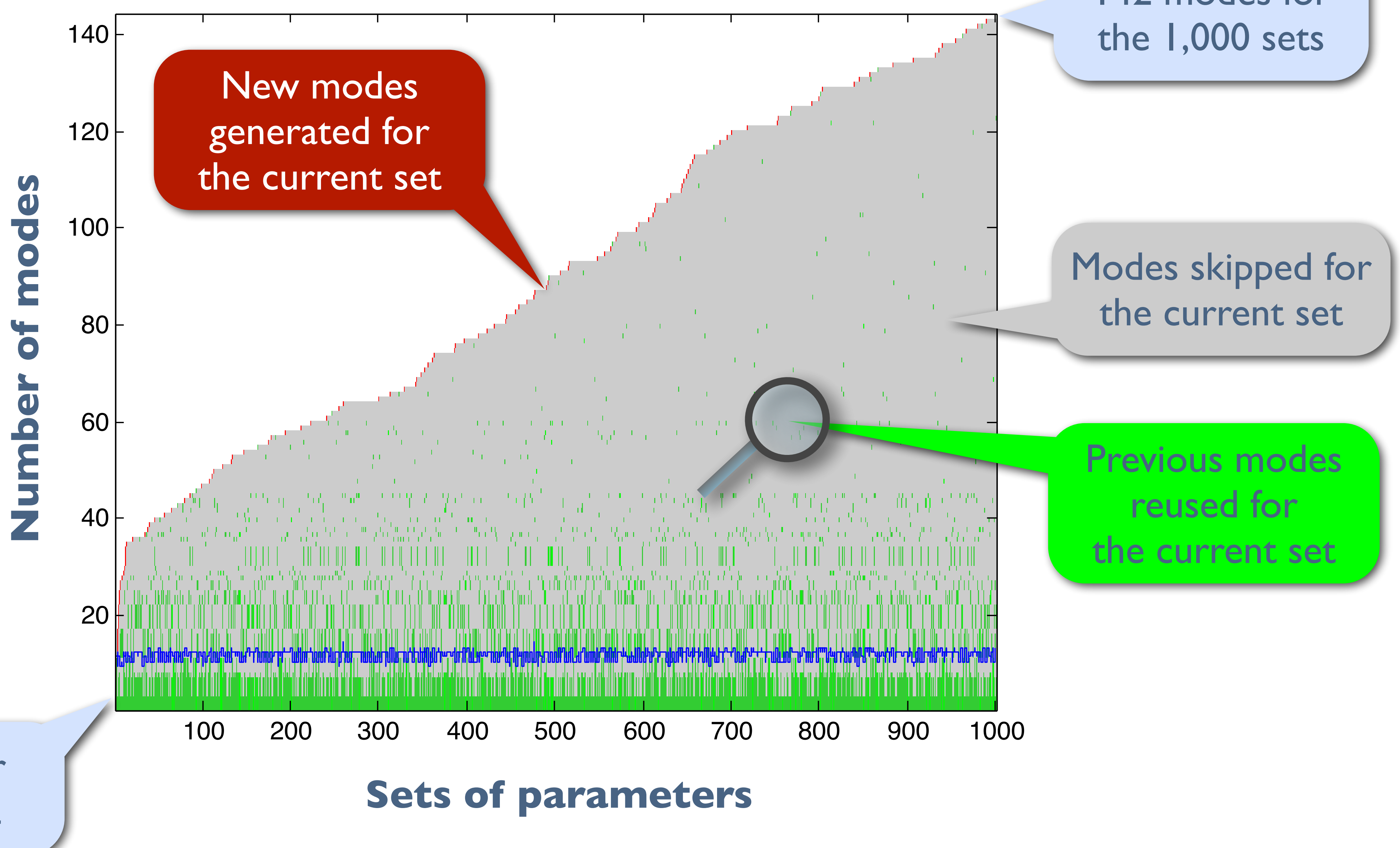
Vessel head of nuclear reactor

Parametric study

- parameters: loading amplitude and material characteristics (R_0 ,γ)
- influence on the maximum value of the σ_{mises}
- 1,000 sets of parameters (**range of variation ±30%**)

4 months with ABAQUS

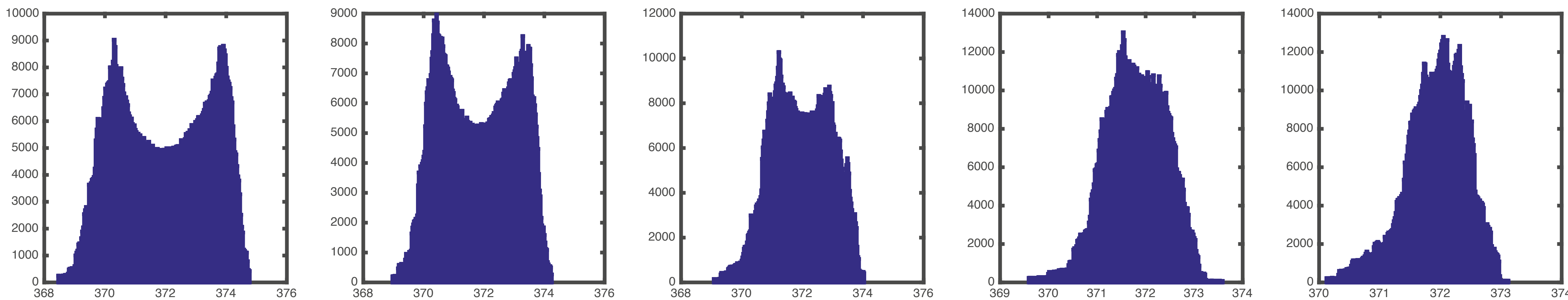
LATIN+PGD 1 week (gain: 12)



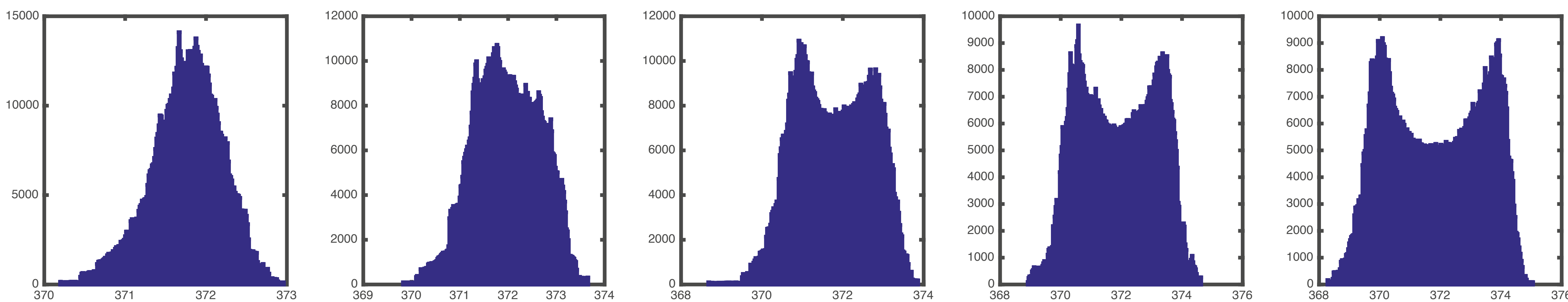
Post-treatment of virtual charts

Uncertainties

- material characteristics (R_0 ,γ) are **stochastic**
- loading parameter is described by an **interval**



PDFs of the maximum of sigma Mises max



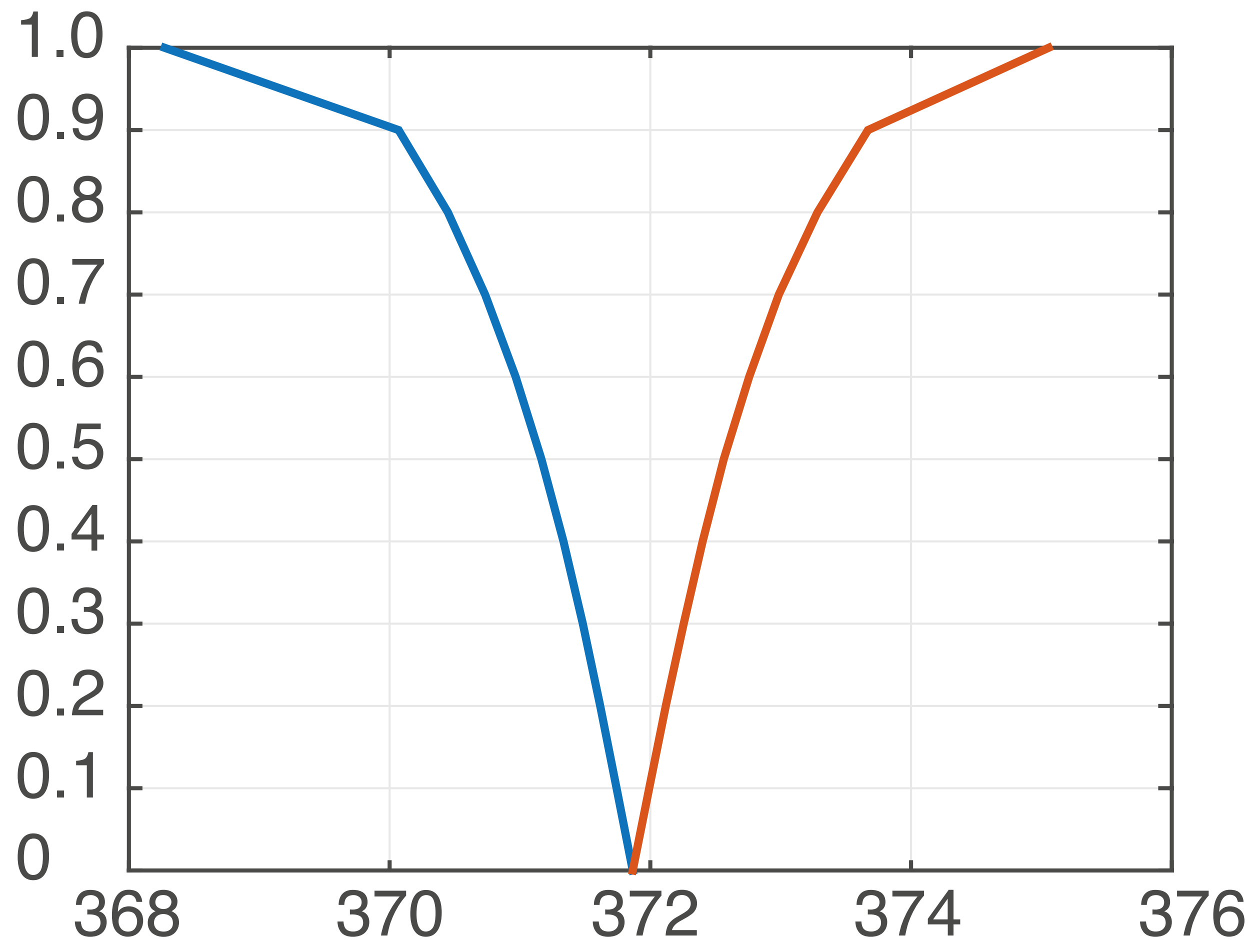
~ 5 s for 10 millions of Monte-Carlo calls

LMPS

Post-treatment of virtual charts

Uncertainties

- material characteristics (R_0 ,γ) are **stochastic**
- loading parameter is described by an **interval**



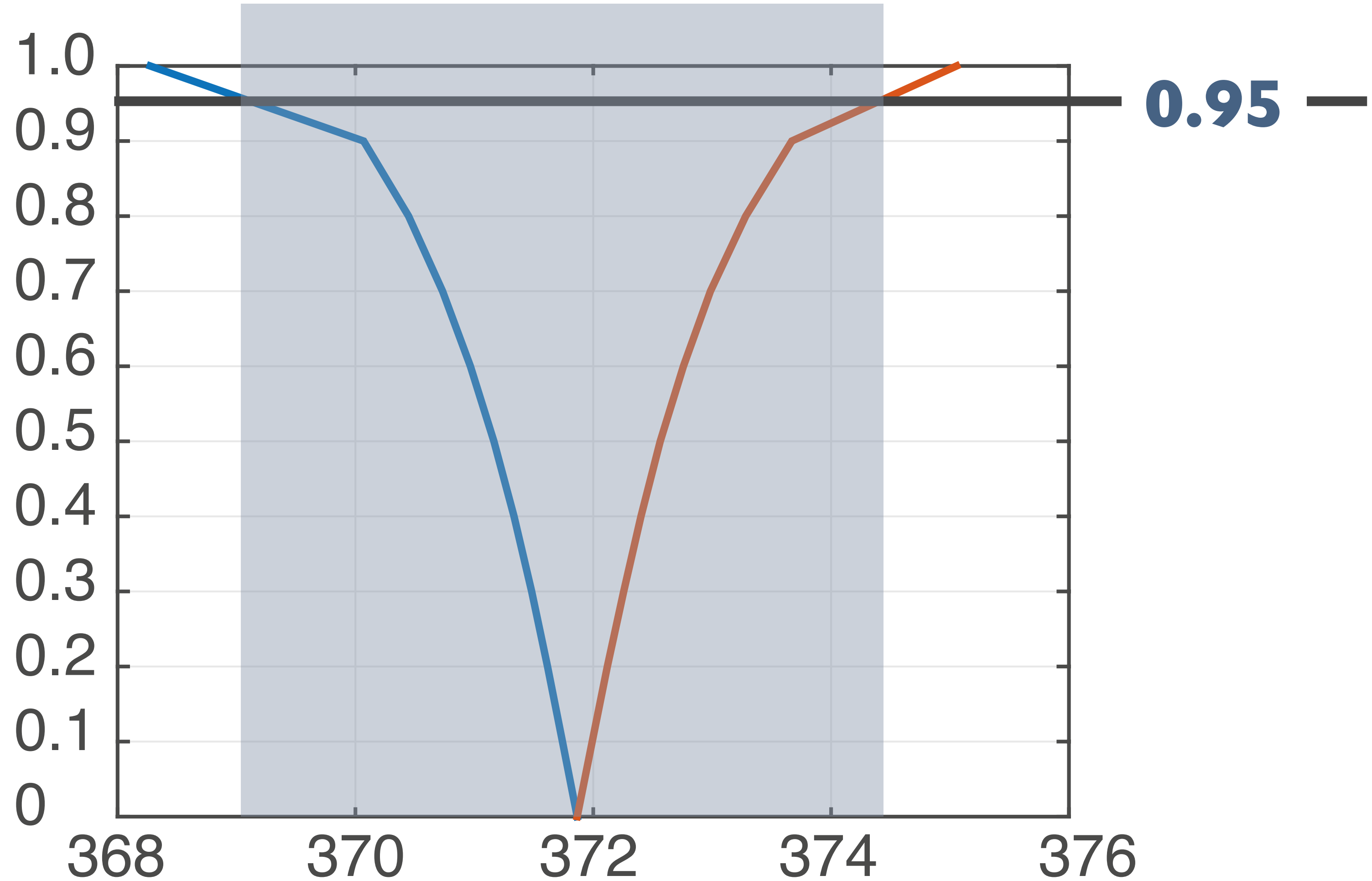
Interval with stochastic bounds
for the maximum of sigma Mises max

Post-treatment of virtual charts

Uncertainties

- material characteristics (R_0, γ) are **stochastic**
- loading parameter is described by an **interval**

369 MPa < σ_{Mises} < 374.5 MPa



Interval with stochastic bounds
for the maximum of sigma Mises max

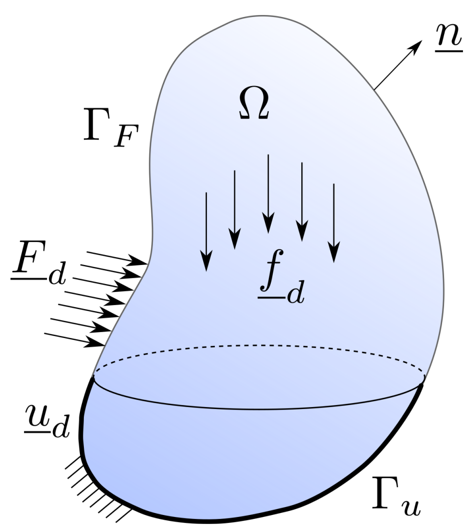
LMPS

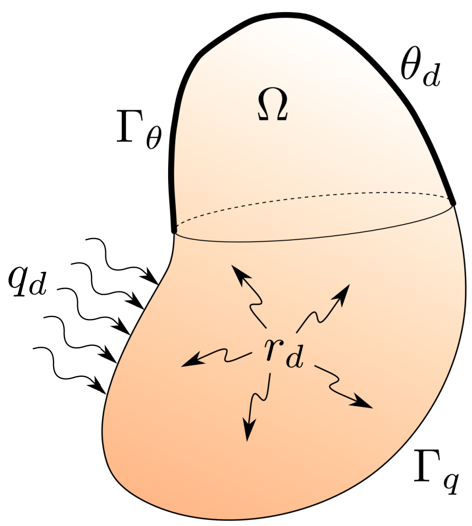
Outline

1. The LATIN method and Proper Generalized Decomposition
2. Solving parametrized problems to build virtual charts
3. Many queries in multiphysics problems
4. Conclusion

LMPS

Simple problem: thermoelasticity





Assumptions: quasi-static evolution, small strains, small temperature changes, homogeneous and isotropic material

Mechanical equilibrium

- Stress equilibrium $\nabla \cdot \boldsymbol{\sigma} + \underline{f}_d = \underline{0}$ in Ω
- Strain compatibility $\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla \underline{u} + {}^{\mathrm{T}}\nabla \underline{u})$ in Ω
- Boundary conditions $\underline{u} = \underline{u}_d$ on Γ_u and $\boldsymbol{\sigma}\,\underline{n} = \underline{F}_d$ on Γ_F

Strongly coupled constitutive equations

- Hooke's law $\boldsymbol{\sigma} = \boldsymbol{\mathcal{K}} : \boldsymbol{\varepsilon} - \beta\theta\boldsymbol{I}$
- Fourier's law $\underline{q} = -k\,\nabla\theta$
- Mechanical heat source $r_m = T_0\beta\,\mathrm{Tr}\,\dot{\boldsymbol{\varepsilon}}$

Thermal equilibrium

- Heat equation $\rho c\dot{\theta} + r_m = -\nabla \cdot \underline{q} + r_d$ in Ω
- Temp. grad. compatibility $\underline{x} = \nabla\theta$ in Ω
- Boundary conditions $\theta = \theta_d$ on Γ_θ and $\underline{q} \cdot \underline{n} = q_d$ on Γ_q

Monolithic coupled resolution

- direct resolution of the system: a priori CPU expensive

Mechanical equilibrium

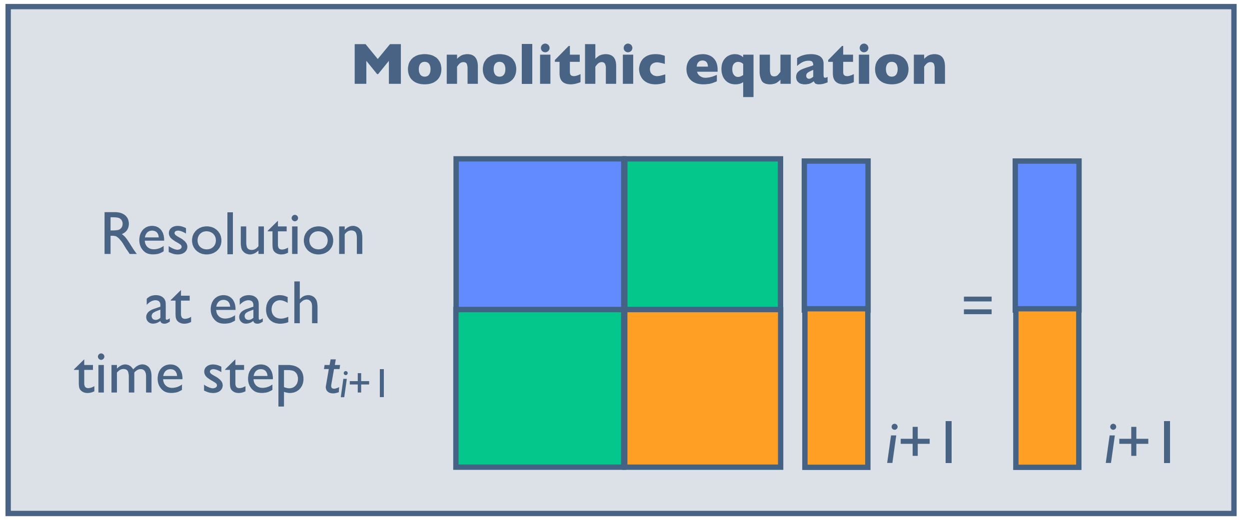
- Stress equilibrium $\nabla \cdot \boldsymbol{\sigma} + \underline{f}_d = \underline{0}$ in Ω
- Strain compatibility $\varepsilon = \frac{1}{2}(\nabla \underline{u} + {}^{\mathrm{T}}\nabla \underline{u})$ in Ω
- Boundary conditions $\underline{u} = \underline{u}_d$ on Γ_u and $\boldsymbol{\sigma}\,\underline{n} = \underline{F}_d$ on Γ_F

Strongly coupled **constitutive equations**

- Hooke's law $\boldsymbol{\sigma} = \boldsymbol{\mathcal{K}} : \varepsilon - \beta\theta \boldsymbol{I}$
- Fourier's law $\underline{q} = -k\,\nabla\theta$
- Mechanical heat source $r_m = T_0 \beta \,\mathrm{Tr}\,\dot{\varepsilon}$

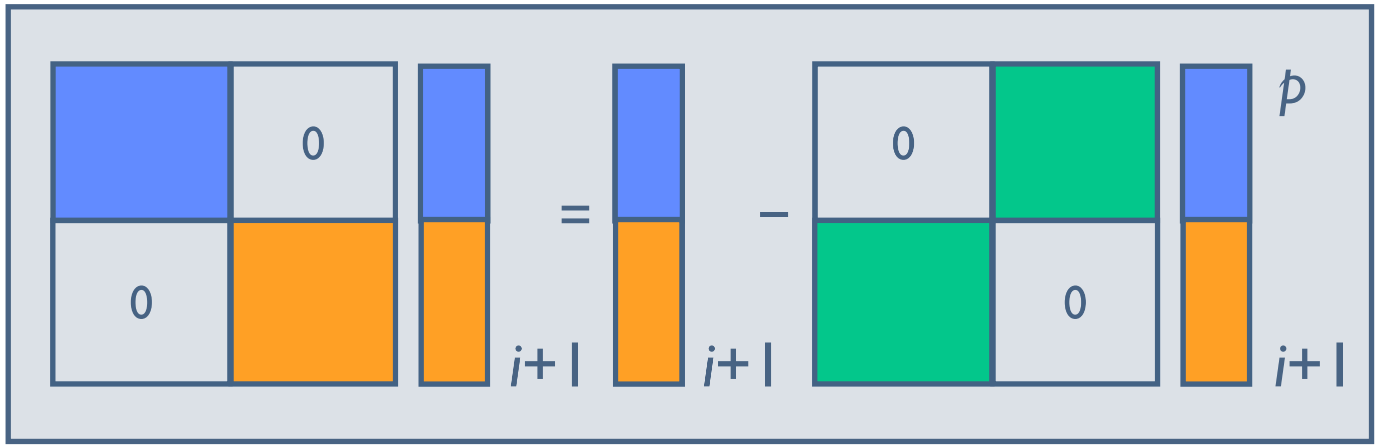
Thermal equilibrium

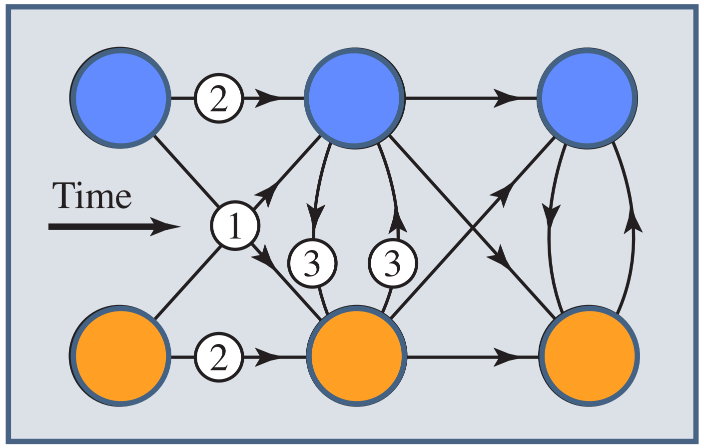
- Heat equation $\rho c \dot{\theta} + r_m = -\nabla \cdot \underline{q} + r_d$ in Ω
- Temp. grad. compatibility $\underline{x} = \nabla\theta$ in Ω
- Boundary conditions $\theta = \theta_d$ on Γ_θ and $\underline{q} \cdot \underline{n} = q_d$ on Γ_q



Partitioned procedures

- decoupled resolution of the physics [Felippa and Park 80, Belytschko and Hugues 83, Shrefler *et al.* 87, Zienkiewicz *et al.* 88, Farhat *et al.* 95, Morand and Ohayon 95, Lewis and Schrefler 98, ...]





Natural separation of the equations

linear decoupled but global-in-space

Mechanical equilibrium

- Stress equilibrium $\nabla \cdot \boldsymbol{\sigma} + \underline{f}_d = \underline{0}$ in Ω
- Strain compatibility $\varepsilon = \frac{1}{2}(\nabla \underline{u} + {}^{\mathrm{T}}\nabla \underline{u})$ in Ω
- Boundary conditions $\underline{u} = \underline{u}_d$ on Γ_u and $\boldsymbol{\sigma}\,\underline{n} = \underline{F}_d$ on Γ_F

Strongly coupled constitutive equations

- Hooke's law $\boldsymbol{\sigma} = \boldsymbol{\mathcal{K}} : \varepsilon - \beta\theta \boldsymbol{I}$
- Fourier's law $\underline{q} = -k\,\nabla\theta$
- Mechanical heat source $r_m = T_0 \beta \,\mathrm{Tr}\,\dot{\varepsilon}$

Thermal equilibrium

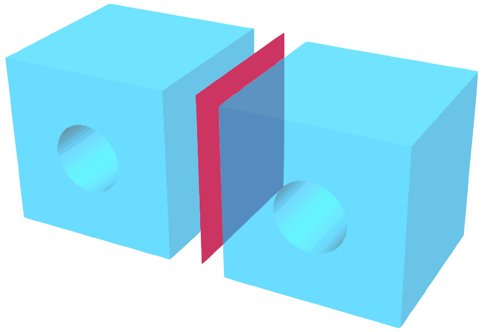
- Heat equation $\rho c \dot{\theta} + r_m = -\nabla \cdot \underline{q} + r_d$ in Ω
- Temp. grad. compatibility $\underline{x} = \nabla\theta$ in Ω
- Boundary conditions $\theta = \theta_d$ on Γ_θ and $\underline{q} \cdot \underline{n} = q_d$ on Γ_q

linear decoupled but global-in-space

Interface between physics

local in space but coupled

similar to



Natural separation of the equations

A_d

Mechanical equilibrium

- Stress equilibrium $\nabla \cdot \sigma + \underline{f}_d = \underline{0}$ in Ω
- Strain compatibility $\varepsilon = \frac{1}{2}(\nabla \underline{u} + {}^{\mathrm{T}}\nabla \underline{u})$ in Ω
- Boundary conditions $\underline{u} = \underline{u}_d$ on Γ_u and $\sigma\,\underline{n} = \underline{F}_d$ on Γ_F

Strongly coupled **constitutive equations**

- Hooke's law $\sigma = \mathcal{K} : \varepsilon - \beta\theta \boldsymbol{I}$
- Fourier's law $\underline{q} = -k\,\nabla\theta$
- Mechanical heat source $r_m = T_0 \beta\, \mathrm{Tr}\, \dot{\varepsilon}$

Γ

Thermal equilibrium

- Heat equation $\rho c \dot{\theta} + r_m = -\nabla \cdot \underline{q} + r_d$ in Ω
- Temp. grad. compatibility $\underline{x} = \nabla\theta$ in Ω
- Boundary conditions $\theta = \theta_d$ on Γ_θ and $\underline{q} \cdot \underline{n} = q_d$ on Γ_q

[DN, Dureisseix 08a, 08b]

LATIN framework

Mechanical equilibrium

- Stress equilibrium $\nabla \cdot \sigma + \underline{f}_d = \underline{0}$ in Ω
- Strain compatibility $\varepsilon = \frac{1}{2}(\nabla \underline{u} + {}^{\mathrm{T}}\nabla \underline{u})$ in Ω
- Boundary conditions $\underline{u} = \underline{u}_d$ on Γ_u and $\sigma\, \underline{n} = \underline{F}_d$ on Γ_F

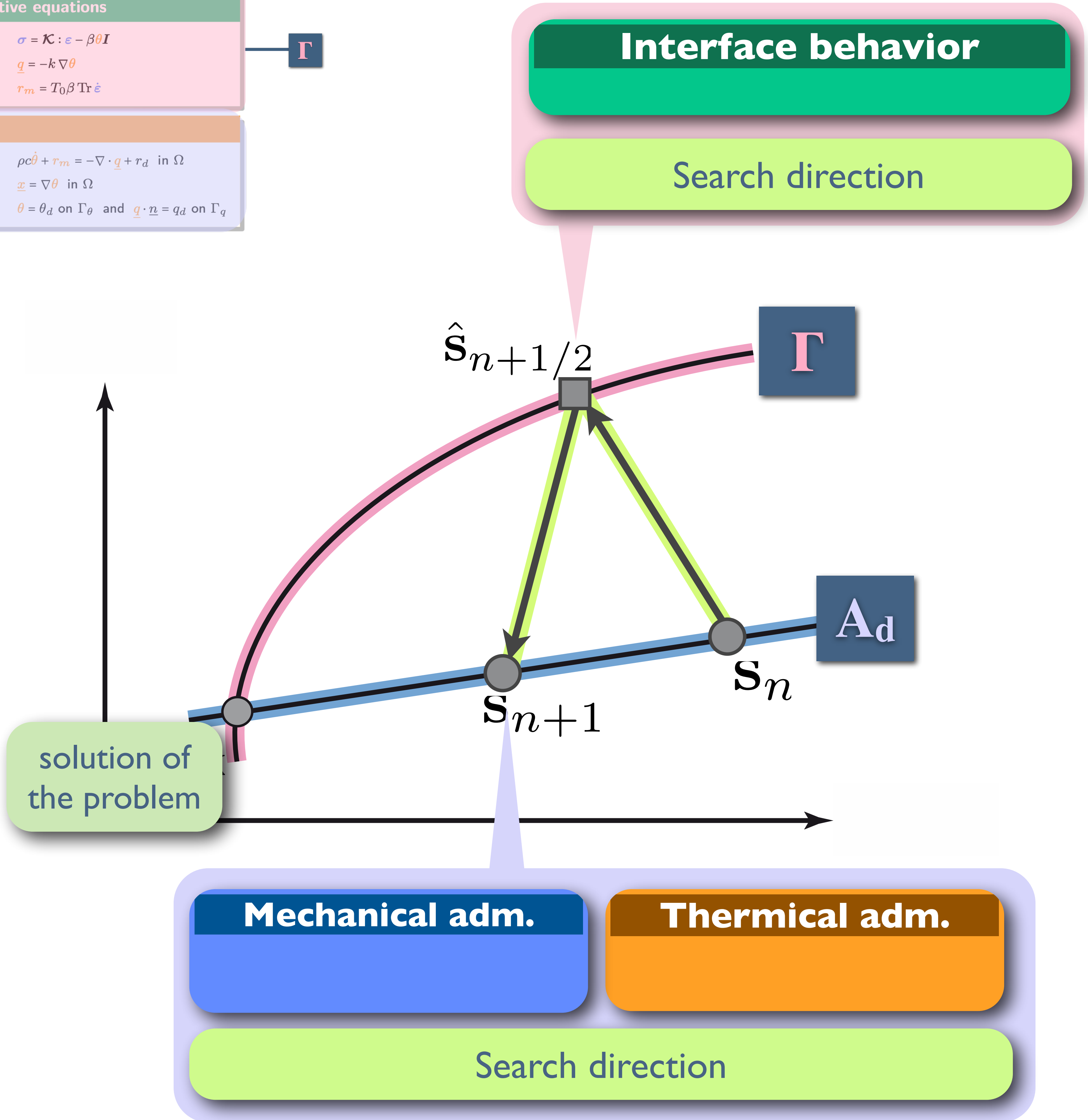
***Strongly coupled* constitutive equations**

- Hooke's law $\sigma = \boldsymbol{\mathcal{K}} : \varepsilon - \beta \theta \boldsymbol{I}$
- Fourier's law $\underline{q} = -k\, \nabla \theta$
- Mechanical heat source $r_m = T_0 \beta\, \mathrm{Tr}\, \dot{\varepsilon}$

Thermal equilibrium

- Heat equation $\rho c \dot{\theta} + r_m = -\nabla \cdot \underline{q} + r_d$ in Ω
- Temp. grad. compatibility $\underline{x} = \nabla \theta$ in Ω
- Boundary conditions $\theta = \theta_d$ on Γ_θ and $\underline{q} \cdot \underline{n} = q_d$ on Γ_q

$\mathrm{A_d}$ Γ



LATIN framework

Mechanical equilibrium

- Stress equilibrium $\nabla \cdot \sigma + \underline{f}_d = \underline{0}$ in Ω
- Strain compatibility $\varepsilon = \frac{1}{2}(\nabla \underline{u} + {}^{\mathrm{T}}\nabla \underline{u})$ in Ω
- Boundary conditions $\underline{u} = \underline{u}_d$ on Γ_u and $\sigma\, \underline{n} = \underline{F}_d$ on Γ_F

Strongly coupled **constitutive equations**

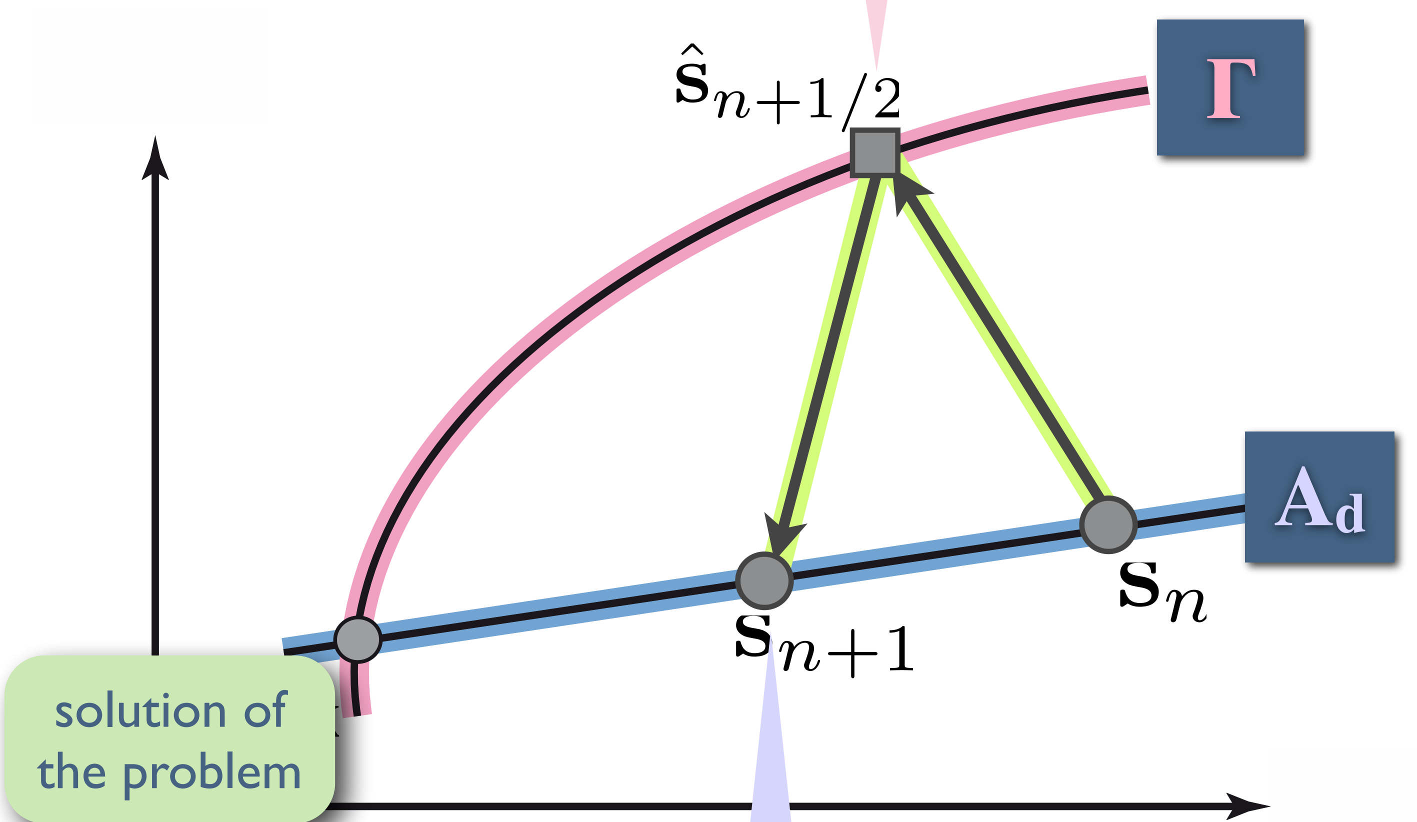
- Hooke's law $\sigma = \mathcal{K} : \varepsilon - \beta\theta \boldsymbol{I}$
- Fourier's law $\underline{q} = -k\,\nabla\theta$
- Mechanical heat source $r_m = T_0 \beta\, \mathrm{Tr}\, \dot{\varepsilon}$

Thermal equilibrium

- Heat equation $\rho c \dot{\theta} + r_m = -\nabla \cdot \underline{q} + r_d$ in Ω
- Temp. grad. compatibility $\underline{x} = \nabla\theta$ in Ω
- Boundary conditions $\theta = \theta_d$ on Γ_θ and $\underline{q} \cdot \underline{n} = q_d$ on Γ_q

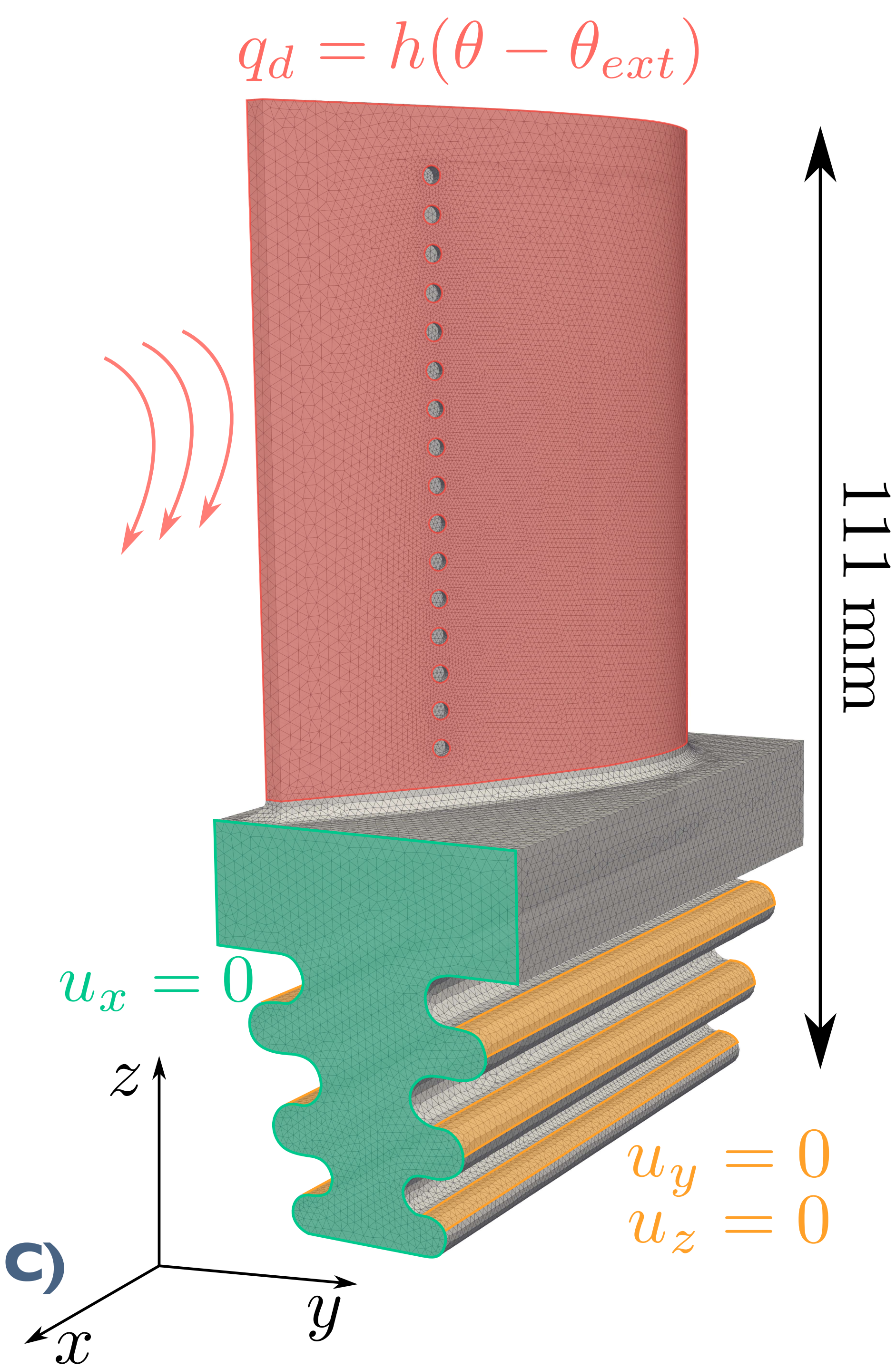
$\mathrm{A_d}$ Γ

Coupled but local-in-space
« **interface** »



Time-space **PGD** for **mechanics**

Time-space **PGD** for **thermics**



Number of DOFs

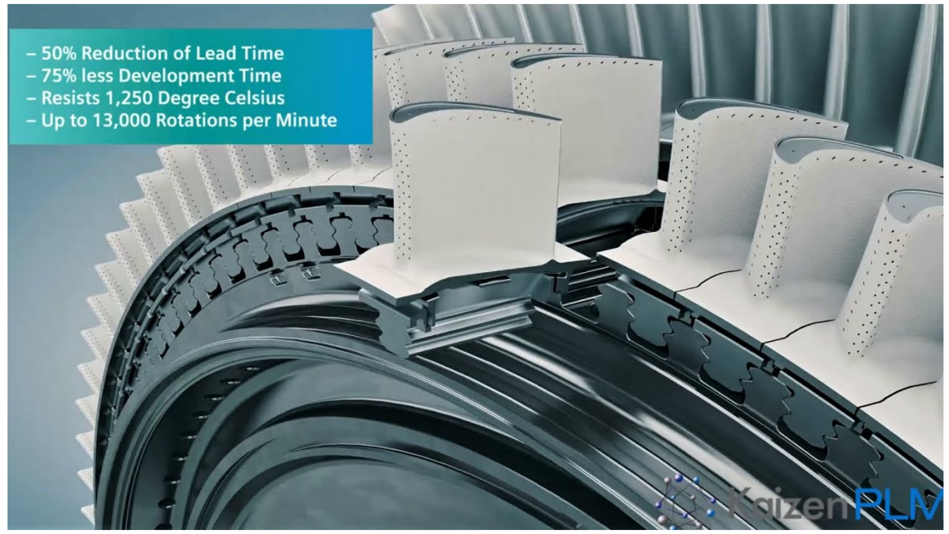
- mechanical part: 2,370,000 DOFs
- thermal part: 118,300 DOFs

Boundary conditions

- from [Kin et al., AIAA Journal, 2018]
- **clamped on the lower tree root**
- **centrifugal load up to 15,000 rpm**
- **forced convective flux on airfoil surface (270°C)**
- forced convective flux on cooling holes (40°C)

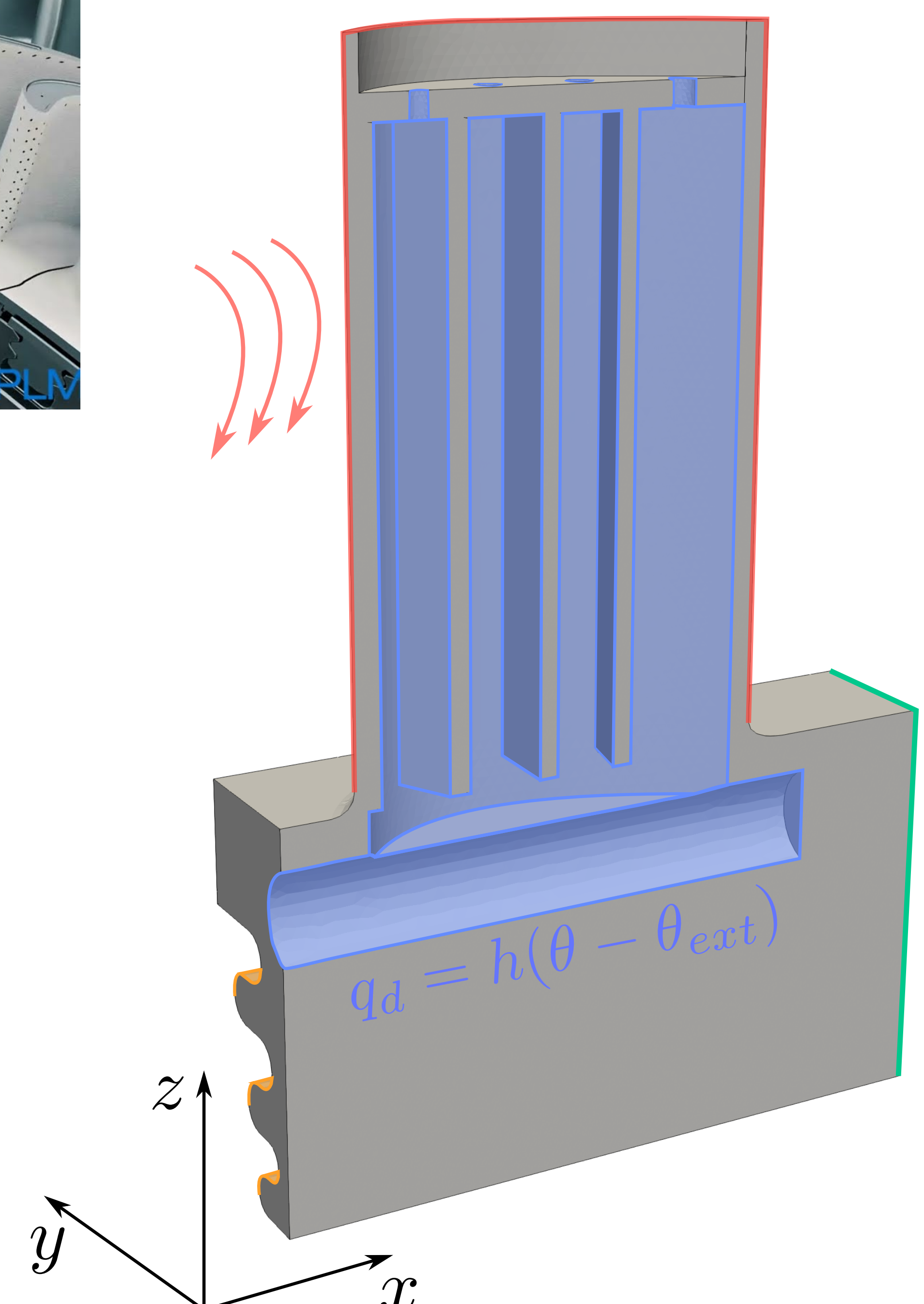
[Wurtzer, DN, Boucard 2024]

Turbine blade



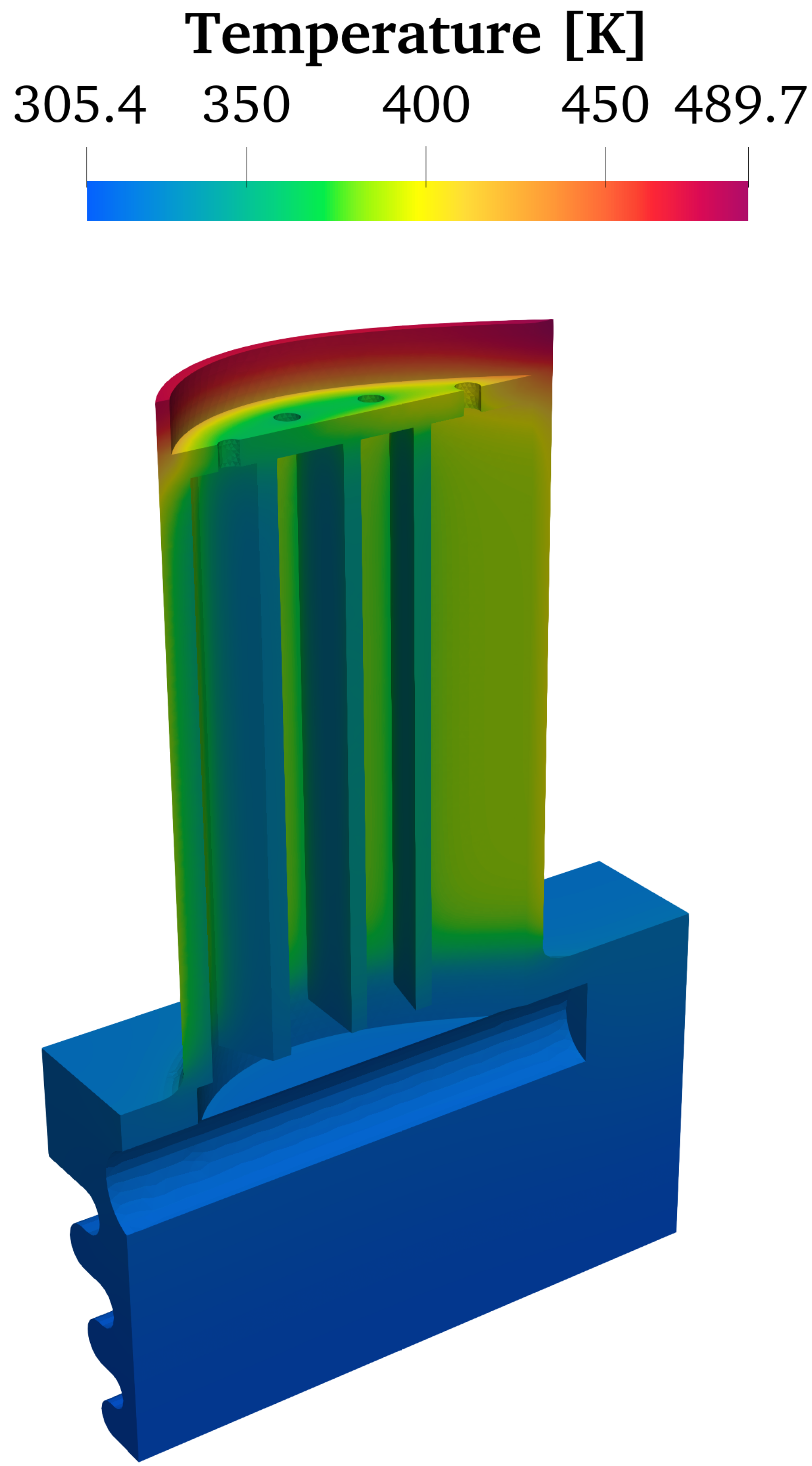
- **Number of DOFs**
  - mechanical part: 2,370,000 DOFs
  - thermal part: 118,300 DOFs

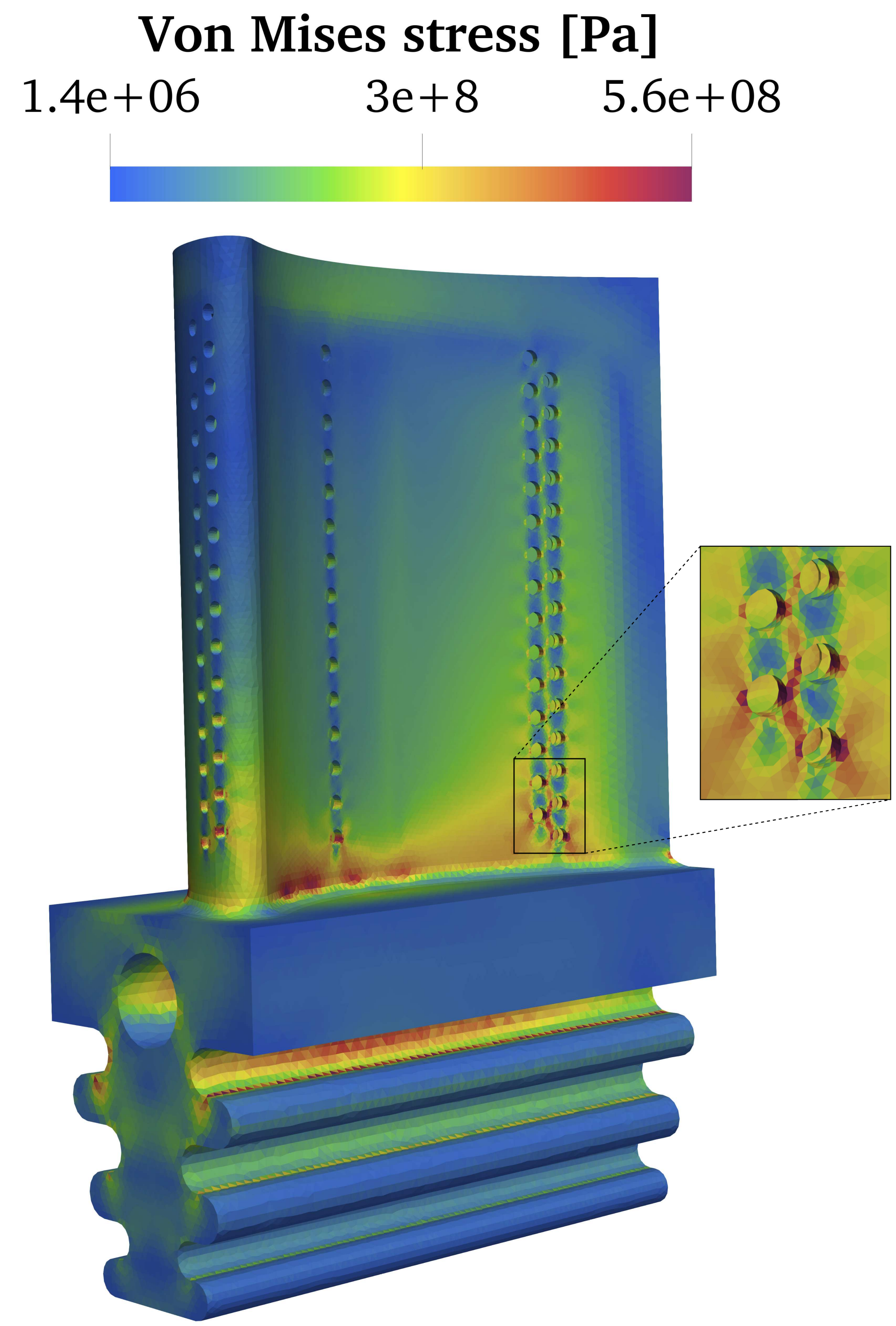
- **Boundary conditions**
  - from [Kin et al., AIAA Journal, 2018]
  - clamped on the lower tree root
  - centrifugal load up to 15,000 rpm
  - forced convective flux on airfoil surface (270°C)
  - **forced convective flux on cooling holes (40°C)**



[Wurtzer, DN, Boucard 2024]

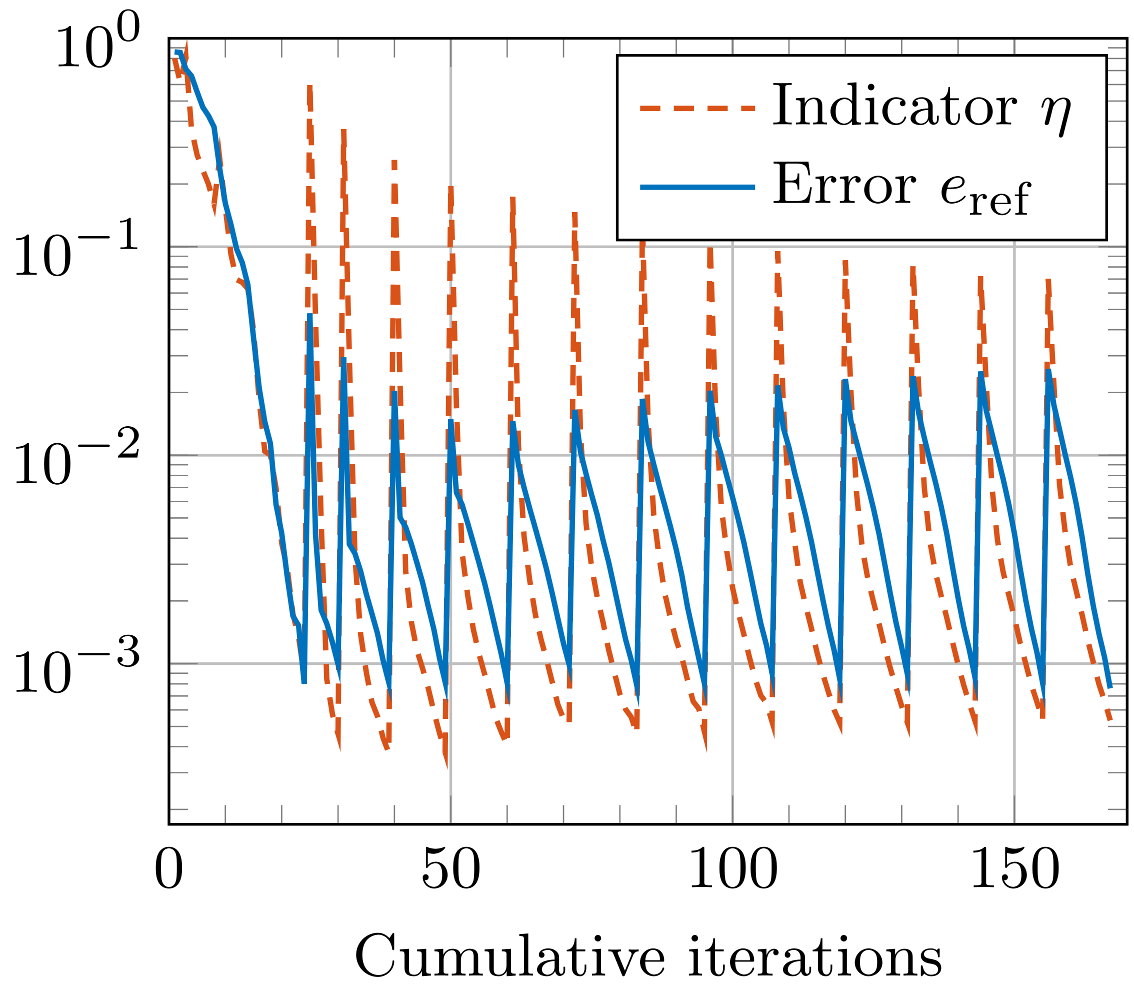
Turbine blade

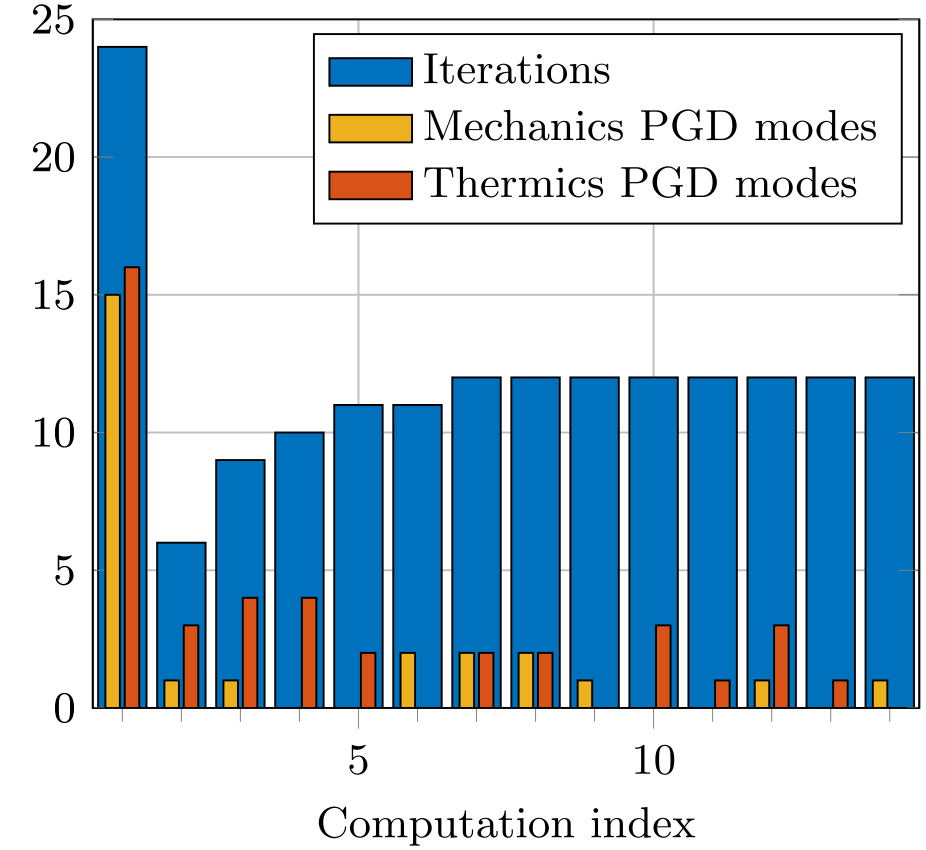


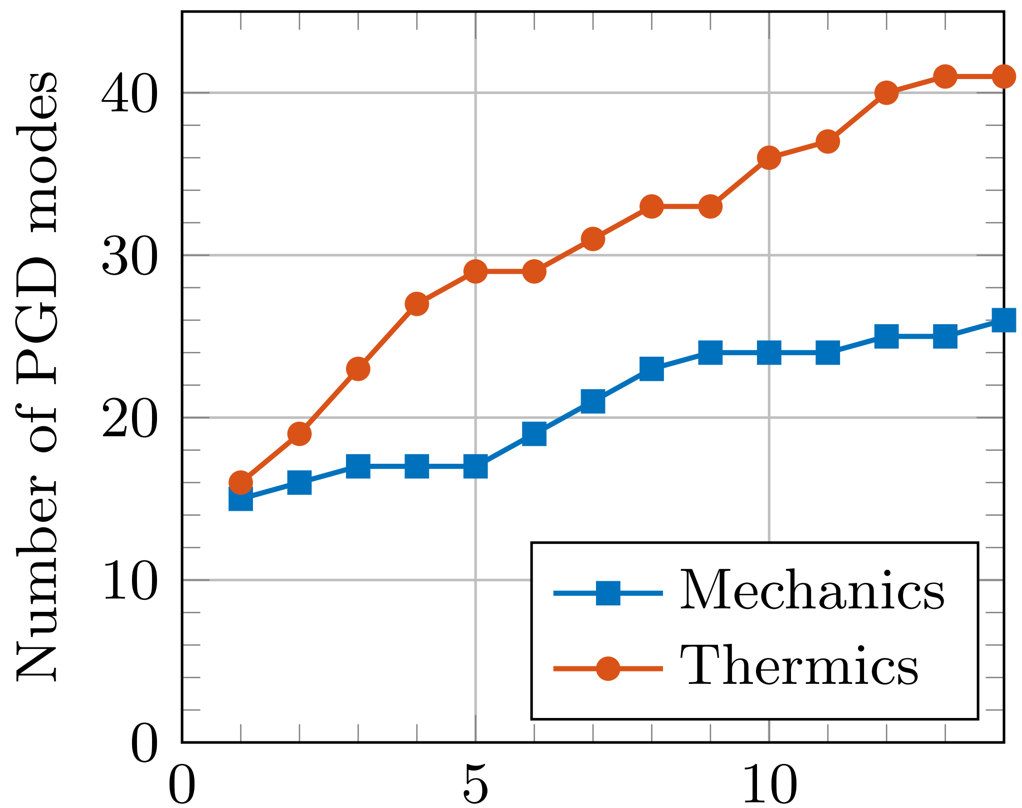


Parametric study

- parameter thermal expansion parameter β (influence the coupling between the 2 physics)
- 14 values of the parameter in the range of variation



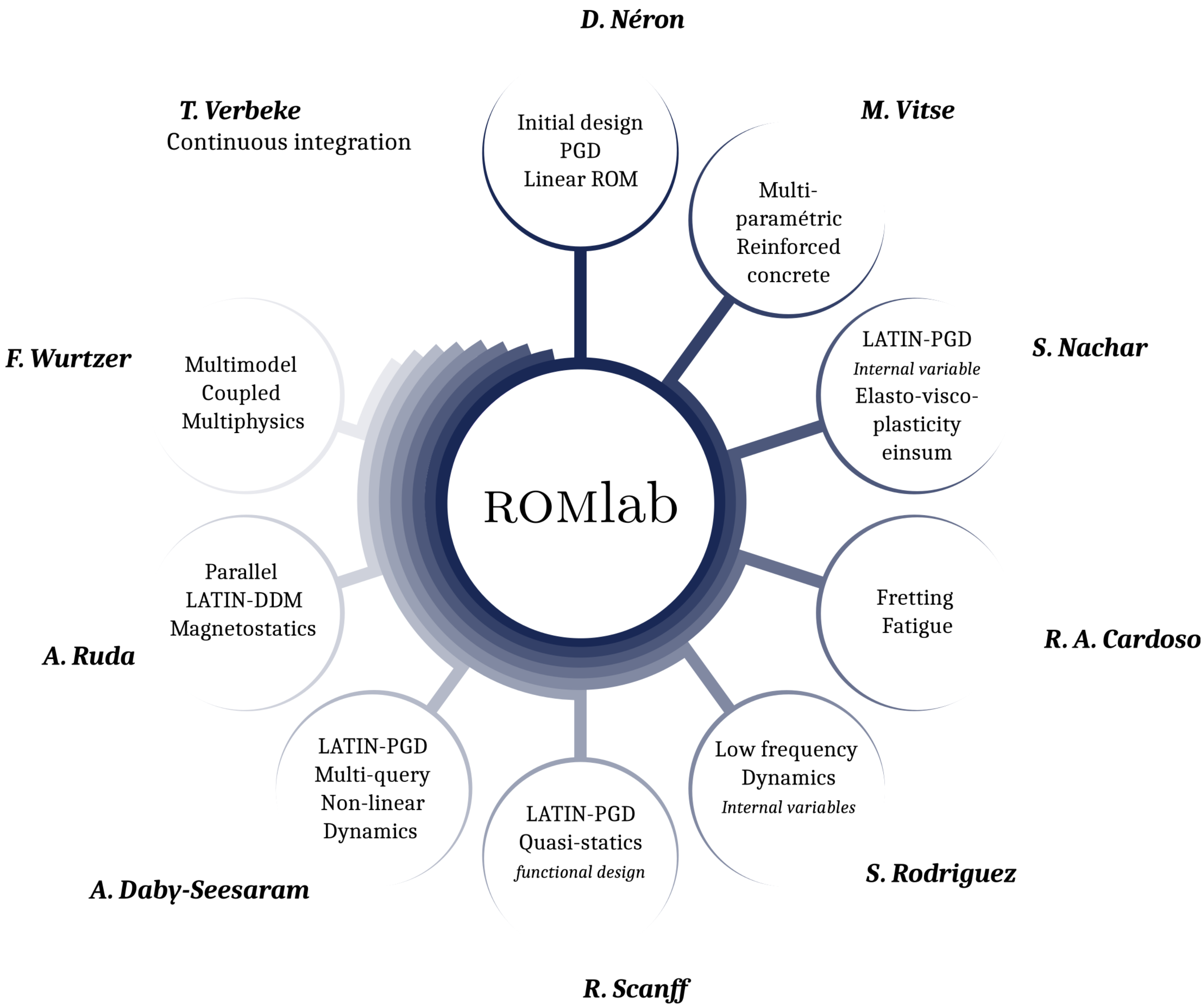




Outline

LMPS

Implementation



Implementation

« Hey guy, what about my favorite industrial finite element software? »

Talk of Pierre-Eliot Malleval
(coll. SIEMENS)

LMPS

Conclusion and prospects

- **LATIN-PGD**
  - to solve a variety of nonlinear problems
- **Reduced-Order Modeling**
  - allows to build new methods for high performance computing
  - concept of virtual charts (overall fields, not only metamodels) opens new perspectives in terms of engineering design
- **Numerical certification using high-fidelity models**
  - available in engineering sciences now reproduce accurately complex physics
  - but direct handling is completely impossible due to CPU time and big data issues
  - implementation or coupling with existing softwares must not be overlooked!
- **Recent works in the nonlinear context**
  - computation of fragility curves (coll CEA)
  - simulation of frictional contact in wire ropes (coll IFP Energies Nouvelles)
  - native implementation in industrial optimisation software (coll SIEMENS)
  - coupling ROM with AI for non parametrisable geometries (coll SAFRAN)
  - coupling ROM with AI for multiphysics problems
  - …

Merci