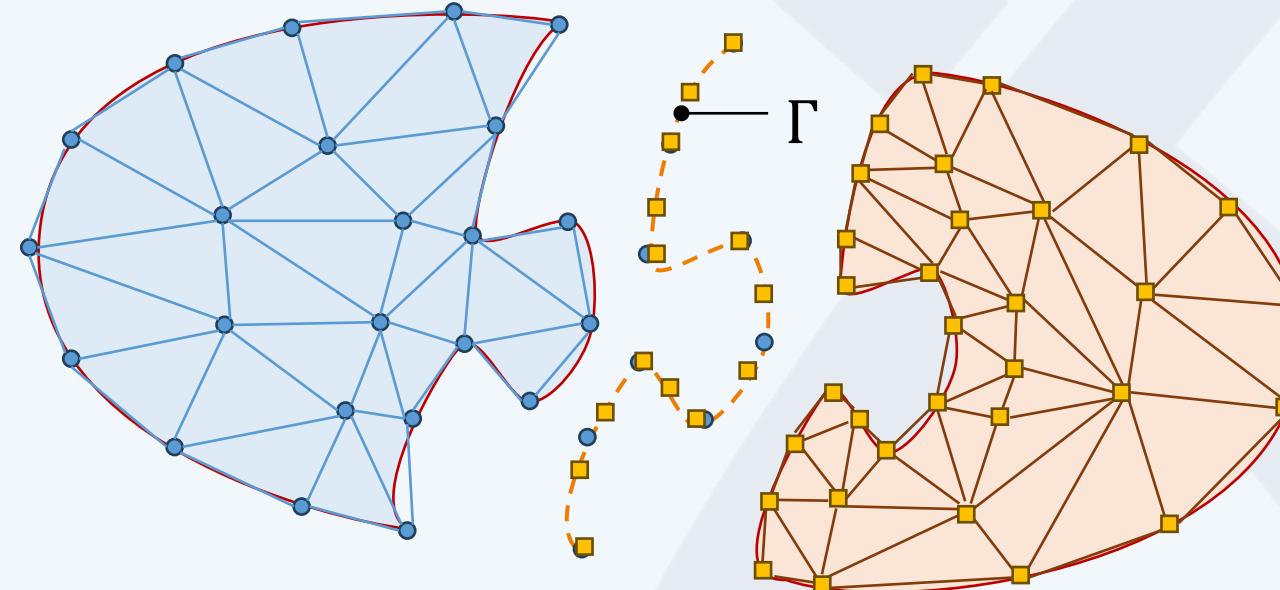
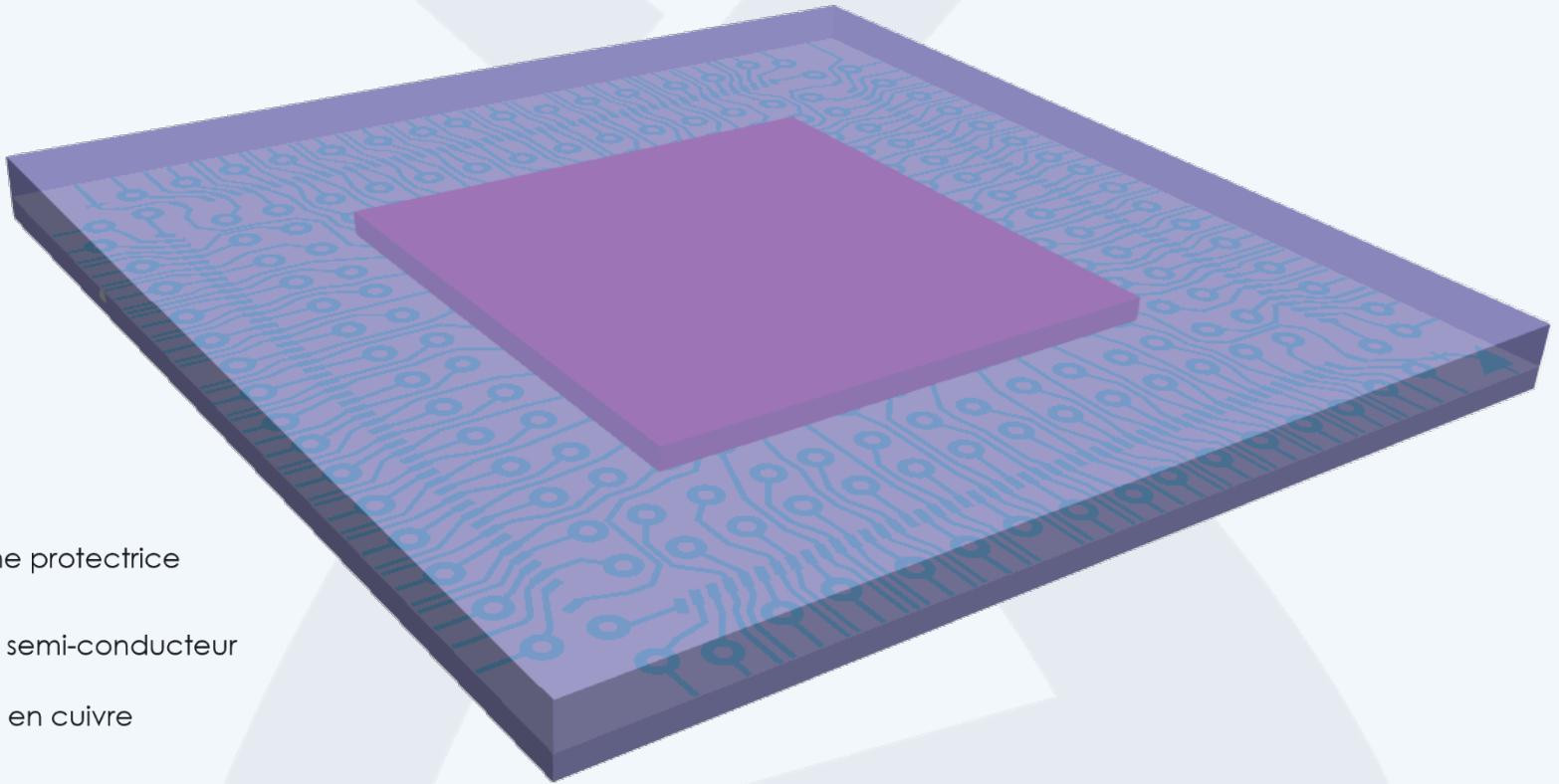
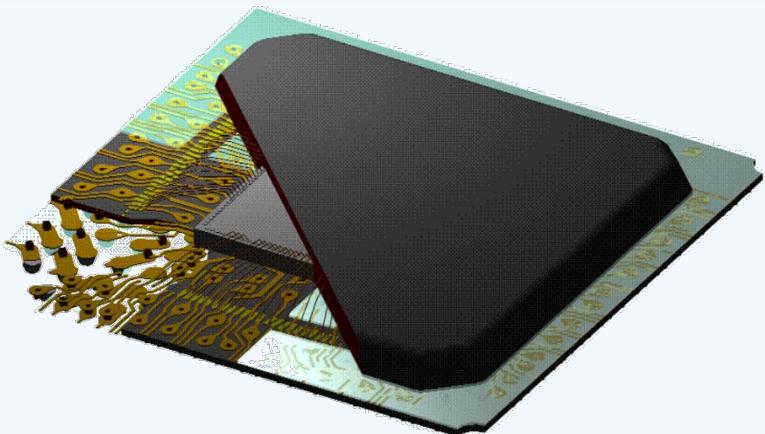


# Sous-structuration modale

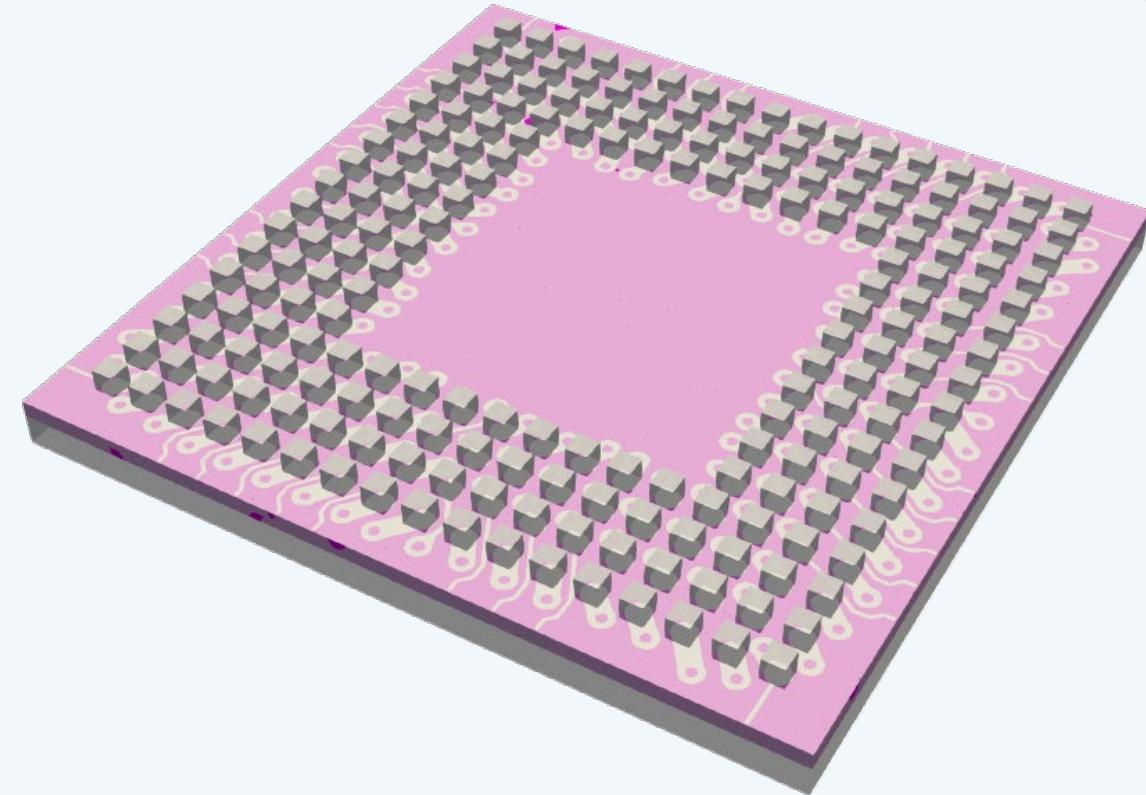


Frédéric Joly

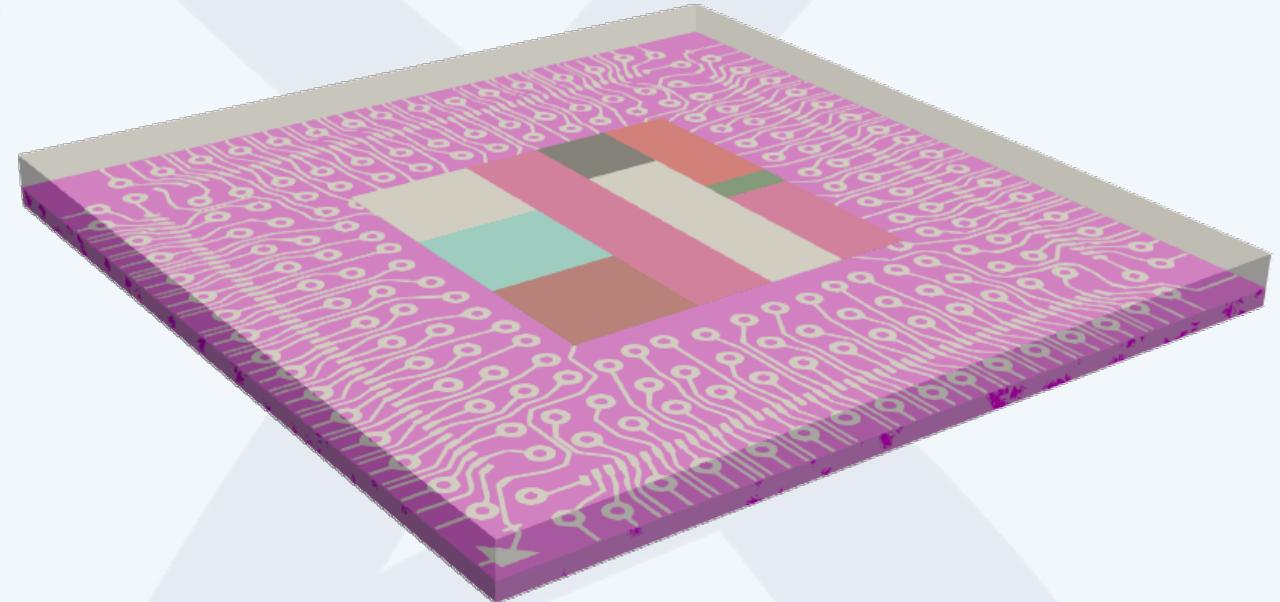
# Problématique : BGA 208



# Problématique : BGA 208



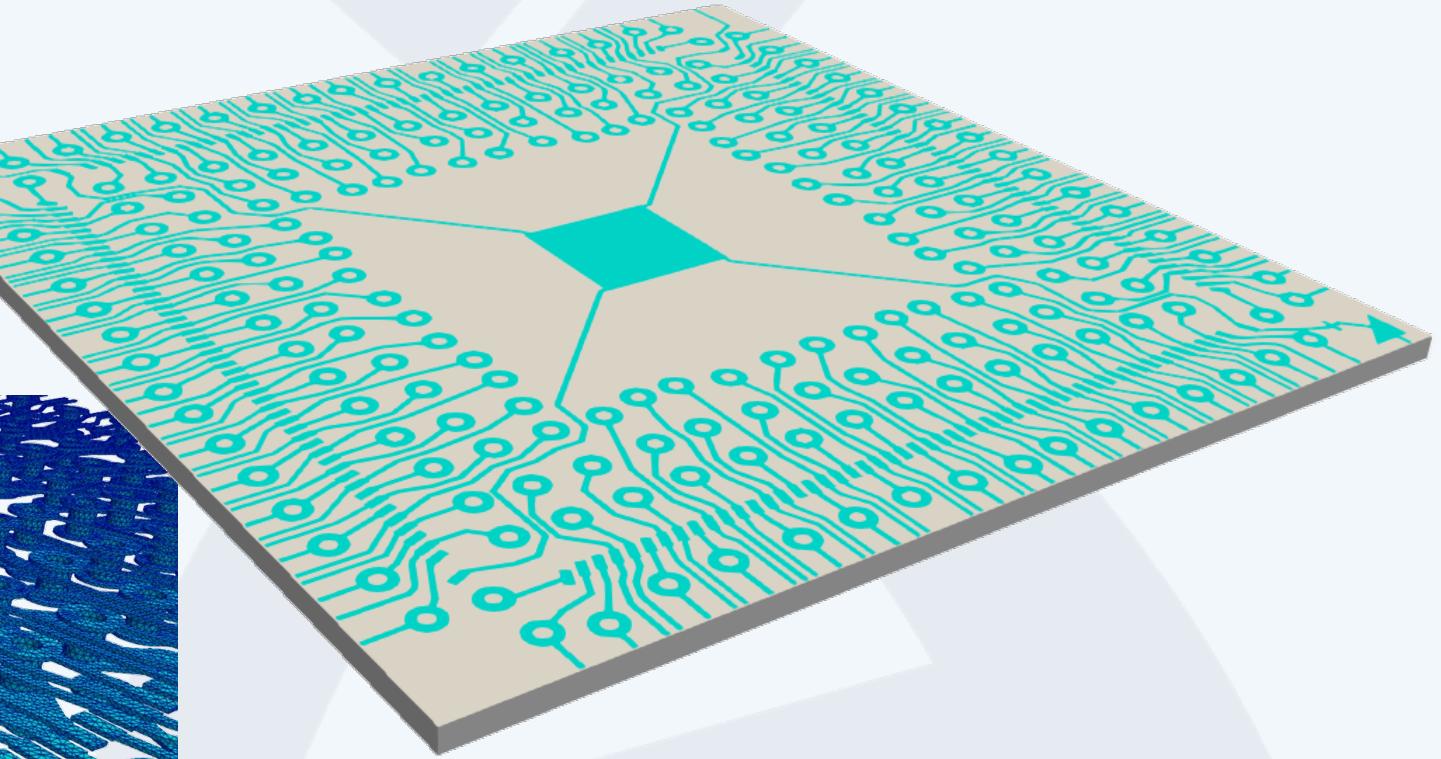
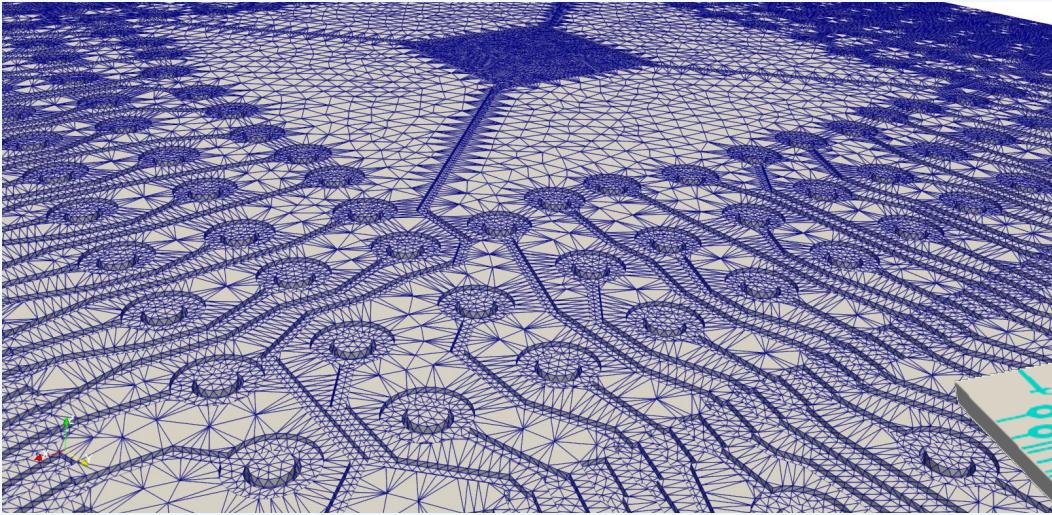
Vue de dessous



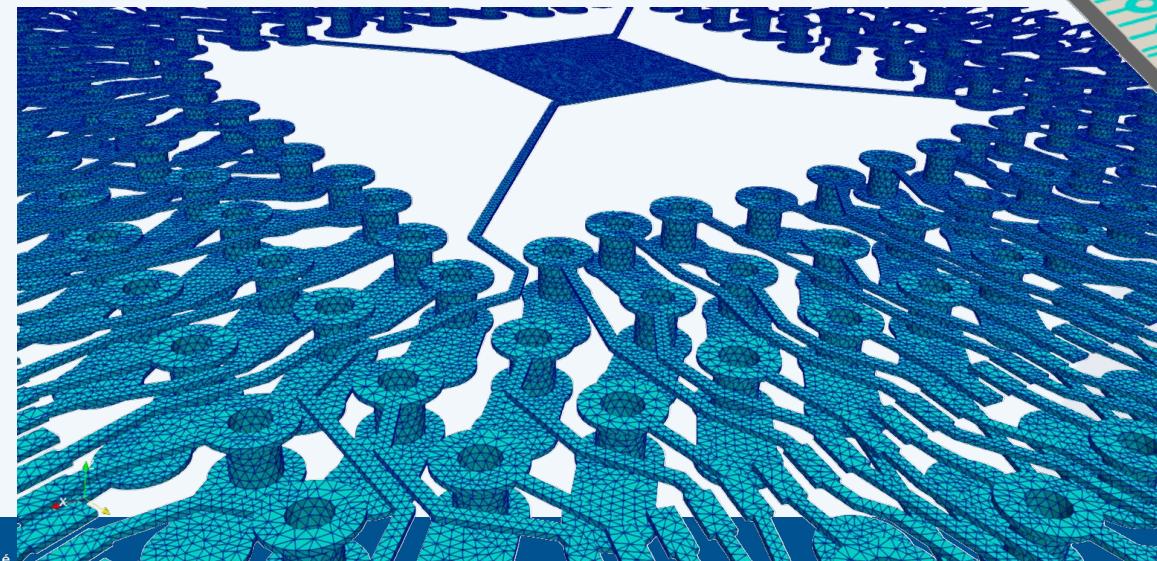
Vue de dessus



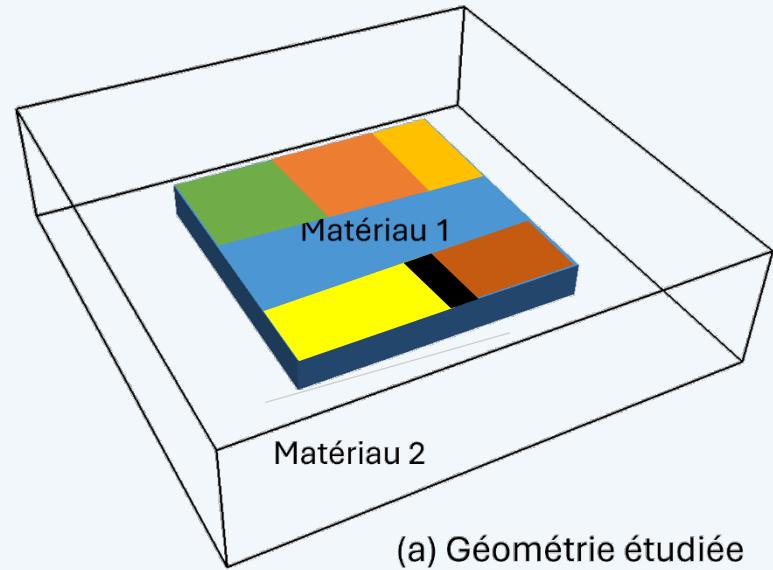
# Problématique : BGA 208



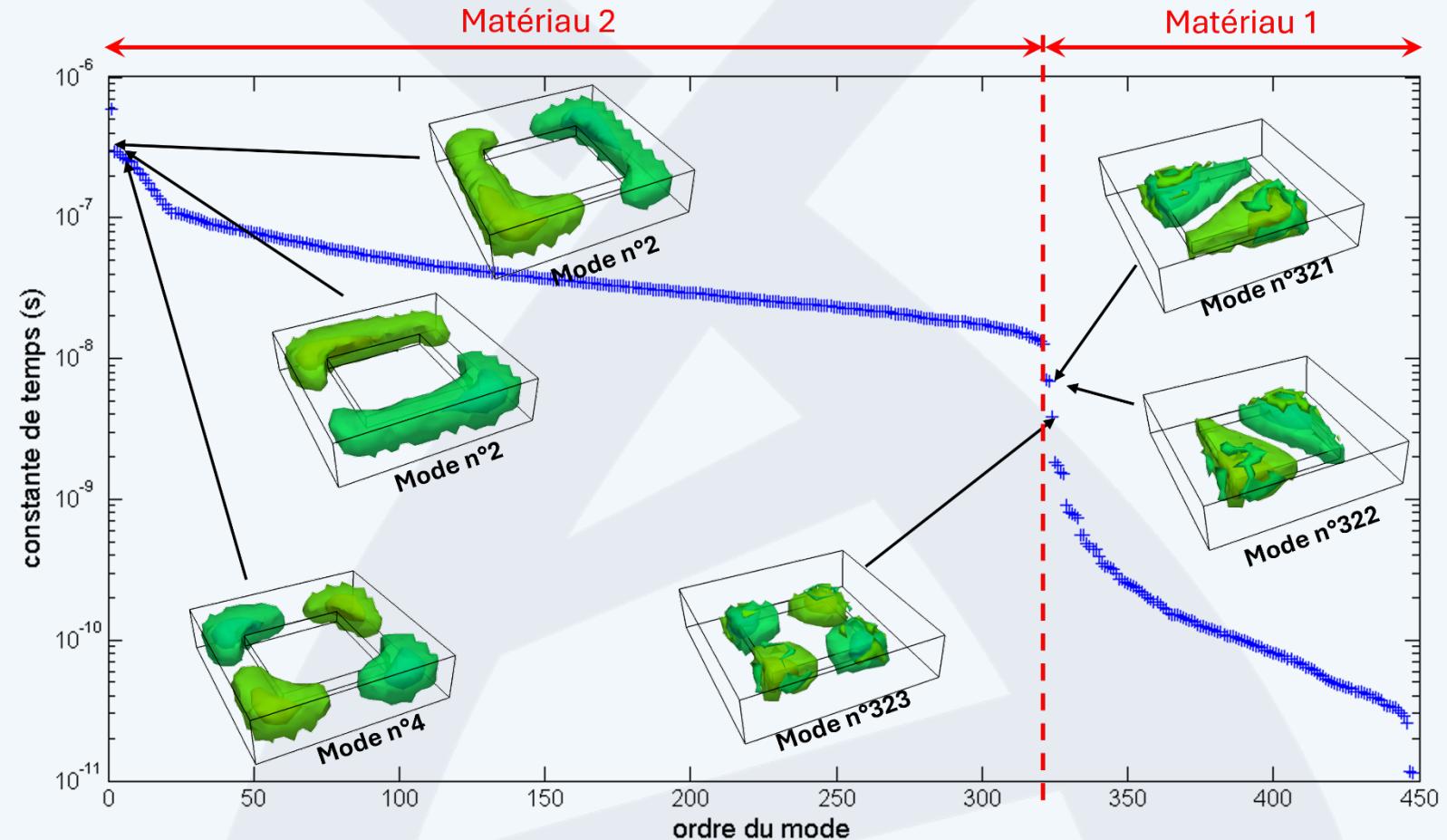
Environ 600 k degrés de liberté



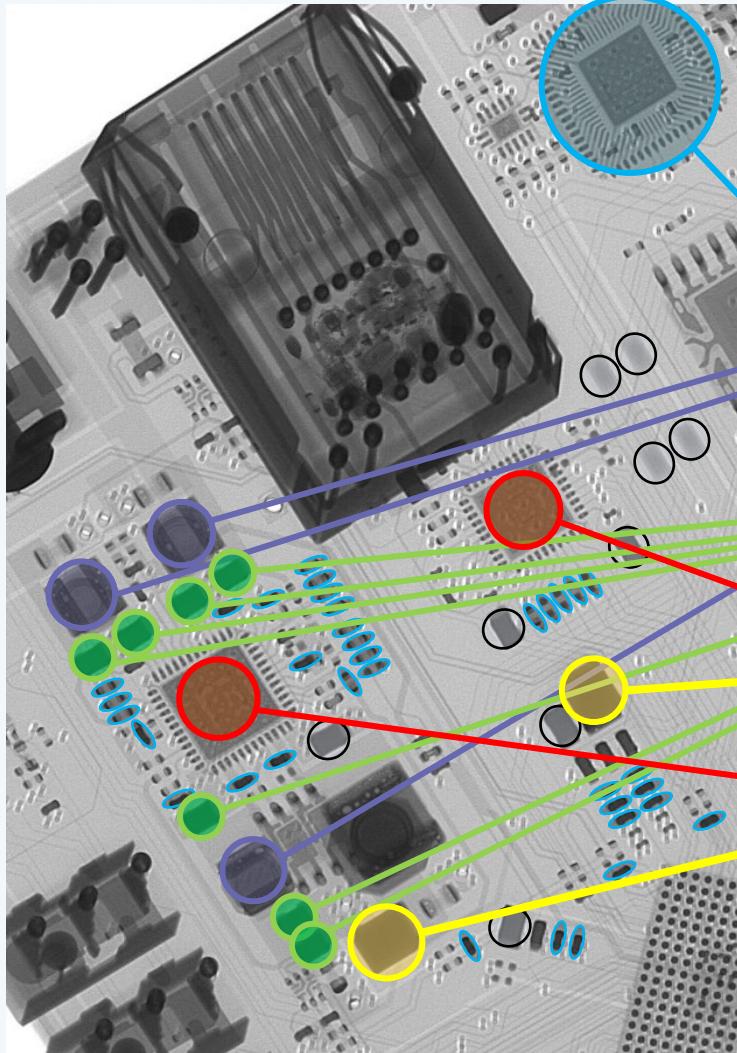
# Problématique : BGA 208



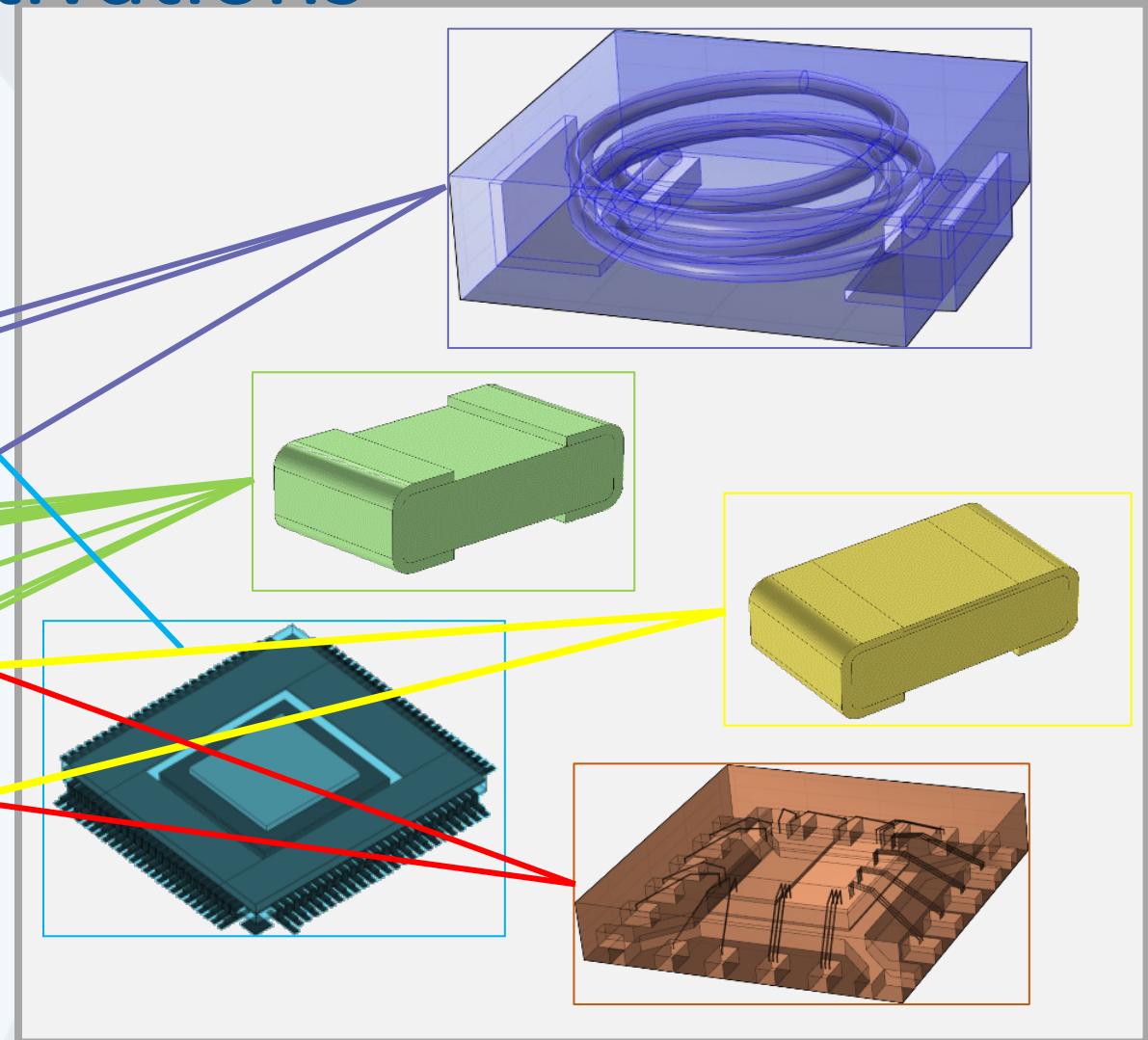
(a) Géométrie étudiée



# Motivations



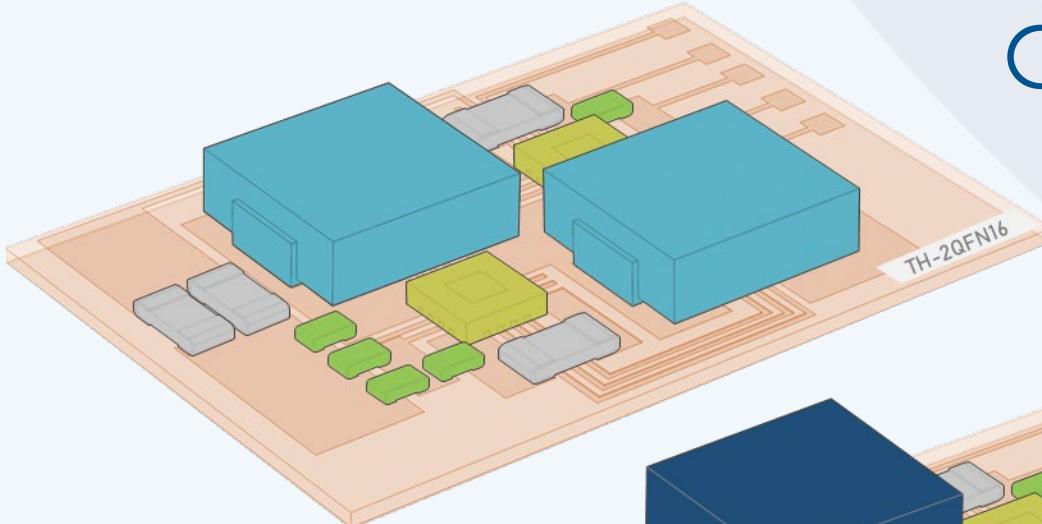
Carte électronique



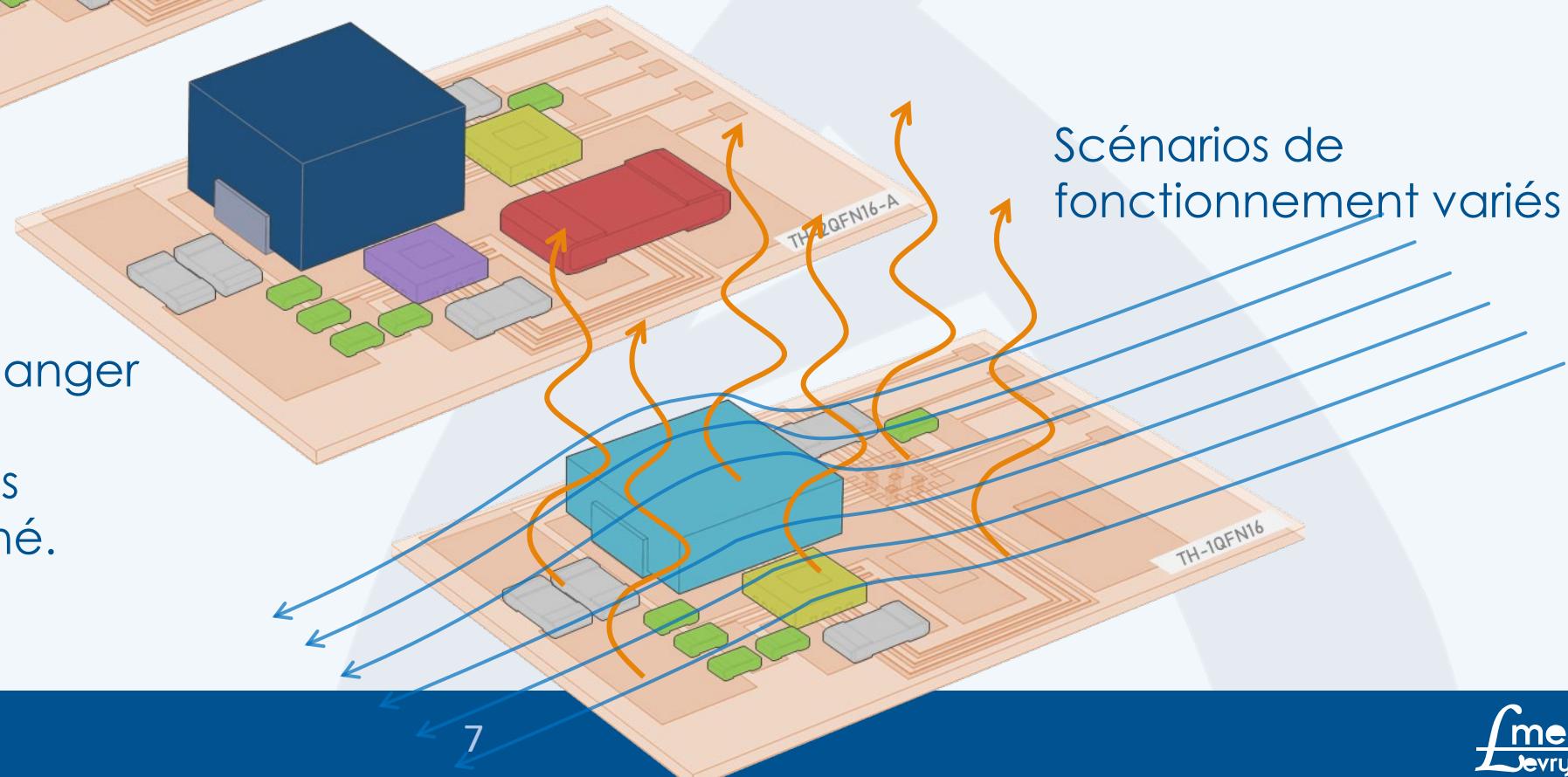
Bibliothèque ?

# Sous-structuration

Grande souplesse  
d'utilisation



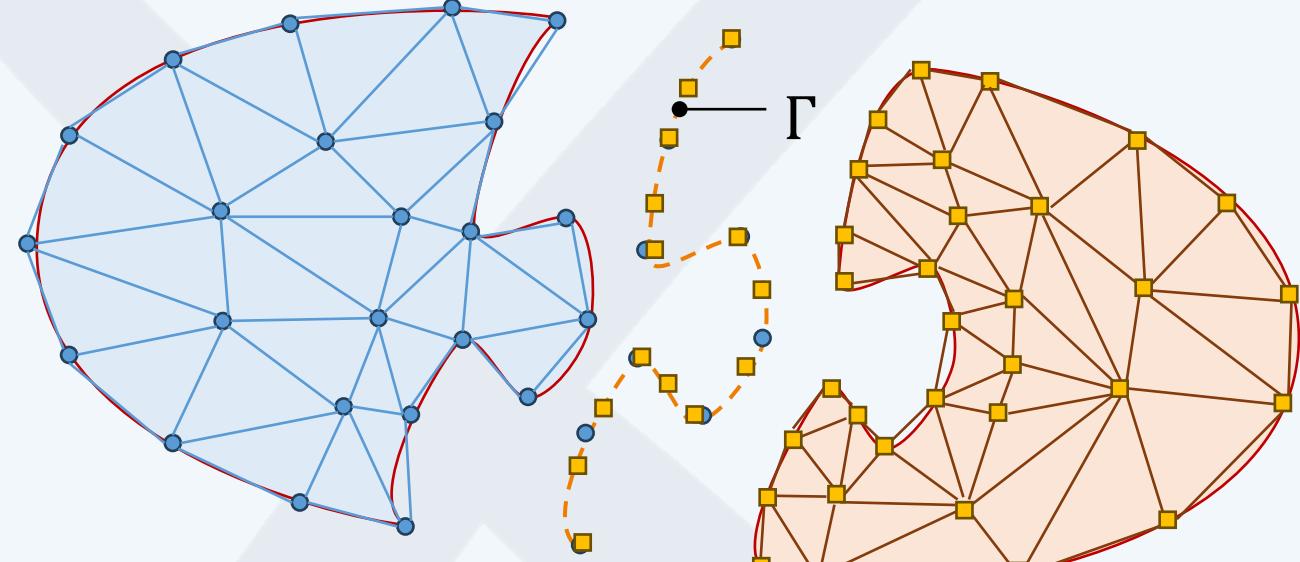
Possibilité de changer  
ou de retirer  
des composants  
sur un PCB donné.



# Principe

Équation de la chaleur :

$$\left\{ \begin{array}{l} c \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T) \\ \kappa \nabla T \cdot \underline{n} = h(T_f - T) \\ \kappa \nabla T \cdot \underline{n}_1 + \kappa \nabla T \cdot \underline{n}_2 = 0 \\ T_1 - T_2 = 0 \end{array} \right. \begin{array}{l} \text{sur } \Omega \\ \text{sur } \partial\Omega \\ \text{sur } \Gamma^{\{1,2\}} \end{array}$$



Formulation variationnelle sur les deux domaines :

$$\begin{aligned} \int_{\Omega_1} c \frac{\partial T}{\partial t} g_1 &= - \int_{\Omega_1} \nabla \cdot (\kappa \nabla T) g_1 + \int_{\partial\Omega} h(\nabla T \cdot \underline{n}) g_1 d\omega + \int_{\Gamma^{1,2}} \psi \underline{g}_1 T d\sigma_1 g_1 d\sigma \\ + \int_{\Omega_2} c \frac{\partial T}{\partial t} g_2 &= \int_{\Omega_2} \nabla \cdot (\kappa \nabla T) g_2 + \int_{\partial\Omega} h(\nabla T \cdot \underline{n}) g_2 d\omega + \int_{\Gamma^{2,1}} \psi \underline{g}_2 T d\sigma_2 g_2 d\sigma \end{aligned}$$

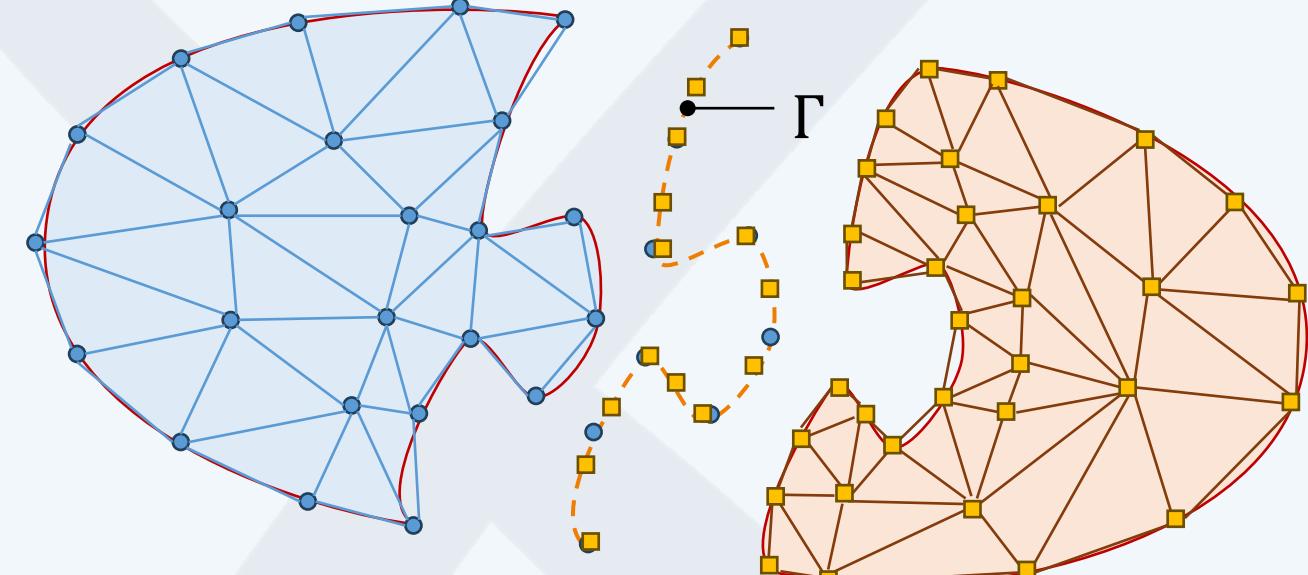
Introduction d'une variable flottante

$$\varphi = \underline{\kappa} \cdot \nabla T \cdot \underline{n}_2$$

# Interfaces non conformes

Équation de la chaleur :

$$\left. \begin{array}{l} c \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T) \\ \kappa \nabla T \cdot \underline{n} = h(T_f - T) \\ \kappa \nabla T \cdot \underline{n}_1 + \kappa \nabla T \cdot \underline{n}_2 = 0 \\ T_1 - T_2 = 0 \end{array} \right\} \begin{array}{l} \text{sur } \Omega \\ \text{sur } \partial\Omega \\ \text{sur } \Gamma^{\{1,2\}} \end{array}$$



Formulation variationnelle sur les deux domaines :

$$\int_{\Omega} c \frac{\partial T}{\partial t} g = - \int_{\Omega} \kappa \nabla T \cdot \nabla g + \int_{\partial\Omega} h(T_f - T) g d\sigma + \int_{\Gamma} \varphi [[g]] d\sigma$$

**continuité au sens faible**

$$\int_{\Gamma} [[T]]_{\Gamma} \phi_i = 0$$

**Saut de fonction test**

**Saut de température**

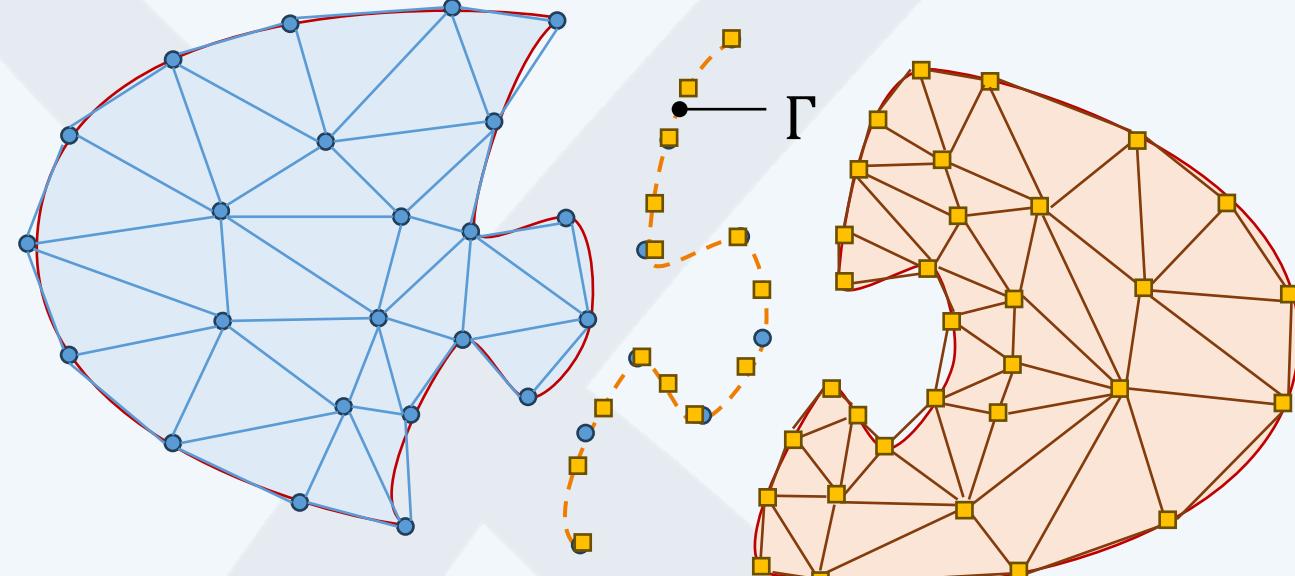
# Interfaces non conformes

Équation de la chaleur :

$$\left. \begin{array}{l} c \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T) \\ \kappa \nabla T \cdot \underline{n} = h(T_f - T) \\ \kappa \nabla T \cdot \underline{n}_1 + \kappa \nabla T \cdot \underline{n}_2 = 0 \\ T_1 - T_2 = 0 \end{array} \right\} \text{sur } \Gamma^{\{1,2\}}$$

sur  $\Omega$

sur  $\partial\Omega$



Formulation variationnelle sur les deux domaines :

$$\int_{\Omega} c \frac{\partial T}{\partial t} g = - \int_{\Omega} \nabla T \cdot \underline{\kappa} \cdot \nabla g + \int_{\partial\Omega} h(T_f - T) g d\sigma + \int_{\Gamma} \varphi \llbracket g \rrbracket d\sigma$$

Nouvelles inconnues

$$\int_{\Gamma} \llbracket T \rrbracket_{\Gamma} \phi_i = 0$$

Fonctions spatiales  
définies sur une interface  
**Fonctions mortiers**

# Interfaces non conformes

$$\int_{\Omega} c \frac{\partial T}{\partial t} g = - \int_{\Omega} \underline{\nabla} T \cdot \underline{\kappa} \cdot \underline{\nabla} g + \int_{\partial\Omega} h(T_f - T) g d\sigma + \int_{\Gamma} \varphi \llbracket g \rrbracket d\sigma$$

$$\int_{\Gamma} \llbracket T \rrbracket_{\Gamma} \phi_i = 0$$

$$\varphi(x, t) = \sum_i \varphi_i(t) \phi_i(x)$$

Nouvelles inconnues

Fonctions spatiales  
définies sur l'interface

**Fonctions mortiers**



# Interfaces non conformes

$$\int_{\Omega} c \frac{\partial T}{\partial t} g = - \int_{\Omega} \underline{\nabla} T \cdot \underline{\kappa} \cdot \underline{\nabla} g + \int_{\partial\Omega} h(T_f - T) g d\sigma + \int_{\Gamma} \varphi \llbracket g \rrbracket d\sigma$$

$$\int_{\Gamma} \llbracket T \rrbracket_{\Gamma} \phi_i = 0$$

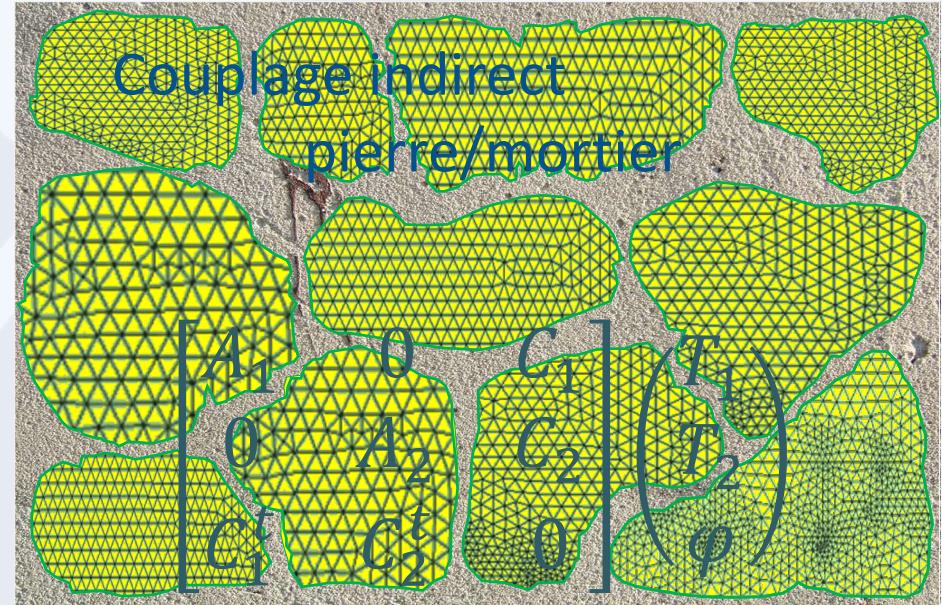
$$\varphi(x, t) = \sum_i \varphi_i(t) \phi_i(x)$$

Nouvelles inconnues

Fonctions spatiales  
définies sur l'interface

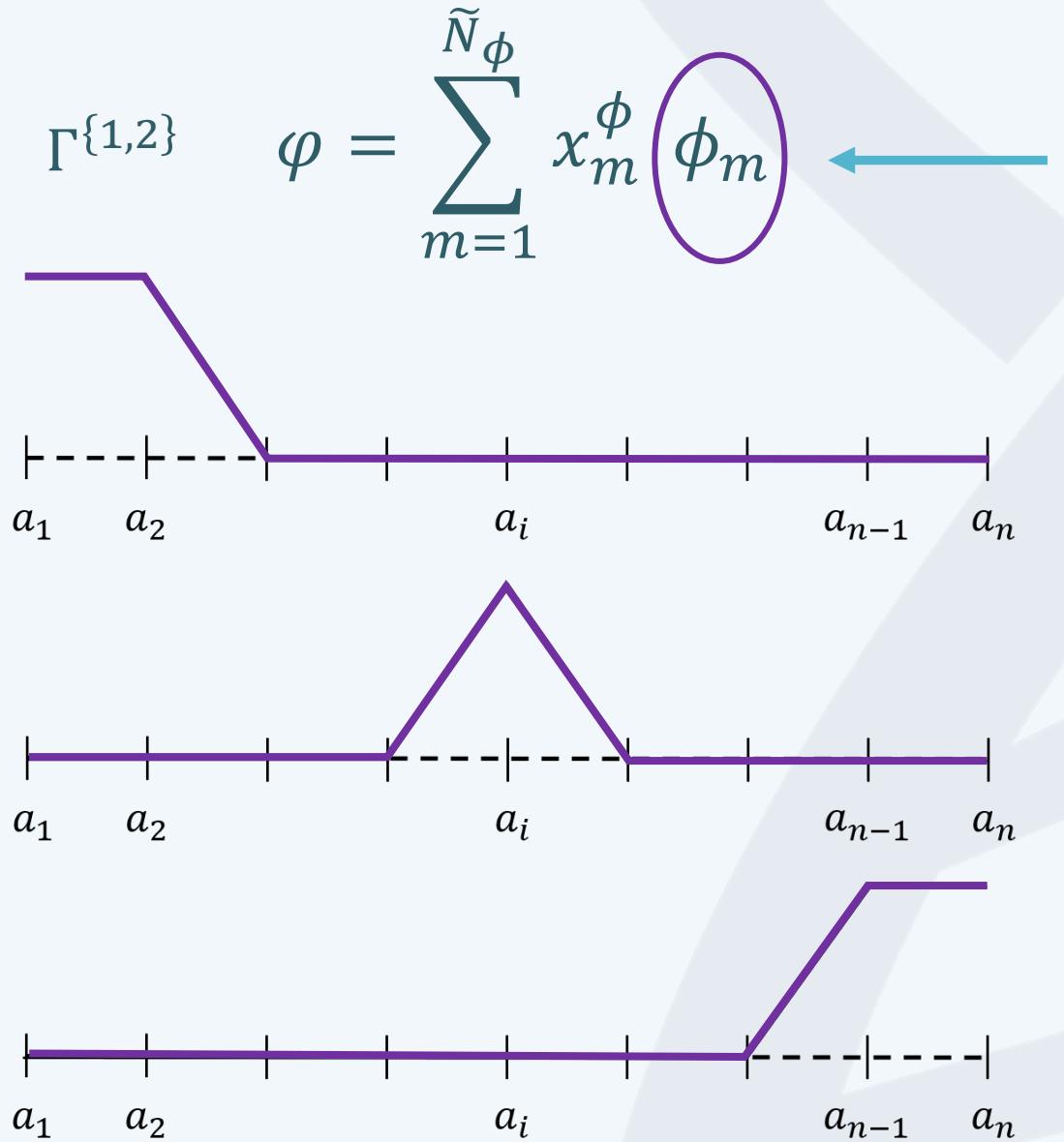
**Fonctions mortiers**

Ajout de matière



Formulation matricielle

# Quelles fonctions mortiers prendre ?

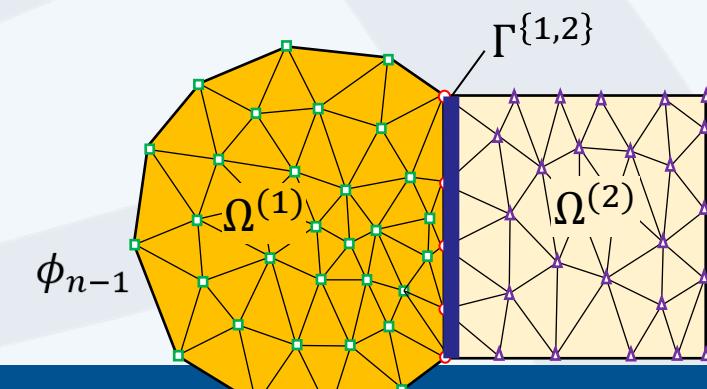


Fonctions ***mortiers***

$\phi_2$

Frontière 1D :  
Fonctions à flux nul  
sur le bord de  $\Gamma^{\{1,2\}}$

$\phi_i$



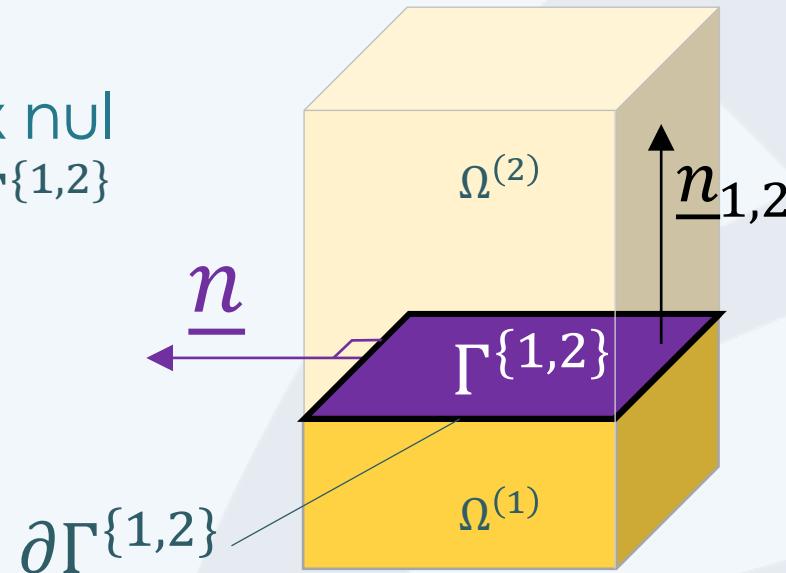
# Quelles fonctions mortiers prendre ?

$$\Gamma^{\{1,2\}} - \Delta V^N = \lambda^N V^N$$

$$\partial\Gamma^{\{1,2\}} \quad \nabla V^N \cdot \underline{n} = 0$$

Problème aux valeurs propres de  
Neumann

Fonctions à flux nul  
sur le bord de  $\Gamma^{\{1,2\}}$



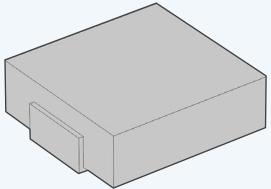
On peut réduire cette famille de modes propres

# Exemples de résultats

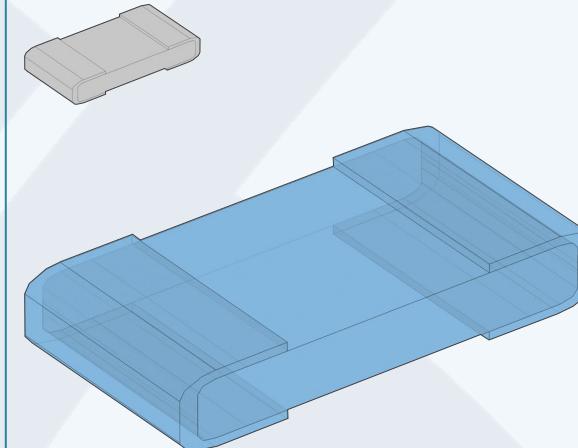
# Création de la bibliothèque

## 4 familles de Composants

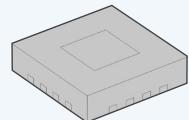
Inductance



Capacité

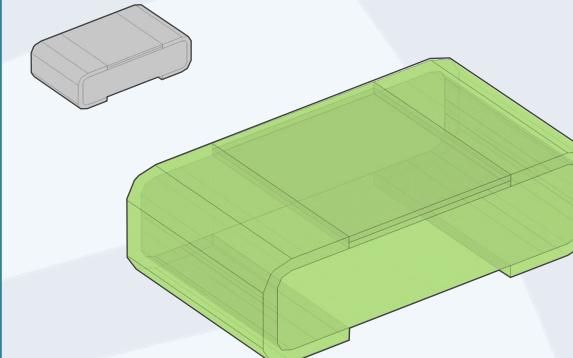


QFN16



Abaisseur de tension à découpage  
ISL 8026A

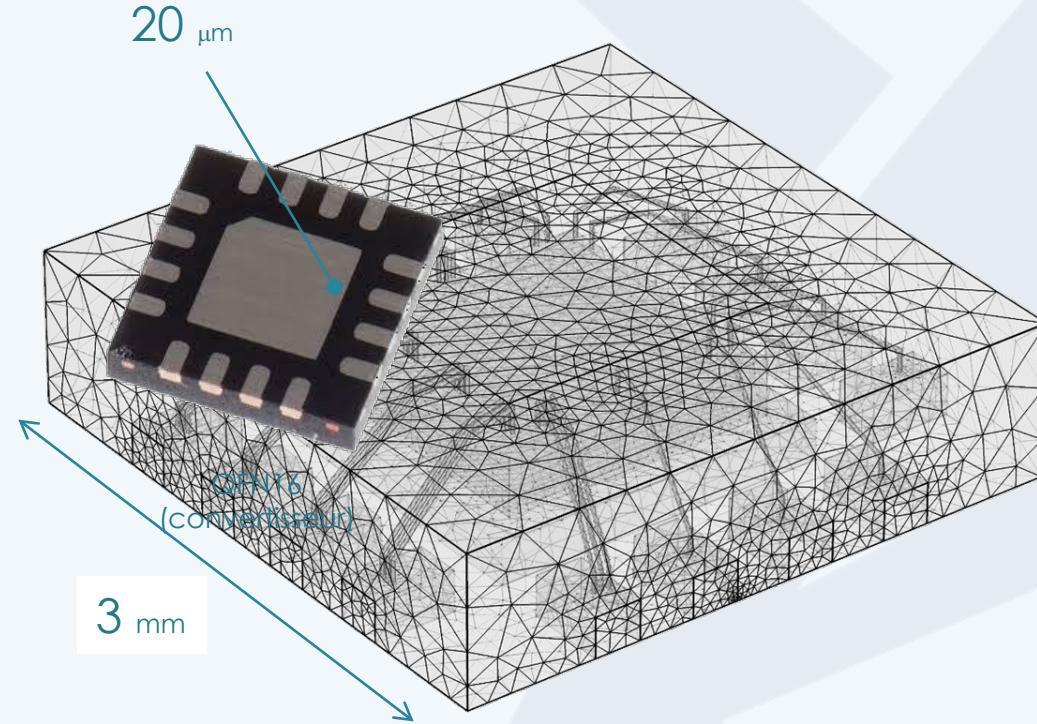
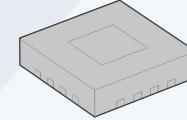
Résistance



# Création de la bibliothèque

Réduction d'un composant

## Abaisseur de tension à découpage

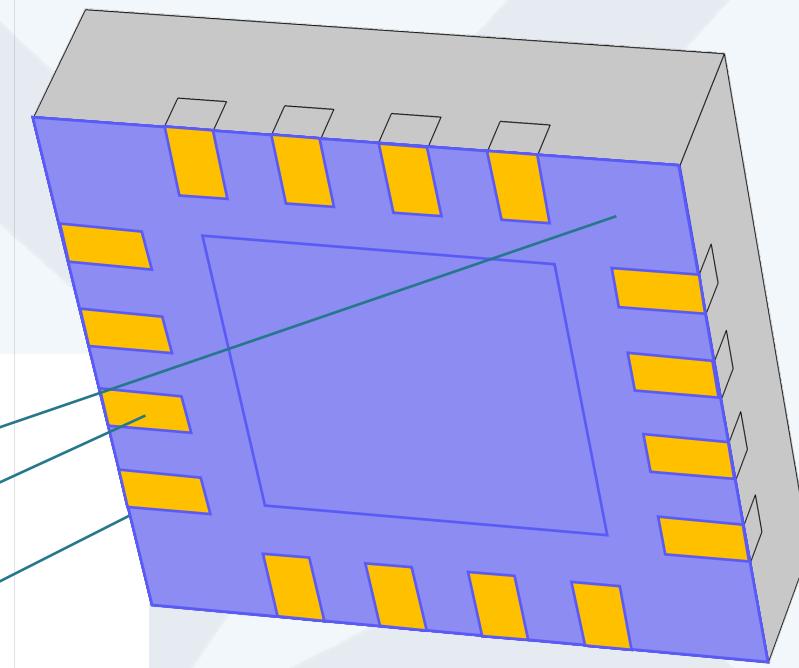
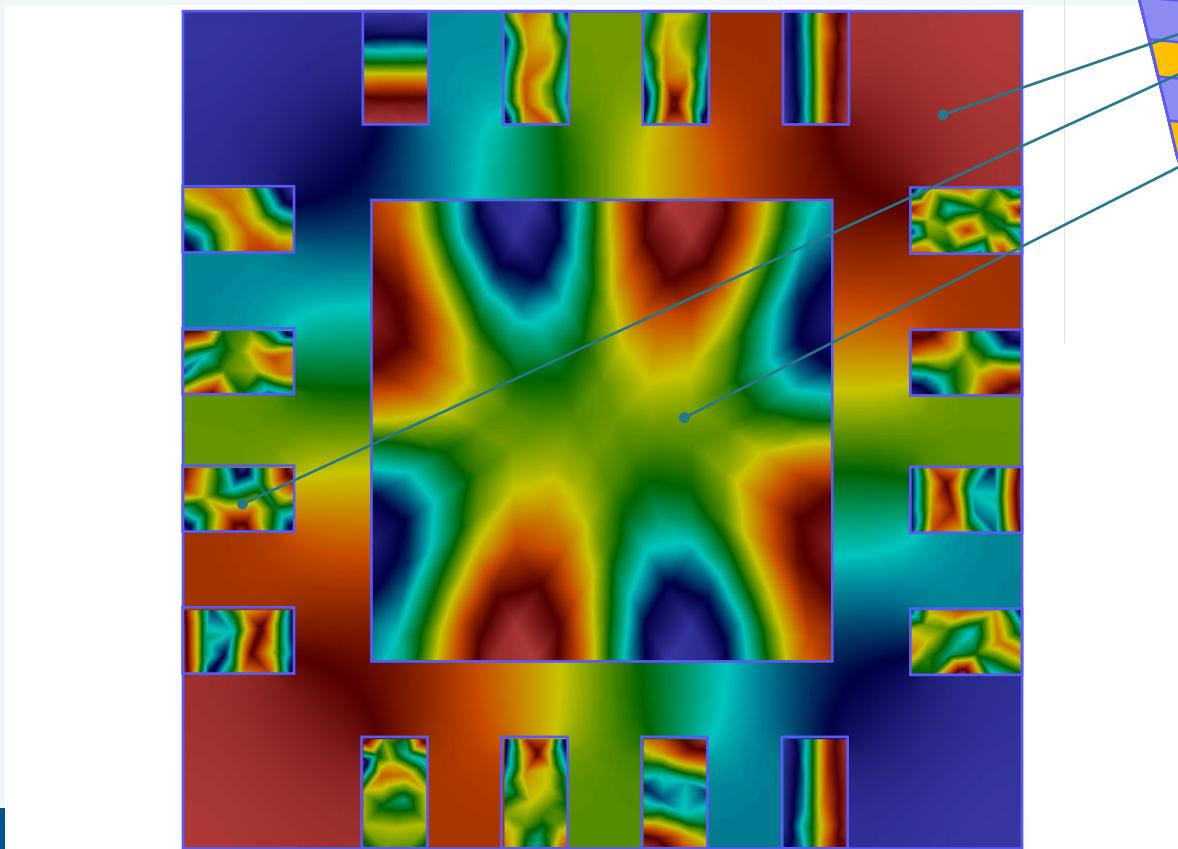


Calcul des modes  $\Rightarrow$  maillage :  $\sim 30\,000$  nœuds

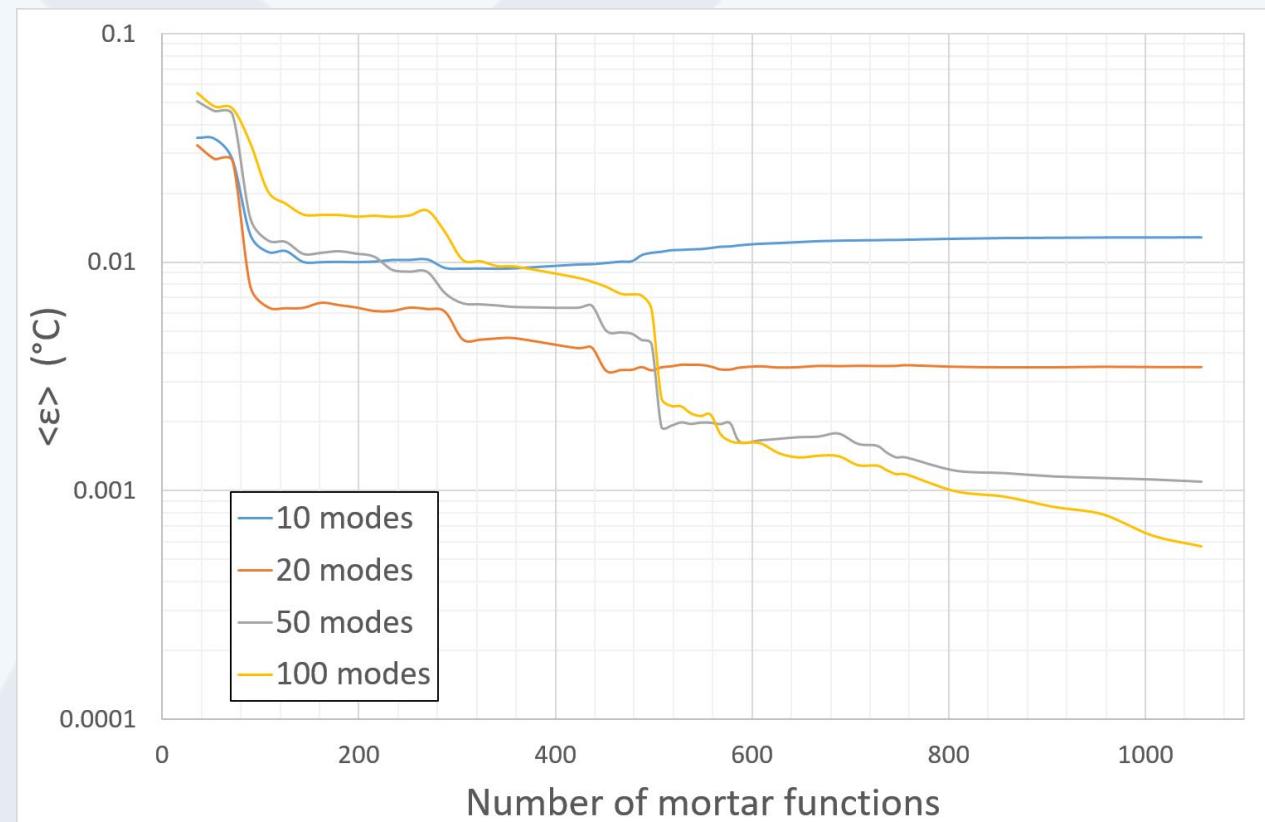
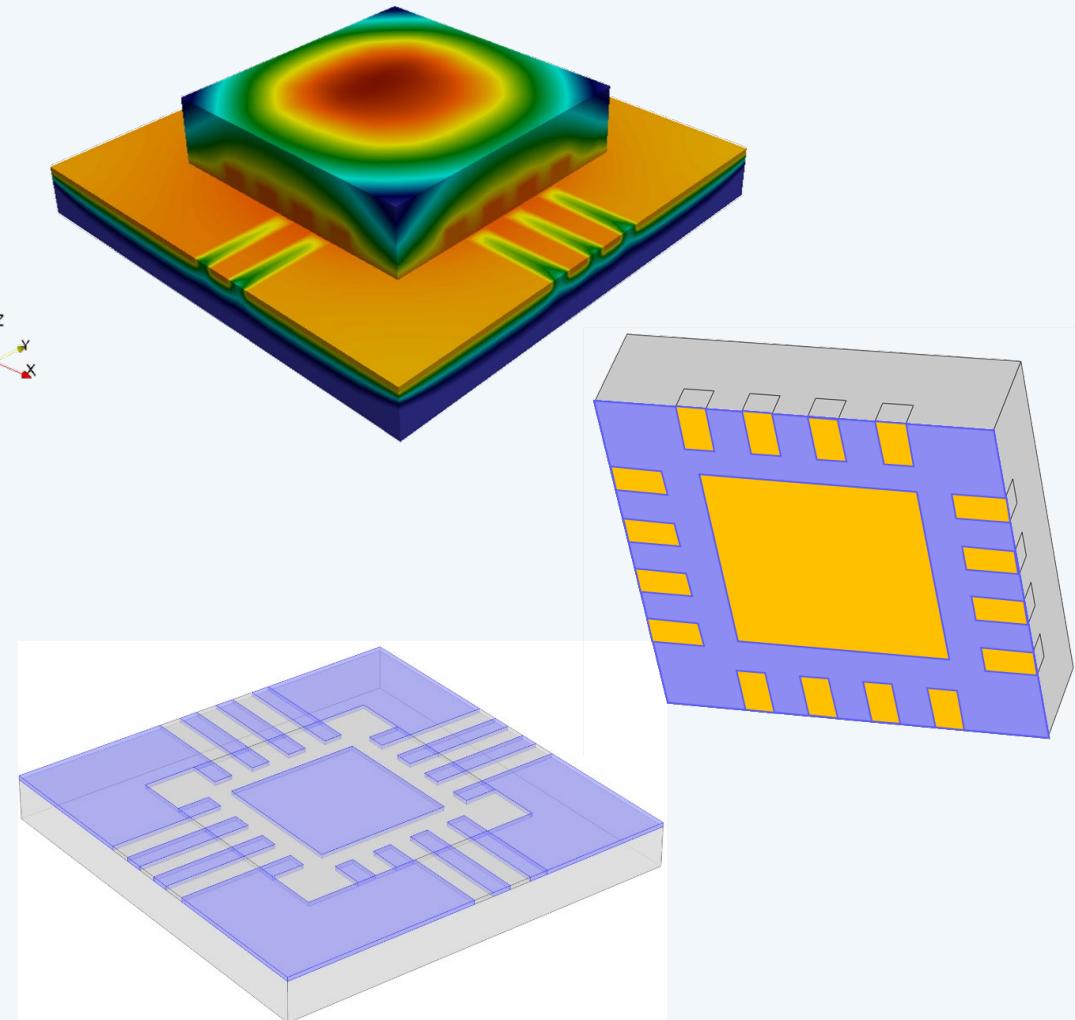
# Réduction d'un composant

Calcul des fonctions mortiers :  
*modes de Neumann*

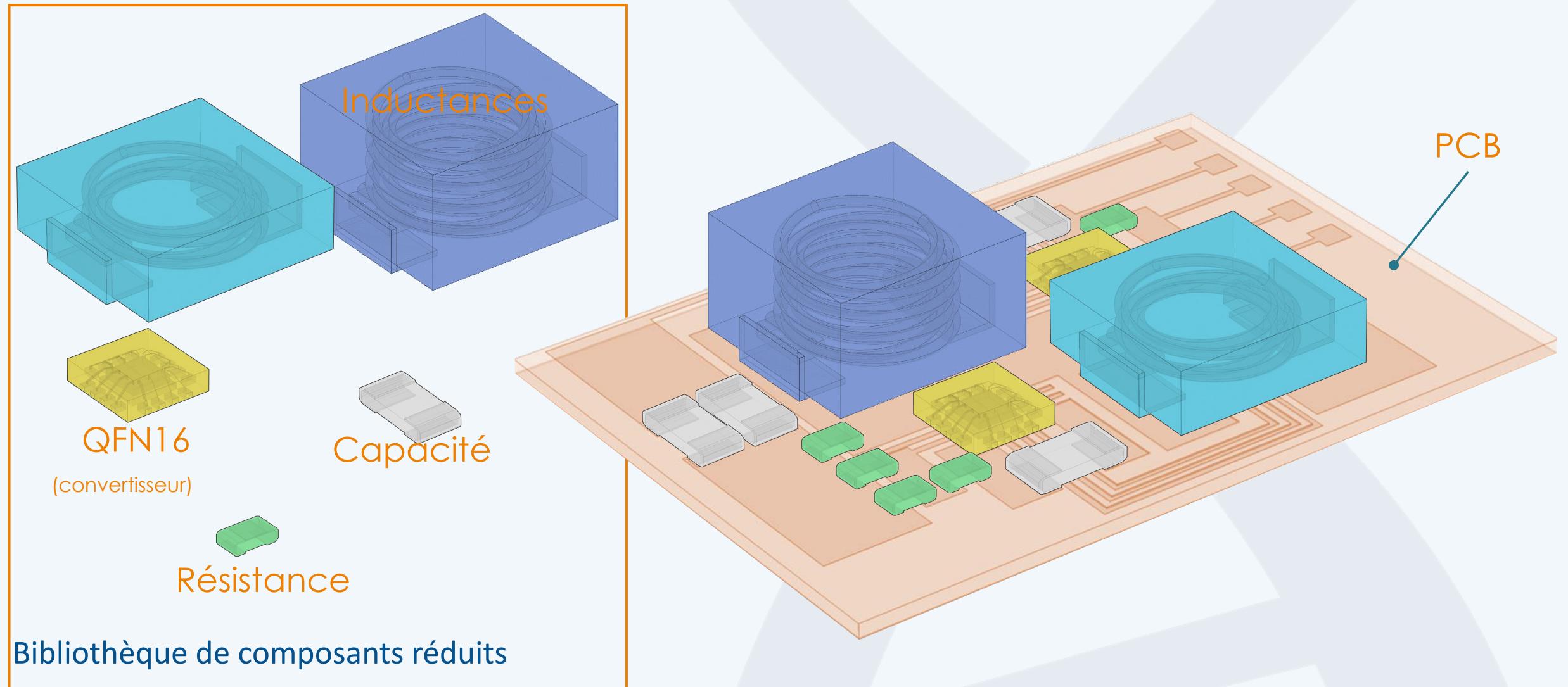
18 interfaces



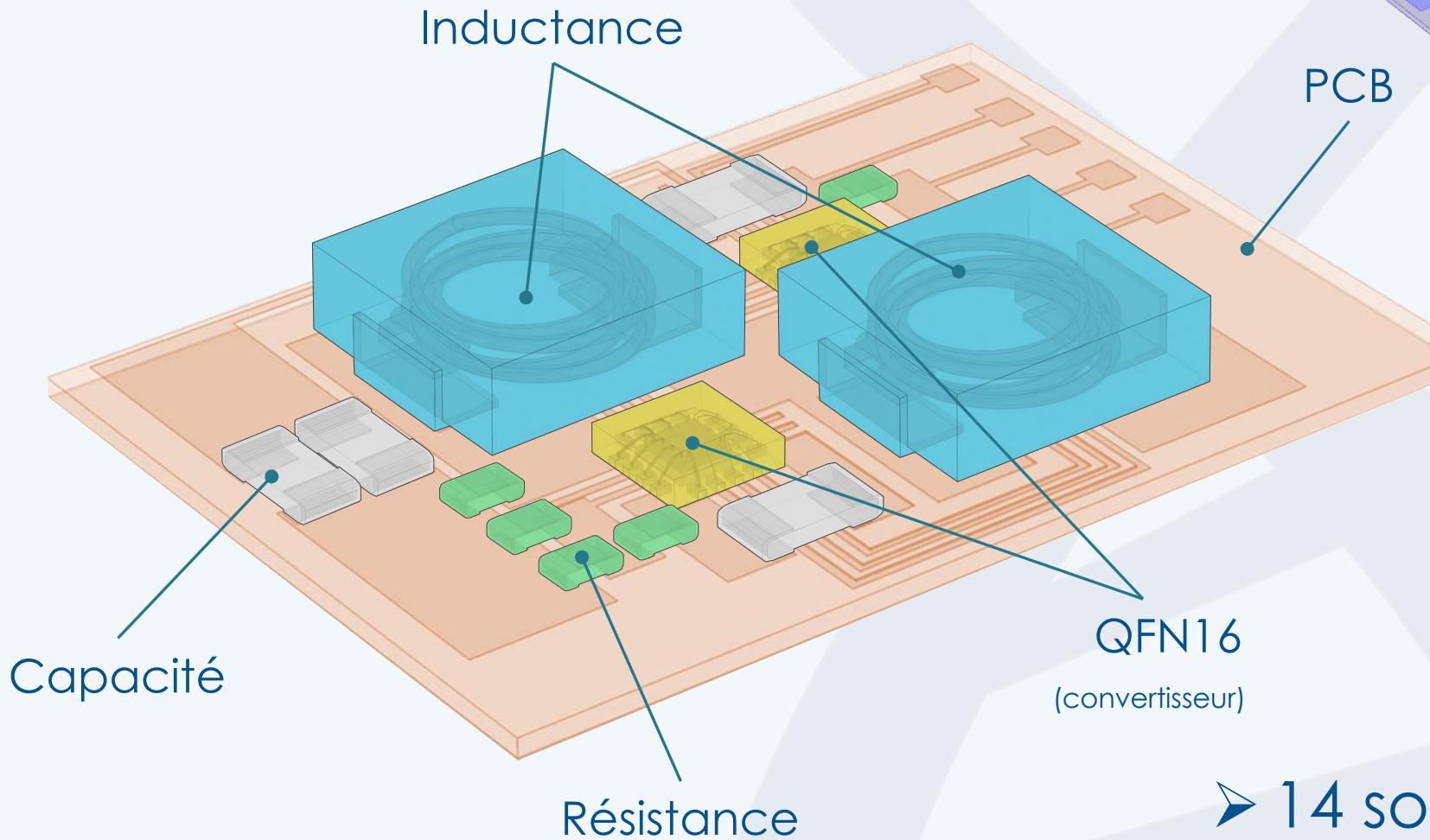
# Mini-carte



# Sous-structuration modale



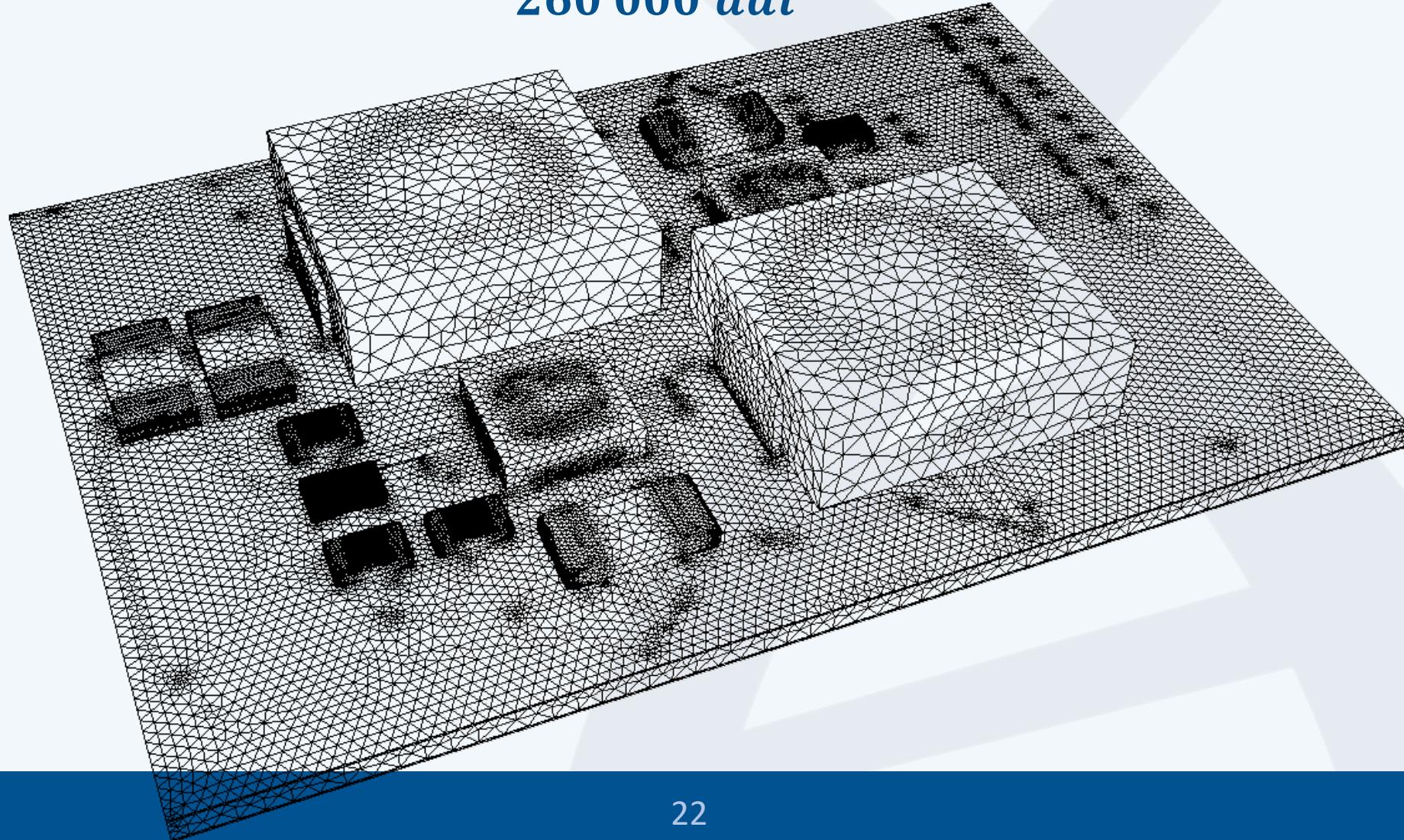
# Construction de la carte



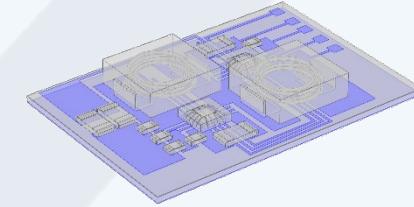
- 14 sous-structures
- 58 interfaces

# Construction de la carte

Nœuds + mortiers (fonctions de forme EF) :  
**260 000 *ddl***



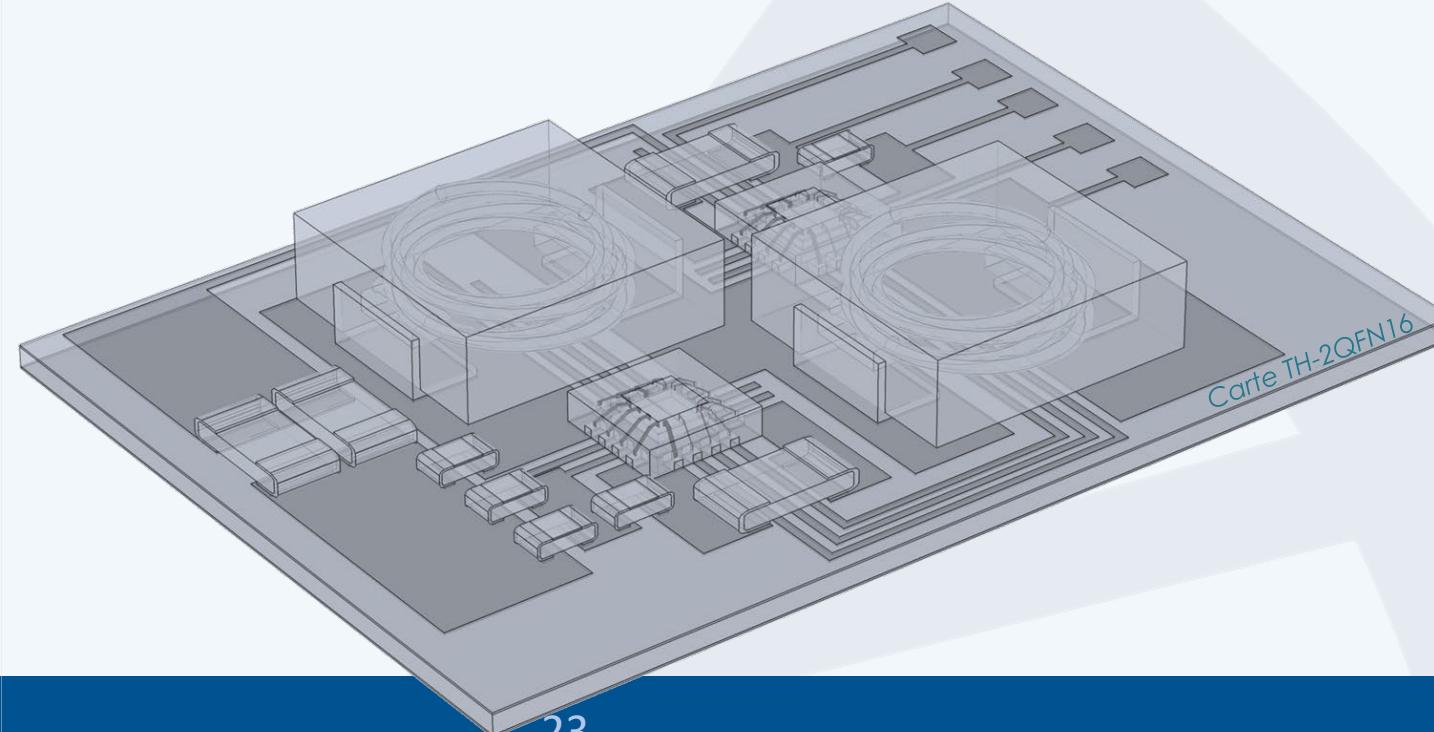
# Simulation modale sous-structurée



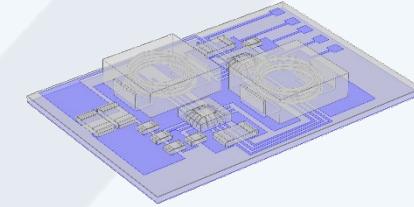
Taille du modèle réduit :

- 100 modes par composant actif et PCB
- 10 par composant passif
- 2 modes mortiers (Neumann) par interface :

**706 *ddl* vs 260 000 *ddl***

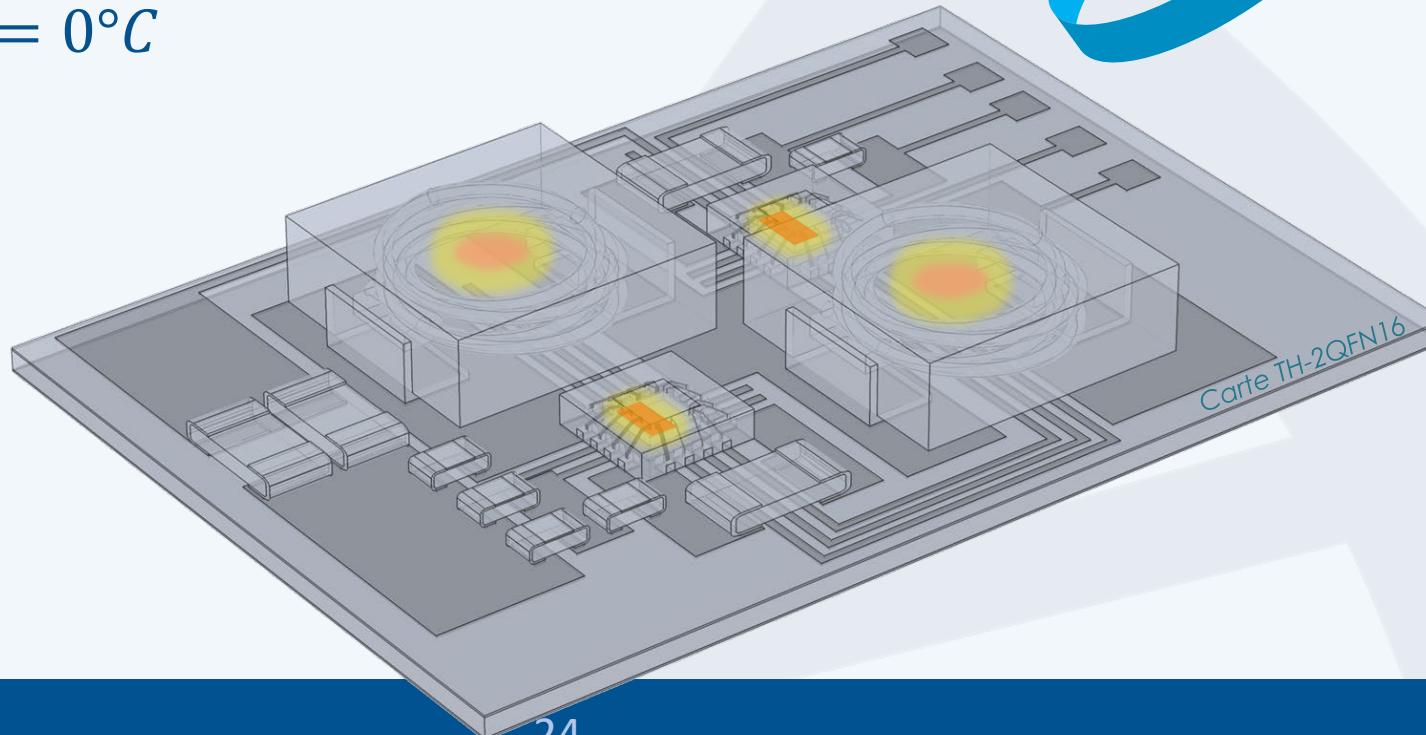


# Simulation modale sous-structurée

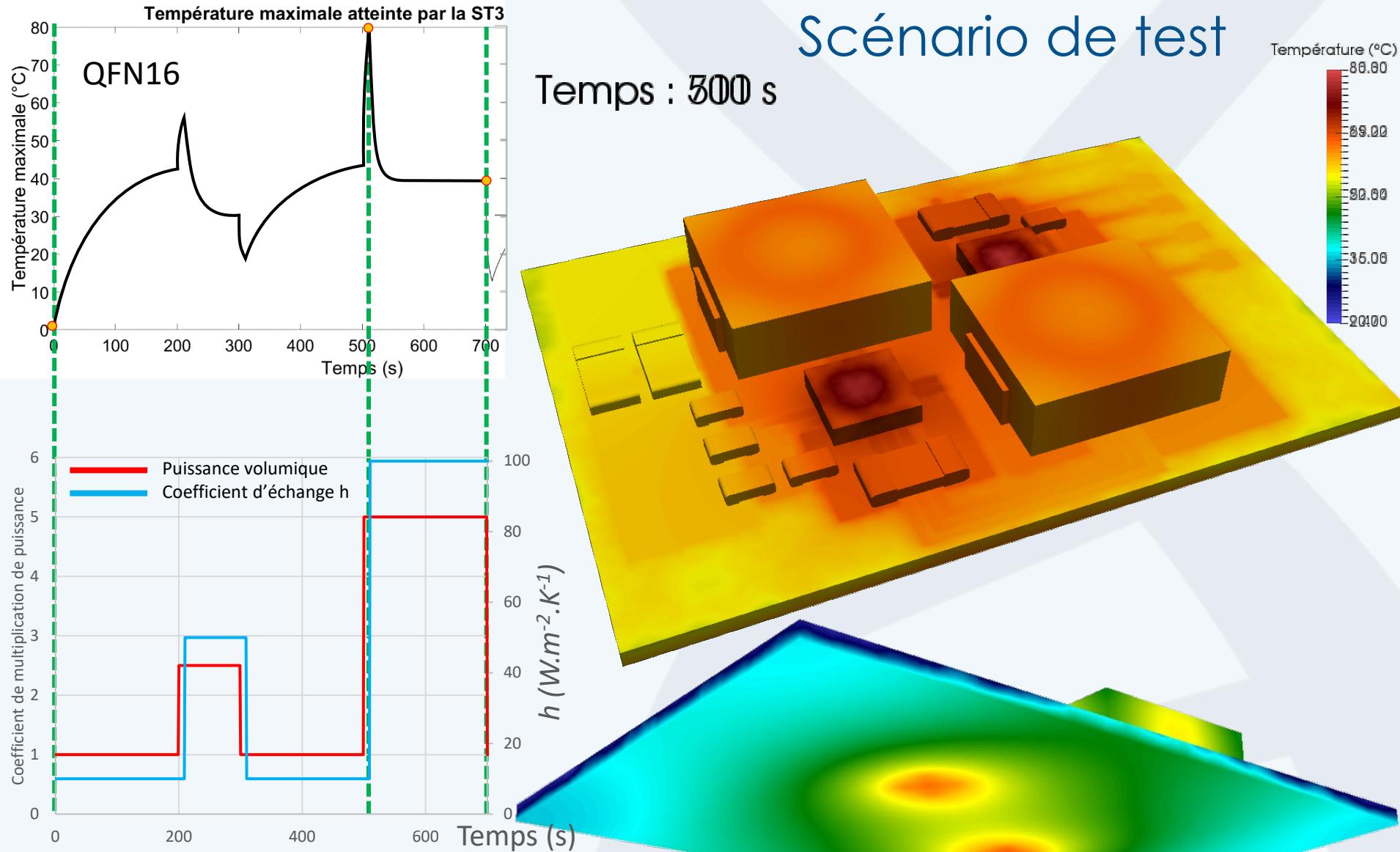


Paramètres de simulation : transitoire de 1000s

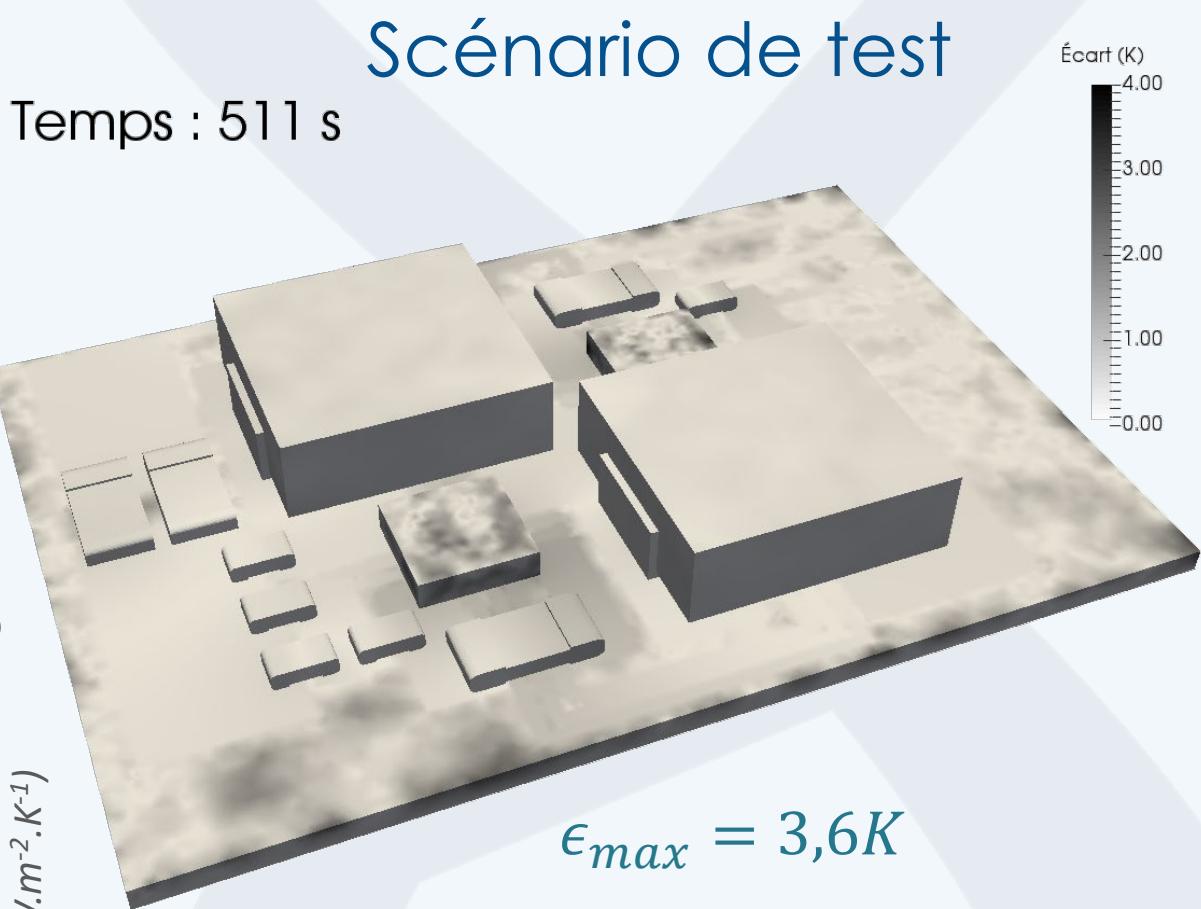
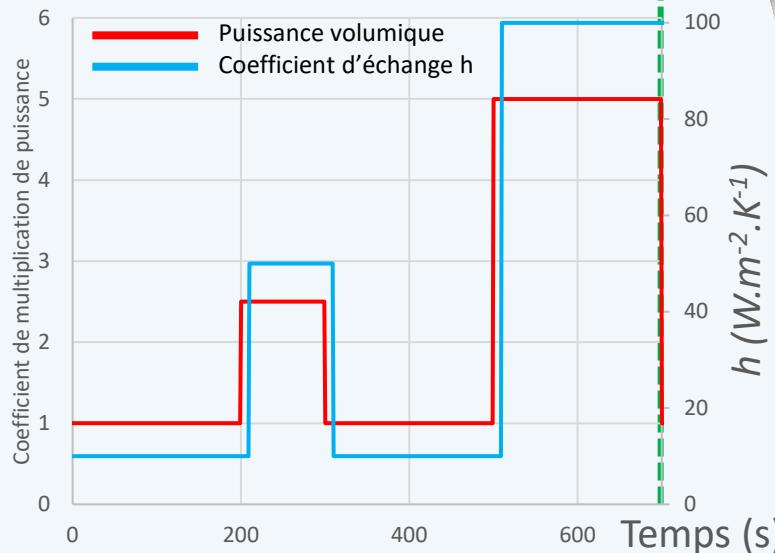
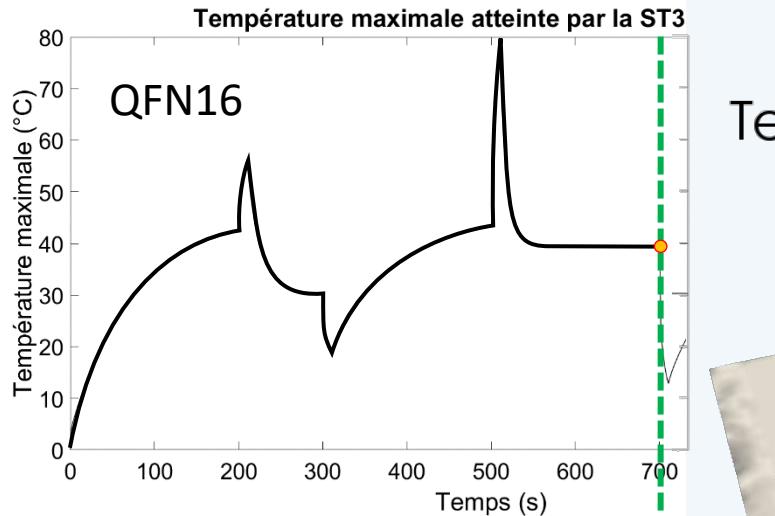
- Puissances initiales dissipées :
  - 0,18 W par QFN16
  - 0,1 W par inductance
- $h$  initial :  $10 \text{ W.m}^{-2}.\text{K}^{-1}$
- $T_{ext} = 0^\circ\text{C}$



# Simulation modale sous-structurée



# Simulation modale sous-structurée



# Evaluation du modèle sous-structuré réduit

Quelques chiffres :

Résolution du modèle détaillé : **851s**

Résolution du modèle réduit : **0,31s**  $\Rightarrow$  temps / 2700

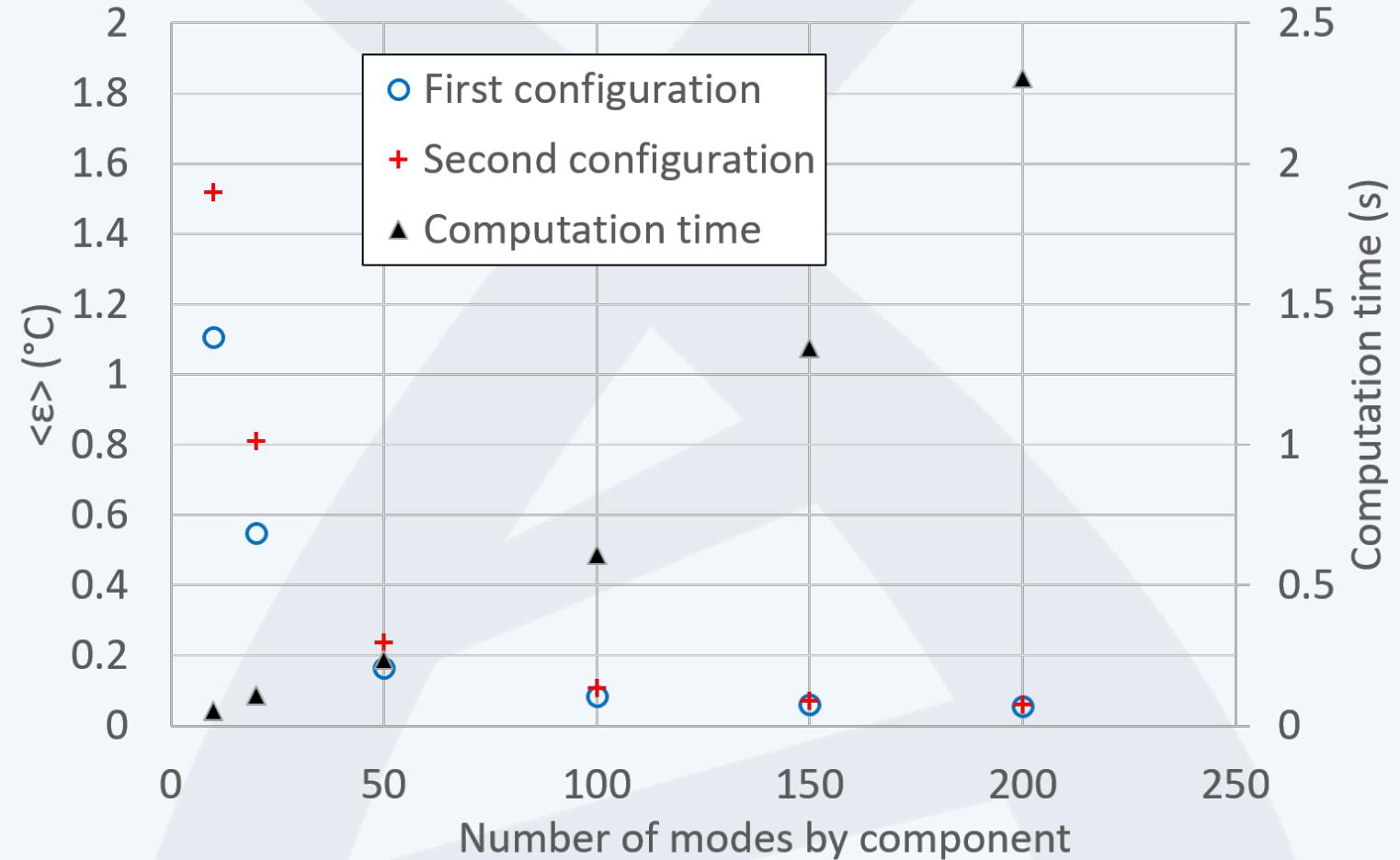
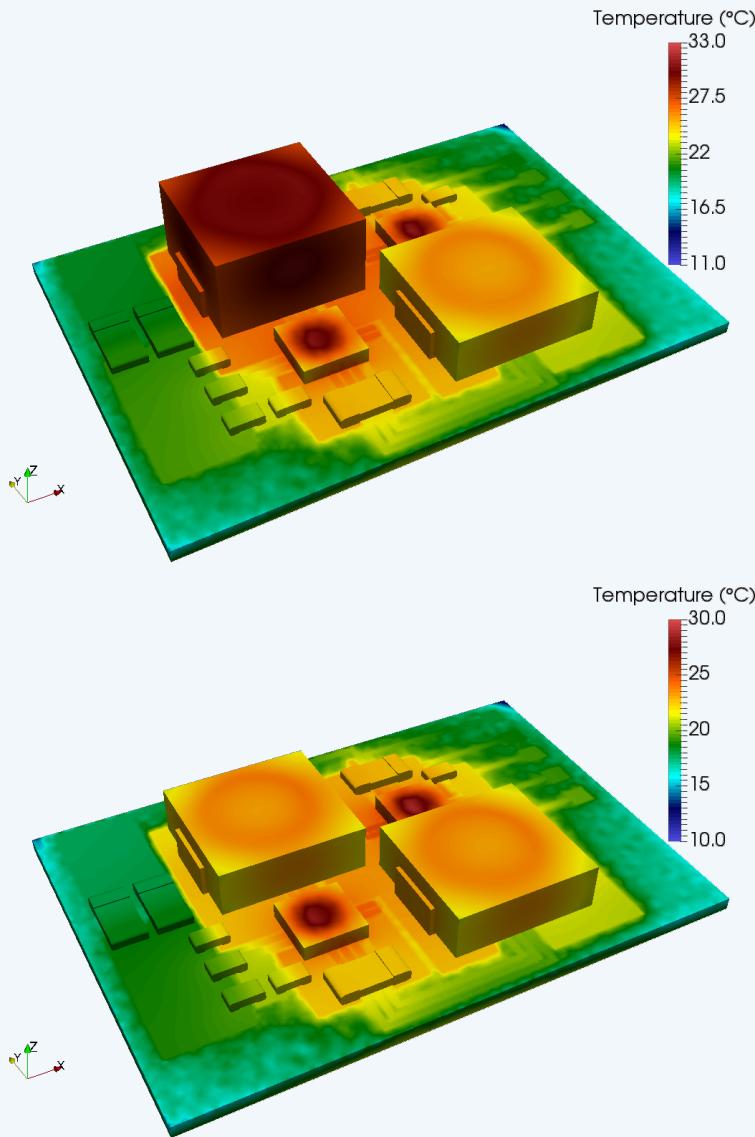
Écart moyen global :  **$\sim 0,2\%$  (0,14 K)**

Écart moyen ST2 (QFN16) :  **$\sim 0,3\%$  (0,23 K)**

Écart moyen ST9 (capacité) :  **$< 0,5\%$  (0,4K)**

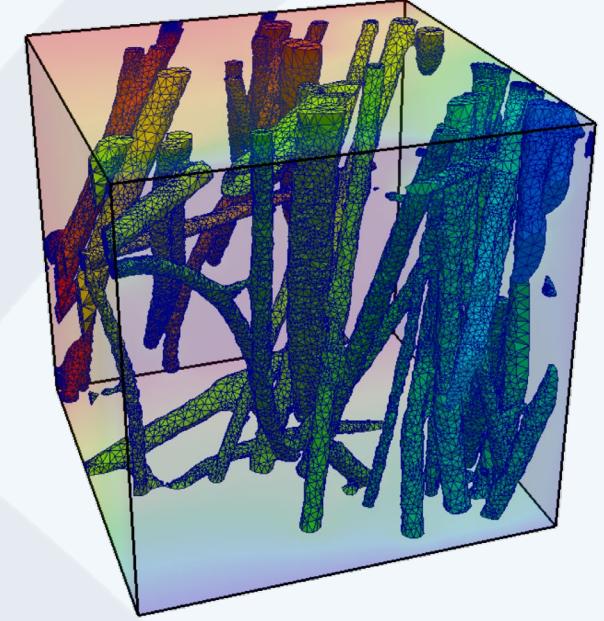
Écart moyen ST14 (résistance) :  **$< 0,4\%$  (0,32K)**

Écart sur les  $T_{max}$  :  **$< 0,5\%$  (0,4 K)**



# Bilan

- Sous-structuration modale
  - Problématique théorique
    - Est-ce que les modes de Neumann sont pertinents ?
    - Comment les réduire ?
    - Interaction modes de température / modes de flux
  - Problématique logicielle : comment décrire les frontières et les couplages quand le nombre de frontières devient trop importants ?



# FIN