Modèles d'ordre réduit pour les transferts dans les matériaux poreux du bâtiment

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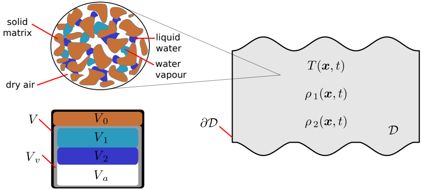
Introduction

Heat and Mass transfer in porous material

Mass conservation and energy balance

$$\frac{\partial \rho_i}{\partial t} = -\nabla \cdot \boldsymbol{j}_i + I_i \,, \forall i \in \{1, 2\}$$

$$\frac{\partial e}{\partial t} = -\nabla \cdot \boldsymbol{q} + \dot{q} \,,$$



Challenges:

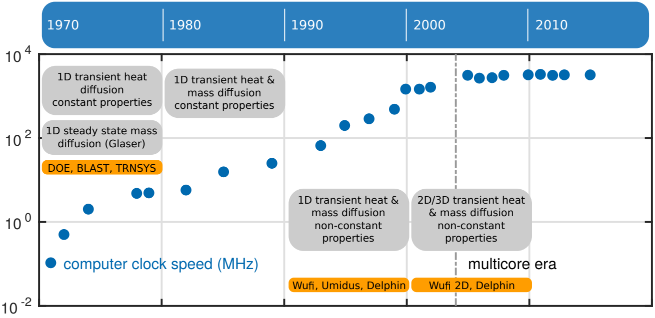
- complex interfaces with resistances
- BC $= f(\boldsymbol{x}, t, T)$
- multi-dimensional transfer
- uncertain material properties $= g(\boldsymbol{x}, t, T, \rho_i)$
- dynamic adsorption and desorption cycles with hysteresis

object of interest:

Objective: Build reliable models = fast & accurate

Introduction

why model reduction?



computer clock speeds [1]

vs

model development in building porous wall [2]

Mathematical problem

For the sake of simplicity, 1D governing equation:

$$c(T, x, t)\,\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda(T, x, t)\,\frac{\partial T}{\partial x}\right), \quad x \in \left[0, L\right], t > 0, \qquad (1)$$

with

- c $\left[\mathrm{J\,.\,m^{-3}\,.\,K^{-1}}\right]$ the volumetric heat capacity
- λ $\left[\mathrm{W\,.\,m^{-1}\,.\,K^{-1}}\right]$ the thermal conductivity

ROBIN-type boundary conditions:

$$\lambda(T, x, t)\frac{\partial T}{\partial x} = h_L(T, t)\,\left(T - T_L(t)\right) - q_L(T, t), \quad x = 0, t \geqslant 0,$$

$$-\lambda(T, x, t)\frac{\partial T}{\partial x} = h_R(T, t)\,\left(T - T_R(t)\right) - q_R(T, t), \quad x = L, t \geqslant 0$$

Initial condition:

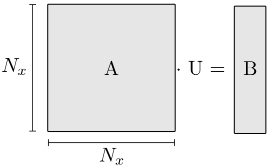
$$T(x, t) = T_0(x), \quad x \in \left[0, L\right], t = 0,$$

Numerical Model

Two approaches:

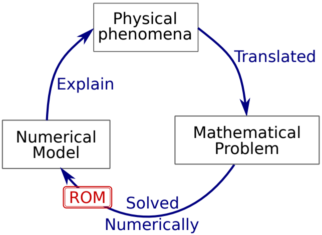
Large (Complete) original model (LOM):

- solution $T(x, t)$
- FDM, FVM, FEM, etc.
- degree of freedom Dof $= N_x \cdot N_t$



Reduced Order model (ROM):

- solution $\widetilde{T}(x, t)$,
- Dof $= N \ll N_x \cdot N_t$,
- the fidelity of the physical model is not degraded,



Reduced order model $\neq$ Lumped (RC) model

Model reduction techniques

Definitions ?

1. Numerical method that enables to compute the solution with a lower degree of freedom than the large original model
2. Compute the solution with reduced computational cost without lowering the accuracy

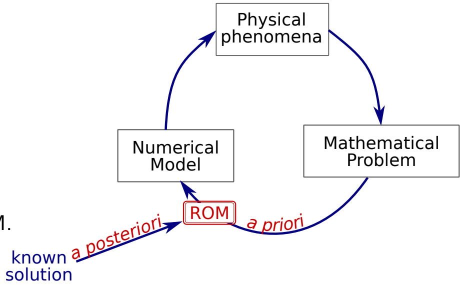
Two approaches:

- a posteriori: requires a *known* solution (or experimental data)
- a priori: no requirement

Basic idea: Decomposition of the solution

$$\widetilde{T}(x, t) \approx \sum_{n=1}^{N} \varphi_n(\bullet)\, a_n(t)$$

4 methods: MIM, POD, PGD, Spectral ROM.



Modal Identification Method

Description [3–5]

- *a posteriori* technique
- use experimental data to build the ROM
- solution is approximated by:

$$\frac{\partial a_n}{\partial t} = \mathcal{F}_{n\,n}\, a_n + \sum_{k=1}^{N_k} \mathcal{G}_{n\,k}\, \mathcal{Q}_k ,$$

$$\widetilde{T}(x_j, t) = \sum_{n=1}^{N} \varphi_{j\,n}\, a_n(t) ,$$

where

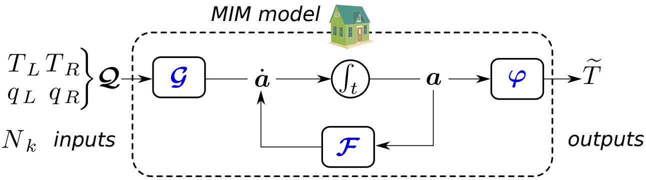
- a_n are the internal states, **no** physical meaning,
- N is the order of the reduced model ($\simeq 10$ hopefully),
- $\widetilde{T}(x_j, t)$ is the output of the system with physical meaning,
- $\mathcal{Q}_k(t)$ are the N_k inputs of the system,
- Dof $= N \cdot N_t$.

Modal Identification Method

Basic steps

offline phase:

a. **Assume** a reduced model based on a state-space representation:



b. Build the reduced model

- using (standardized) experimental data of the system $\widehat{T}(x_j, t^m)$,
- estimate (incrementally) the $N(2 + N_k)$ unknowns through least square estimator:

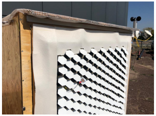
$$\left(\mathcal{F}_{nn}, \ldots, \mathcal{G}_{nk}, \ldots, \varphi_n\right) = \arg\min \sum_{m=1}^{N_t} \left(\widetilde{T}(x_j, t^m) - \widehat{T}(x_j, t^m)\right)^2$$

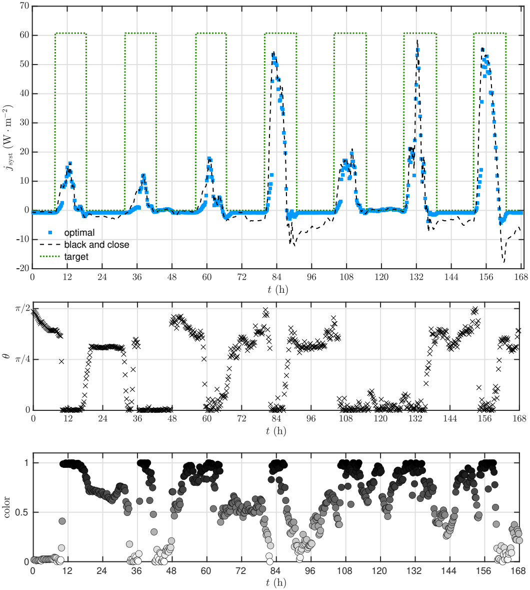
online phase:

c. Use the model for control & command, inverse problem, etc.

Modal Identification Method

Illustration: bio-inspiring wall [6]



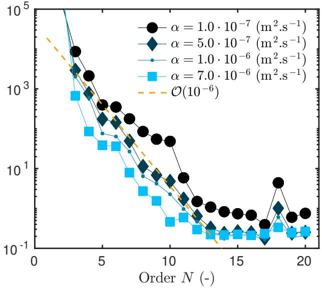


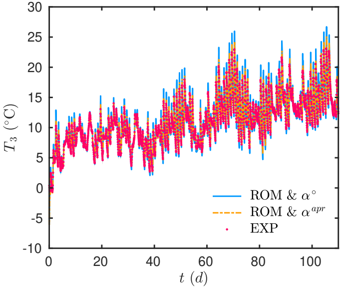
MIM model

- Inputs: irradiance, inside, outside T
- Outputs: rear face heat flux
- 2 control parameters (angle & color flap)
- Offline: 4 × 1 days
- $N = 10$, $\varepsilon_2 = 5$ W . m^{-2}
- Online (validation): 47 days

Modal Identification Method

Illustration: thermal diffusivity estimation [7, 8]





MIM model

- Inputs: inside, outside T
- Outputs: Temperature & sensitivity (3 points)
- 1 control parameter (thermal diffusivity)
- Offline: 3 days
- $N = 7$, $\varepsilon_2 = 0.24$ °C
- Parameter estim.: 3 days (OED)
- Online (validation): 111 days
- $0.19 \cdot t_{\mathrm{cpu}}(\mathrm{LOM})$

Proper Orthogonal Decomposition

Description

- *a posteriori* technique
- use previous computation of the LOM
- solution is approximated by:

$$\widetilde{T}(x,t) = \sum_{n=1}^{N} \varphi_n(x)\, a_n(t),$$

where

- N is the order of the reduced model ($\simeq 10$ hopefully).

Two possibilities:

1. known space function basis $\left\{ \varphi_n \right\}_{n=1}^{N} \rightsquigarrow$ IVP to compute $\left\{ a_n \right\}_{n=1}^{N}$,
   most of works [9–12]
   $\text{Dof} = \left(N_x + N \right) \cdot N_t$
2. known time function basis $\left\{ a_n \right\}_{n=1}^{N} \rightsquigarrow$ BVP to compute $\left\{ \varphi_n \right\}_{n=1}^{N}$ [13]
   $\text{Dof} = \left(N_t + N \right) \cdot N_x$

Proper Orthogonal Decomposition

Basic steps (IVP approach)

offline phase:

a. **Assume** a known solution T of the problem (computed with the LOM)

- Compute covariance matrix:

$$C \simeq T^{\mathrm{t}} \cdot T$$

- Compute eigenvectors A & eigenvalues λ
- Extract the main components by sorting the N highest eigenvalues:

$$\frac{\lambda_1}{\sum_i \lambda_i} > \ldots > \frac{\lambda_{N_t}}{\sum_i \lambda_i} \leadsto \left\{ a_n(t) \right\}_{n=1}^{N}$$

b. Insert decomposed solution

$$\widetilde{T}(x_j, t) = \sum_{n=1}^{N} \varphi_n(x)\, a_n(t) \longrightarrow \text{Eq (1)}$$

c. Project the result on each time function $\left\{ a_n(t) \right\}_{n=1}^{N}$

d. Obtain the boundary value problem:

$$\sum_{m=1}^{N} c\, \alpha_{nm}\, \varphi_n - \lambda \frac{\mathrm{d}^2 \varphi_n}{\mathrm{d}x^2} = 0, \qquad \forall n \in \left\{ 1, \ldots, N \right\}, \tag{2}$$

online phase: Solve the boundary value problem (2).

Proper Orthogonal Decomposition

Parametric solutions

Solution depends on extra-parameters of the problem (diffusivity, surface transf. coeff., etc.):

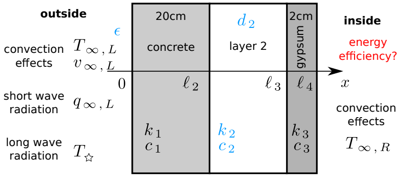
$$\widetilde{T}(x, t, p) = \sum_{n=1}^{N} \varphi_n(x)\, a_n(t, p),$$

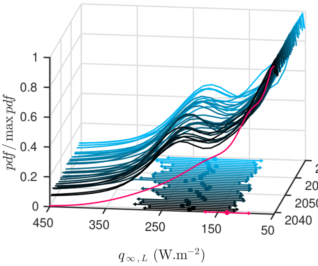
where the time function basis $\left\{ a_n(t, p) \right\}_{n=1}^{N}$:

- is computed for a set of parameters (offline phase)
- is interpolated on a tangent space of GRASSMANN manifold for a new parameter [14] (online phase)

Proper Orthogonal Decomposition

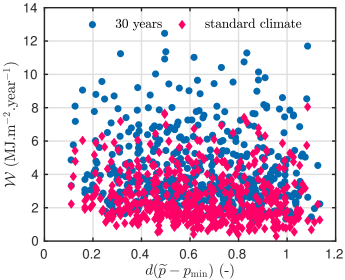
Illustration: thermal design under climate change [13]





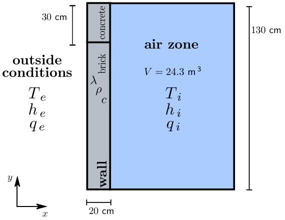
POD model

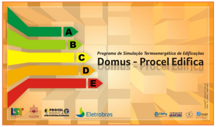
- 4 parameters solution
- Offline: 5 basis generated
- $N = 10$, $\varepsilon_2 = 0.01$ °C
- Online: 30 years, 500 parameters
- $\underbrace{5 \cdot t_{\mathrm{cpu}}(\mathrm{LOM})}_{\text{offline}} + \underbrace{10^{-3} \cdot t_{\mathrm{cpu}}(\mathrm{LOM})}_{\text{online}}$



Proper Orthogonal Decomposition

Illustration (IVP): building 2D heat transfer [15, 16]



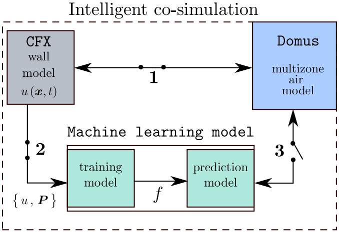


Model	POD	NN
Order	$N = 8$	5 nodes
Error	$\varepsilon_2 = 0.01\ ^{\circ}\mathrm{C}$	
Offline	2 days	8 days
Online	365 days	
t_{cpu}	$\underbrace{34\ \text{min}}_{\text{offline}} + \underbrace{54\ \text{min}}_{\text{online}}$	$\underbrace{133\ \text{min}}_{\text{offline}} + \underbrace{2\ \text{min}}_{\text{online}}$

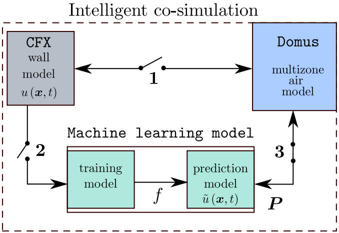
$t_{\mathrm{cpu}}(\mathrm{LOM}) = 14$ h

$t_{\mathrm{cpu}}(\mathrm{LOM} - 1\mathrm{D}) = 2$ min

online



offline



Proper Generalized Decomposition

Description [17–19]

- *a priori* technique
- no-need of known solution
- solution is approximated by:

$$\widetilde{T}(x, t) = \sum_{n=1}^{N} \varphi_n(x)\, a_n(t),$$

where

- the functions space $\varphi_n(x)$ and $a_n(t)$ are unknown,
- N is the order of the reduced model ($\simeq$ 30 hopefully),
- $\text{Dof} = N \cdot \left(N_x + N_t \right)$.

Proper Generalized Decomposition

Basic steps

$$\widetilde{T}(x,t) = \sum_{n=1}^{N} \varphi_n(x)\, a_n(t),$$

Online phase: Set $N = 1$
while $\mathrm{Res}(\mathit{Eq}\ (1)) \leqslant \varepsilon_1$ **do**
  $i = 1$
  initial guess $\varphi_N^i,\ a_N^i$
  while $\left|\left| \varphi_N^{i+1} - \varphi_N^i \right|\right|_2,\ \left|\left| a_N^{i+1} - a_N^i \right|\right|_2 \leqslant \varepsilon_2$ **do**
    $\langle \mathit{Eq}\ (1),\ \varphi_N^i(x) \rangle_x \rightsquigarrow \alpha_1^i \frac{\mathrm{d} a_N^i}{\mathrm{d} t} - \beta_1^i a_N^i = \gamma_1$
    $\langle \mathit{Eq}\ (1),\ a_N^i(t) \rangle_t \rightsquigarrow \alpha_2^i \varphi_N^i - \beta_2^i \frac{\mathrm{d}^2 \varphi_N^i}{\mathrm{d} x^2} = \gamma_2$
    $i = i + 1$
  end while
  Update $\varphi_N = \varphi_N^i,\quad a_N = a_N^i$
  Increment $N = N + 1$
end while

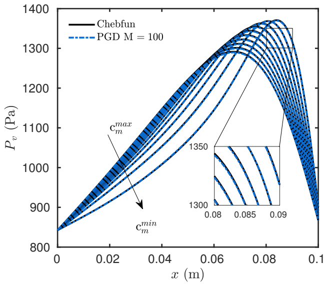
Proper Generalized Decomposition

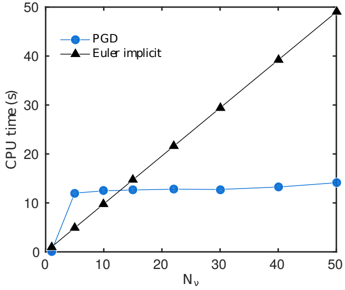
Parametric solutions [12, 20–25]

Solution depends on extra-parameters of the problem (diffusivity, surface transf. coeff., etc.):

$$\widetilde{T}(x, t, \textcolor{red}{p}) = \sum_{n=1}^{N} \varphi_n(x)\, a_n(t)\, f_n(\textcolor{red}{p}),$$

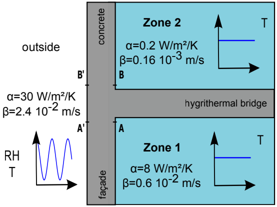
where the solution is computed at once by multiple projection of Eq. (1) [18]

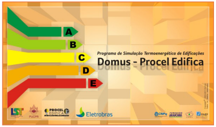


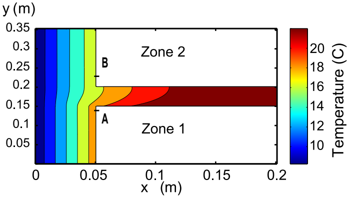


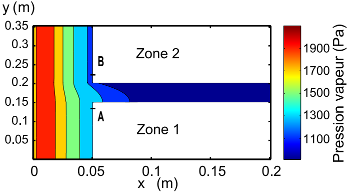
Proper Generalized Decomposition

Illustration: building 2D heat and mass transfer [26]









PGD model

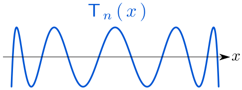
- $N = 30\,,\ \varepsilon_2 = 0.1\ ^\circ\mathrm{C}$
- $\underbrace{0.08 \cdot t_{\mathrm{cpu}}(\,\mathrm{LOM}\,)}_{\text{online}}$
- coupling issues

Spectral Reduction Method

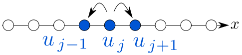
Description [27–31]

- *a priori* technique
- no-need of known solution
- solution is approximated by:

$$\widetilde{T}(x,t) = \sum_{n=1}^{N} \varphi_n(x)\, a_n(t),$$



High order global representation



Local representation

where

- the space functions $\varphi_n(x) \equiv \mathrm{T}_n(x)$

  the CHEBYSHEV polynomials,

  $\hookrightarrow$ Global representation of the solution
- a_n are the only unknowns,
- N is the order of the reduced model ($\simeq 10$ hopefully),
- $\mathrm{Dof} = N \cdot N_t$.

Spectral Reduction Method

Basic steps

a. Insert decomposed solution into the Equation:

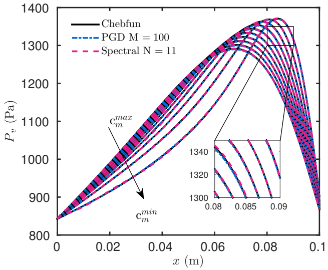
$$\widetilde{T}(x,t) = \sum_{n=1}^{N} \mathrm{T}_n(x)\, a_n(t) \longrightarrow \text{Eq. (1)}$$

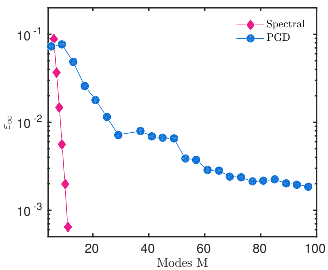
b. Minimization of the residual (Galerkin-Tau method):

$$\langle \mathrm{Res}(Eq\ (1)), \mathrm{T}_n \rangle = \int \frac{R(x,t)\,\mathrm{T}_n(x)}{\sqrt{1-x^2}}\,\mathrm{d}x = 0$$

c. Obtain a reduced system of ODE to solve:

$$\dot{a}_n(t) = \mathcal{A} \cdot a_n(t), \qquad \forall n \in \{1, \ldots, N\}.$$





Spectral Reduction Method

Illustration: parametric solution [32] (SFT 2025)

Solution depends on extra-parameters:

$$\widetilde{T}(x,t,p) = \sum_{n=1}^{N} \sum_{m=1}^{M} \mathrm{T}_n(x)\, a_{nm}(t)\, \mathrm{P}_m(p),$$

and the sensitivity coefficient:

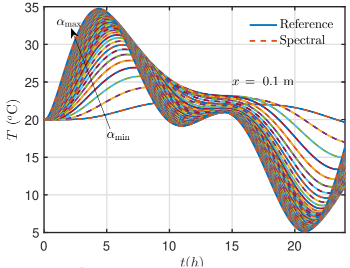
$$X(x,t,p) = \frac{\partial T}{\partial \bar{p}}$$

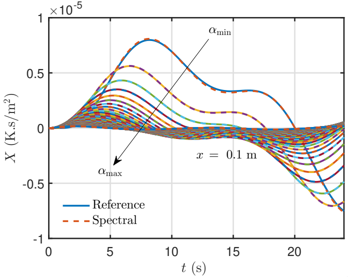
$$= \sum_{n=1}^{N} \sum_{m=1}^{M} \mathrm{T}_n(x)\, a_{nm}(t)\, \frac{\mathrm{dP}_m}{\mathrm{d}p}(p).$$

$\rightsquigarrow$ Gradient-based algorithm for Inverse Problem

$$\mathcal{J}(p) = \left|\left| \widetilde{T}(x_j, t^m) - \widehat{T}(x_j, t^m) \right|\right|_2 \longrightarrow \min$$

and freely $\nabla_p \mathcal{J}$!





Conclusion

4 Model Reduction Techniques: MIM, POD, PGD, Spectral ROM

- requires preliminary (mathematical) developments,
- verified with reference solutions,
- compared with experimental observations,
- bench-marked with complete models.

ROM	reduction	accuracy	whole T	offline cost	coupling	parametric
MIM	$+$	-	no	$+$/-	$++$	-
POD	$++$	$++$	yes	$++$	$++$	$+$
PGD	$+$	$+$	yes	-	-	$++$
Spectral	$++$	$++$	yes	-	$++$	$++$

Open topics:

- treatment of non-linearities,
- coupling with parametric solution.

Thank you for your attention

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Merci de votre attention