A reduced-order cylindrical conduction model for transient heat flux reconstruction under LOCA-like conditions

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Journée METTI: Approches originales en inversion de mesures

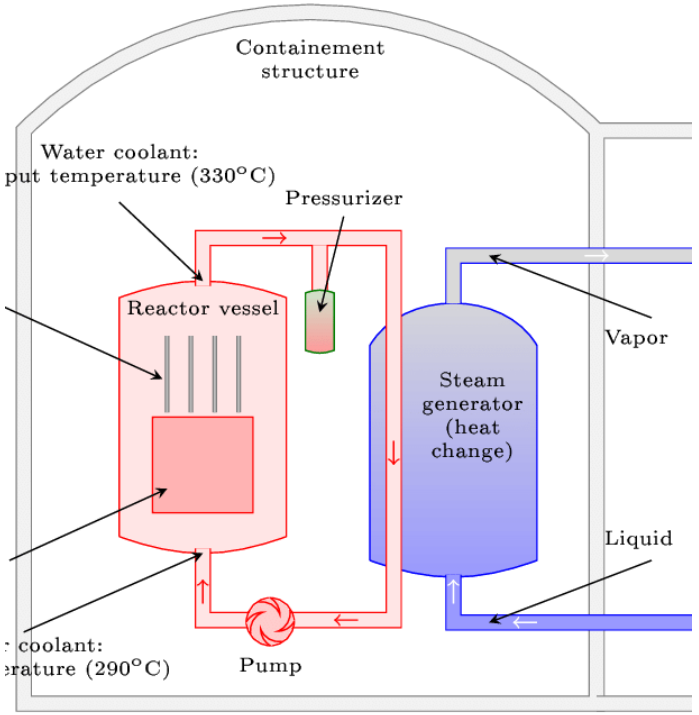
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Industrial context

- Pressurized Water Reactors (PWRs) represent the **majority** of nuclear power plants worldwide

- In the primary circuit, water performs a dual function: it acts as both a neutron moderator and the coolant responsible for **carrying the energy** generated by fission reactions

- This energy is then delivered to the steam generators, where it is used to **vaporize the water** in the secondary circuit



Bernard et al. (2012)

Loss-of-coolant accident (LOCA)

- A loss-of-coolant accident (LOCA) is a hypothetical situation resulting from **breaches in the primary circuit pipes** of a Pressurized Water Reactor (PWR)

- These breaches **lead to rapid depressurization** in the core, causing the coolant to boil and the **cladding temperature to rise**

- Emergency core cooling systems (ECCS) are responsible for injecting borated water into the fuel assembly, thereby promoting the **removal of decay heat**

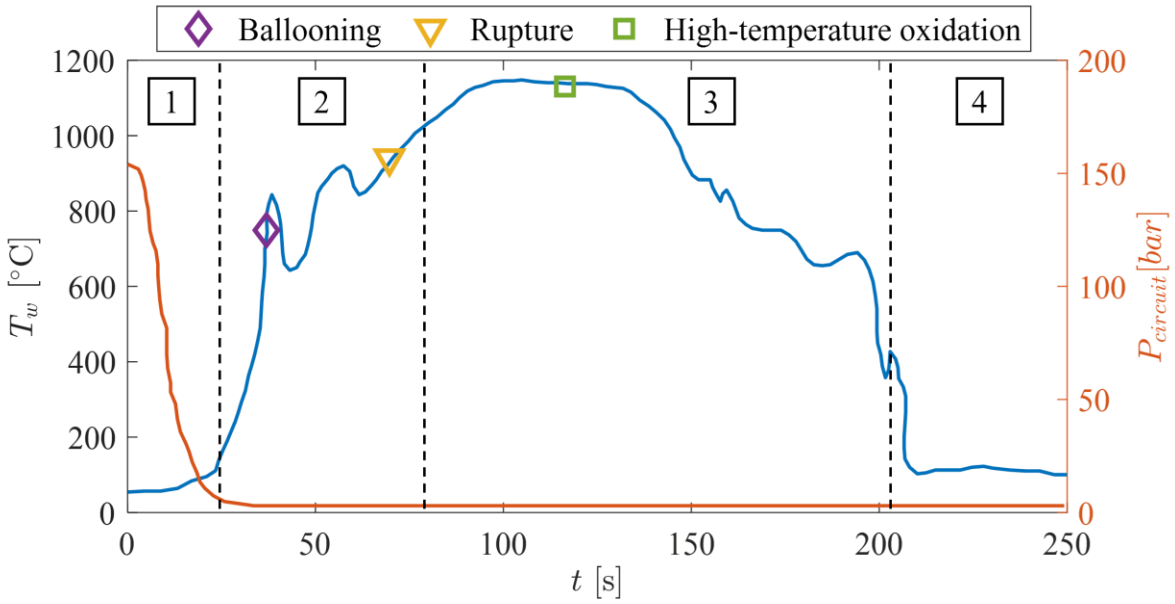
Loss-of-coolant accident (LOCA)

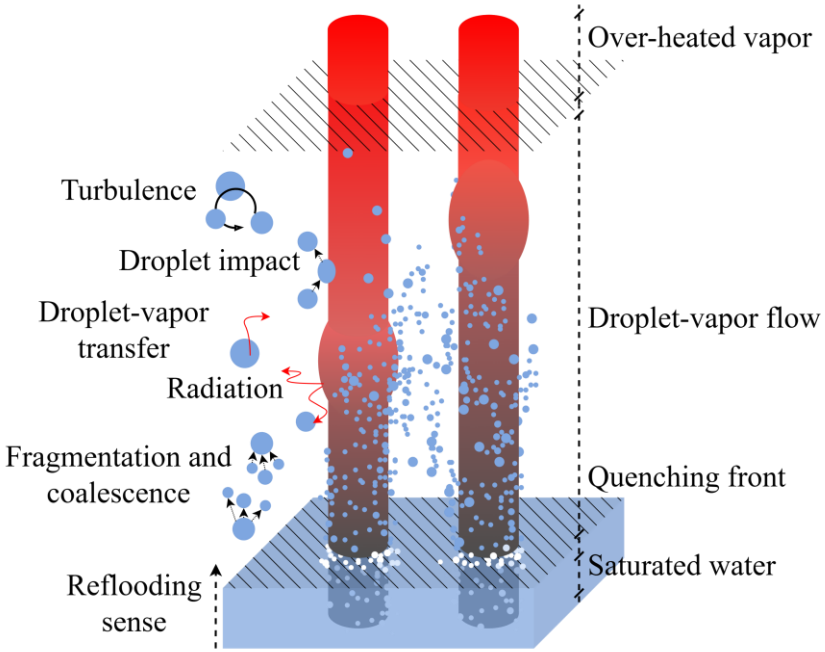
1 Depressurization

2 Ballooning of the fuel cladding

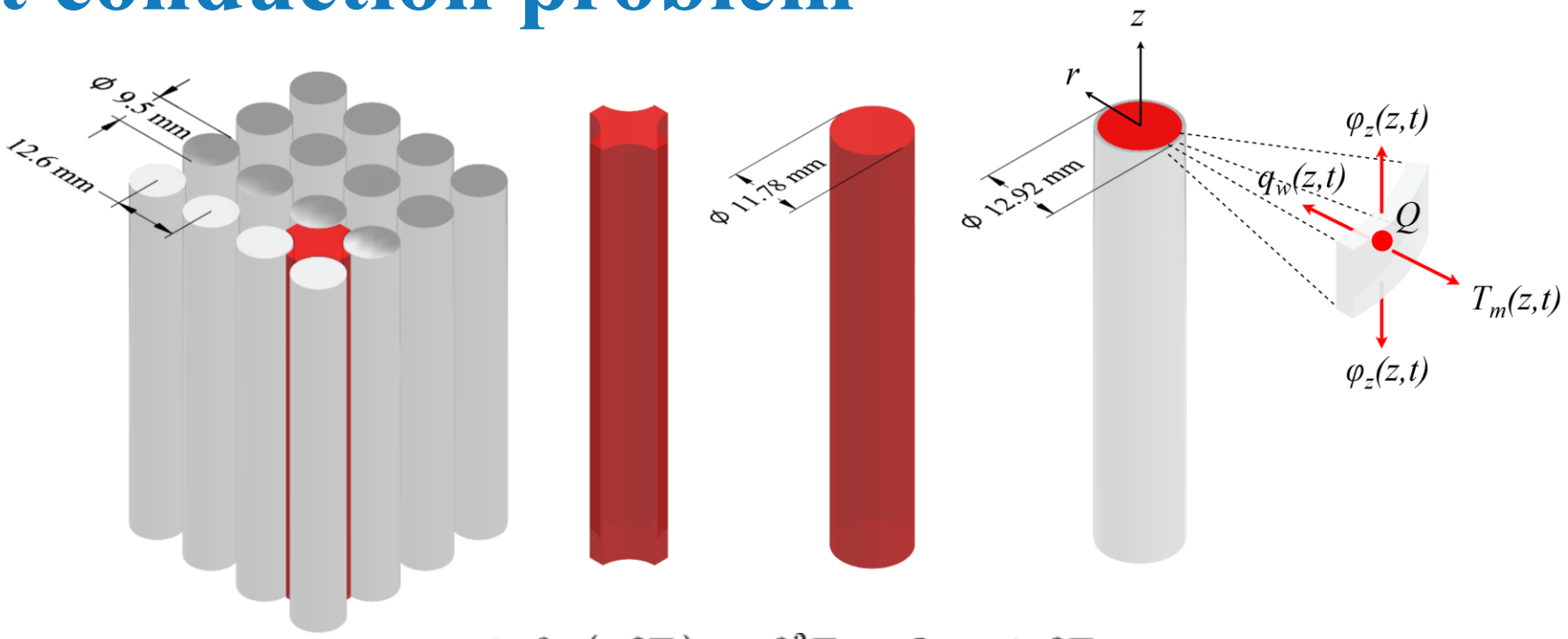
3 Reflooding

4 Post-quench behavior





Heat conduction problem



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}+\frac{Q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=r_i}=\frac{q''_w}{k} \qquad \left.\frac{\partial T}{\partial r}\right|_{r=r_e}=-\frac{h_c(T_m-T_\infty)}{k} \qquad \left.\frac{\partial T}{\partial z}\right|_{z=0}=\left.\frac{\partial T}{\partial z}\right|_{z=z_{\max}}=0$$

Traditional approaches

- Some approaches for heat flux estimation in dispersed flow rely on **lumped assumptions that neglect conduction in one spatial direction** and compute the heat flux algebraically (*Peña Carrillo*, 2018, and *Luna Valencia*, 2023)

- Conversely, rigorous analytical methods, such as the **classical integral transform technique** (CITT) (*Hahn and Özisik*, 2012), provide exact solutions but involve high mathematical complexity and increased computational cost

Improved lumped methodology

The improved lumped methodology (*Cotta and Mikhailov*, 1997) aims to **reduce the order of the heat equation** while **preserving information about the temperature gradients**. First, we define the average temperature in the radial direction:

$$T_{av}(z,t) = \frac{2}{(r_e^2 - r_i^2)} \int_{r_i}^{r_e} rT\, dr$$

$$C_1 q_w'' + C_2 T_\infty - C_2 T_m + \frac{\partial^2 T_{av}}{\partial z^2} + \frac{Q}{k} = \frac{1}{\alpha}\frac{\partial T_{av}}{\partial t}$$

$$C_1 = \frac{-2r_i}{(r_e^2 - r_i^2)k} \qquad C_2 = \frac{-2r_e h_c}{(r_e^2 - r_i^2)k}$$

Hermite approximations

We must obtain **two new equations** relating the average temperature and the boundary conditions. For that, we employ the **Hermite approximations** (*Mennig and Hälg*, 1983):

$$H_{0,0}: \int_0^h y(x)dx \approx \frac{h}{2}[y(0)+y(h)]$$

$$H_{1,1}: \int_0^h y(x)\,dx \approx \frac{h}{2}[y(0)+y(h)] + \frac{h^2}{12}[y'(0)-y'(h)]$$

Hermite approximations

Applying the $H_{(1,1)}$ to the **average temperature** and the $H_{(0,0)}$ to the **average heat flux**:

$$T_{av} = \frac{2}{(r_e^2 - r_i^2)} \int_{r_i}^{r_e} rT\,dr \approx \frac{1}{(r_e + r_i)}[r_e T_m + r_i T_i] + \frac{(r_e - r_i)}{6(r_e + r_i)} \left[T_i - T_m + \frac{(r_i q''_w + h_c r_e T_\infty - h_c r_e T_m)}{k} \right]$$

$$T_m - T_i = \int_{r_i}^{r_e} \frac{\partial T}{\partial r} dr \approx \frac{(r_e - r_i)}{2k} (q''_w - h_c T_\infty + h_c T_m)$$

$$T_{av} = (1 - C_3) T_m + C_3 T_\infty + C_4 q''_w$$

$$C_3 = \frac{h(r_e - r_i)(3r_e + 5r_i)}{12(r_e + r_i)k} \qquad C_4 = -\frac{(r_e - r_i)(r_e + 3r_i)}{12(r_e + r_i)k}$$

How to solve it?

Combining the previous Equations would yield a governing equation relating both boundary conditions. However, this leads to a **non-homogeneous partial differential equation of first order in time and second order in space**!

We may rearrange the governing equation, solving the **average temperature** first and then obtaining the **heat flux** from a previous equation:

$$\frac{\partial^2 T_{av}}{\partial z^2} - \frac{1}{\alpha}\frac{\partial T_{av}}{\partial t} + \frac{C_1}{C_4}T_{av} = \left[\frac{C_1}{C_4}(1 - C_3) + C_2\right]T_m + \left(\frac{C_1 C_3}{C_4} - C_2\right)T_\infty - \frac{Q}{k} = F$$

Finite difference formulation

A numerical solution based on the **finite difference method** is proposed:

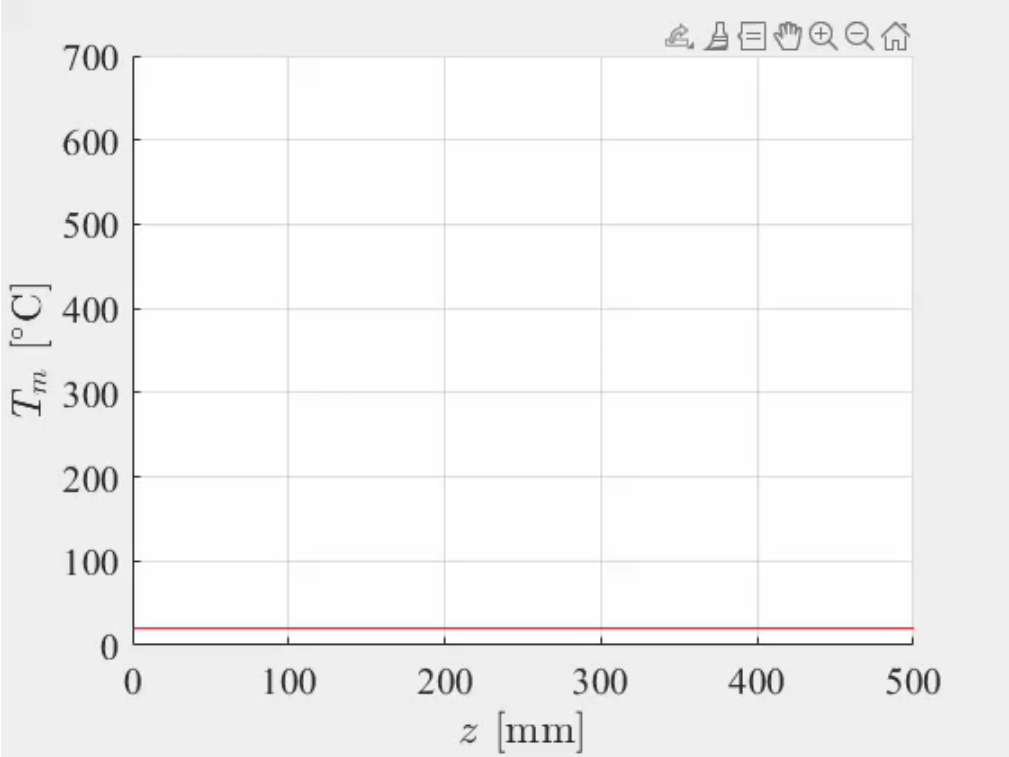
$$\boldsymbol{T}_{av}^{n} = \boldsymbol{M}^{-1}\left(\boldsymbol{F} - \frac{\boldsymbol{T}_{av}^{n-1}}{\alpha \Delta t}\right)$$

$$\boldsymbol{M} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ A & B & A & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & A & B & A \\ 0 & \cdots & 0 & -1 & 1 \end{bmatrix} \qquad A = \frac{1}{\Delta z^2} \qquad B = \frac{C_1}{C_4} - \frac{2}{\Delta z^2} - \frac{1}{\alpha \Delta t}$$

$$T_{av} = (1 - C_3)T_m + C_3 T_\infty + C_4 q_w''$$

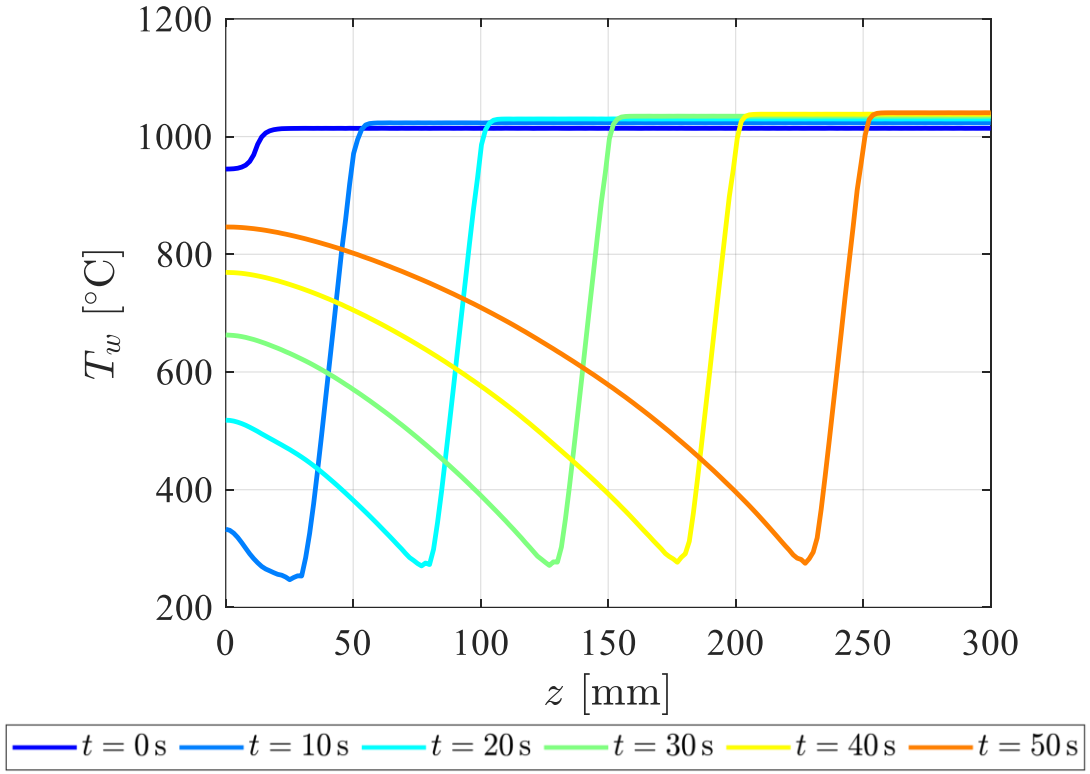
Synthetic data

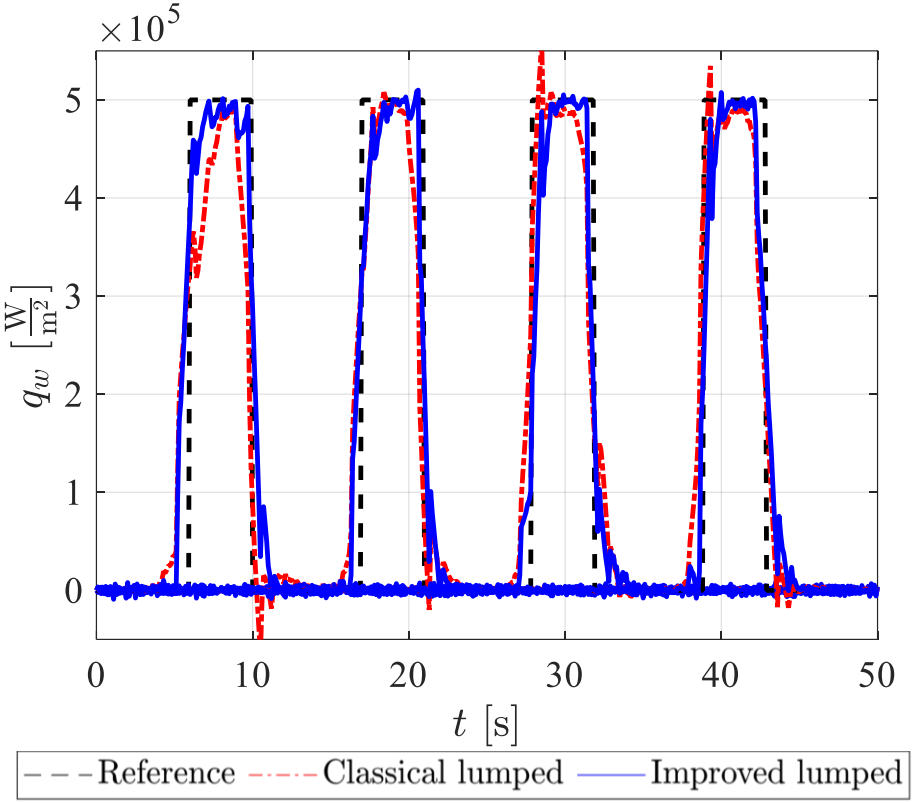
COMSOL Multiphysics® was employed to generate high-fidelity simulations for **model verification**:



Results for model verification

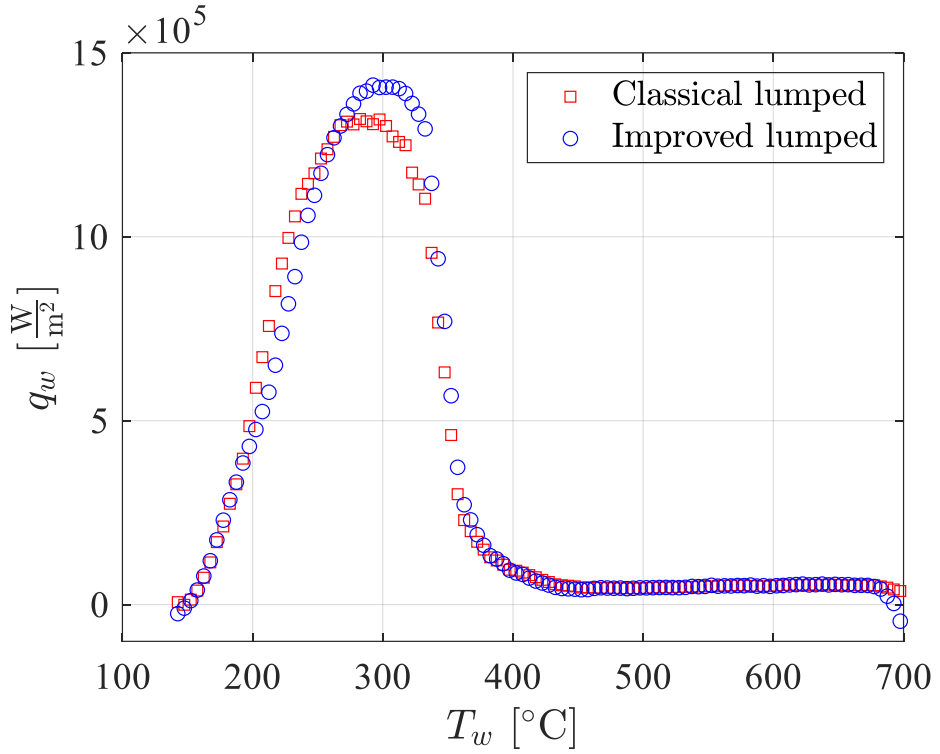
For a heat flux rectangle profile of 0.5 MW/m^2 and a temperature noise of 0.5 °C, the following profile was reconstructed:





Results using real experimental data

Finally, the methodology was also employed for **real experimental data processing**. Data obtained from Experiment T0 (*Luna Valencia et al.*, 2025):



Conclusions and perspectives

- The improved lumped methodology demonstrated **good agreement** with the classical formulation under **low temperature-gradient** conditions

- **New synthetic and experimental (reflooding) data** with **higher temperature gradients** must be produced to investigate how much better the improved version may perform

- Eventually, the method will be also employed for **different geometries and heat transfer conditions**

Thanks for your attention,

(your **questions** are welcome!)

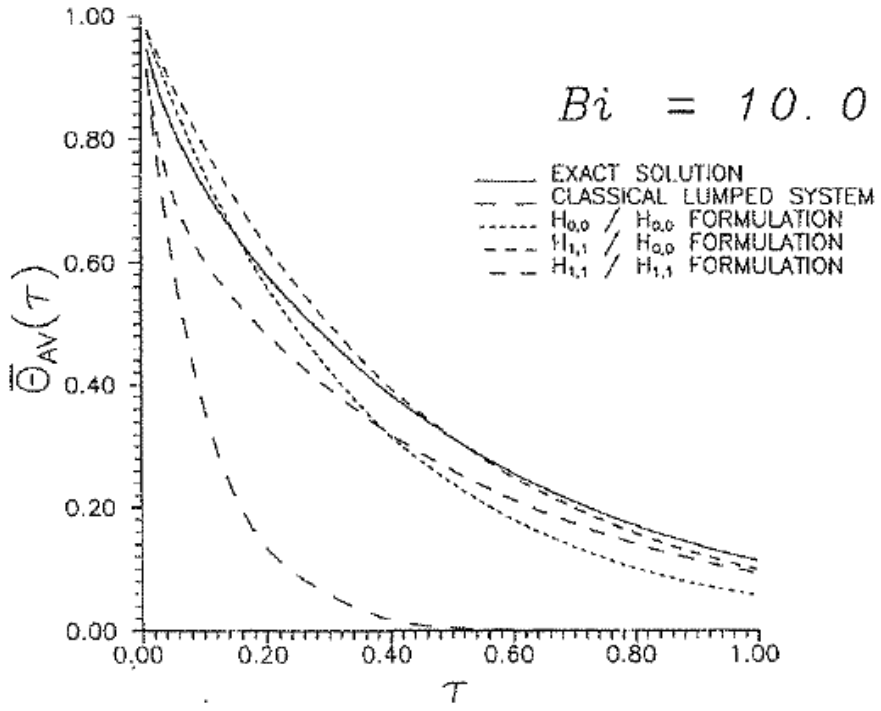
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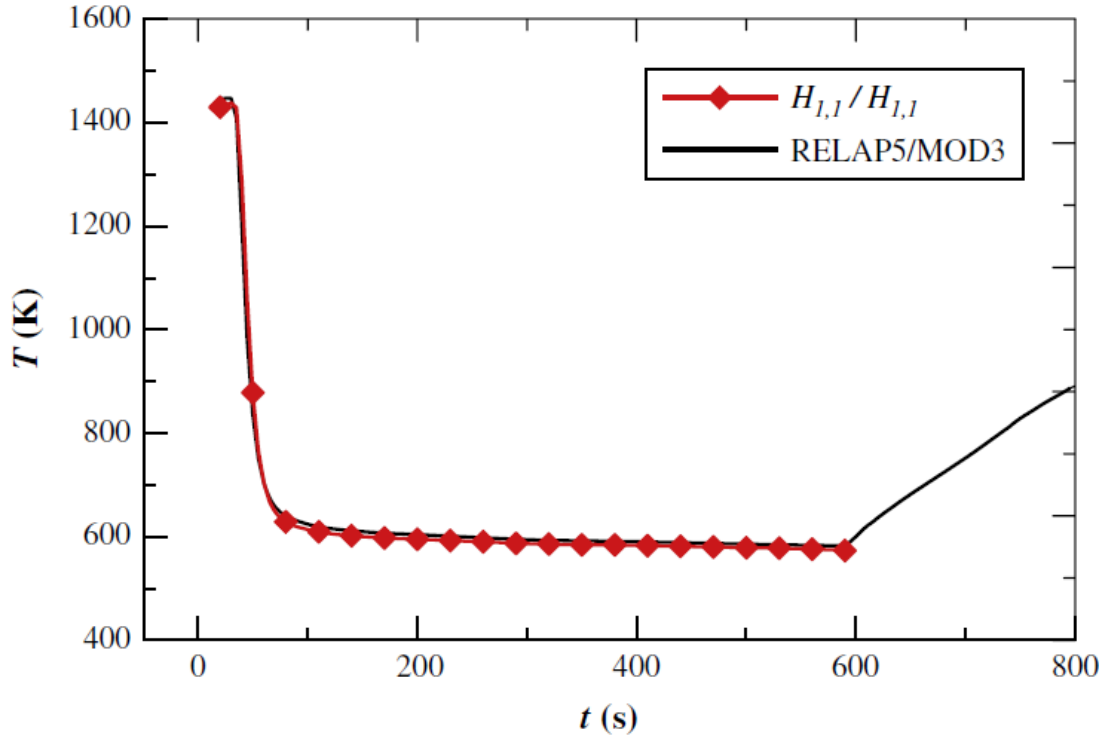
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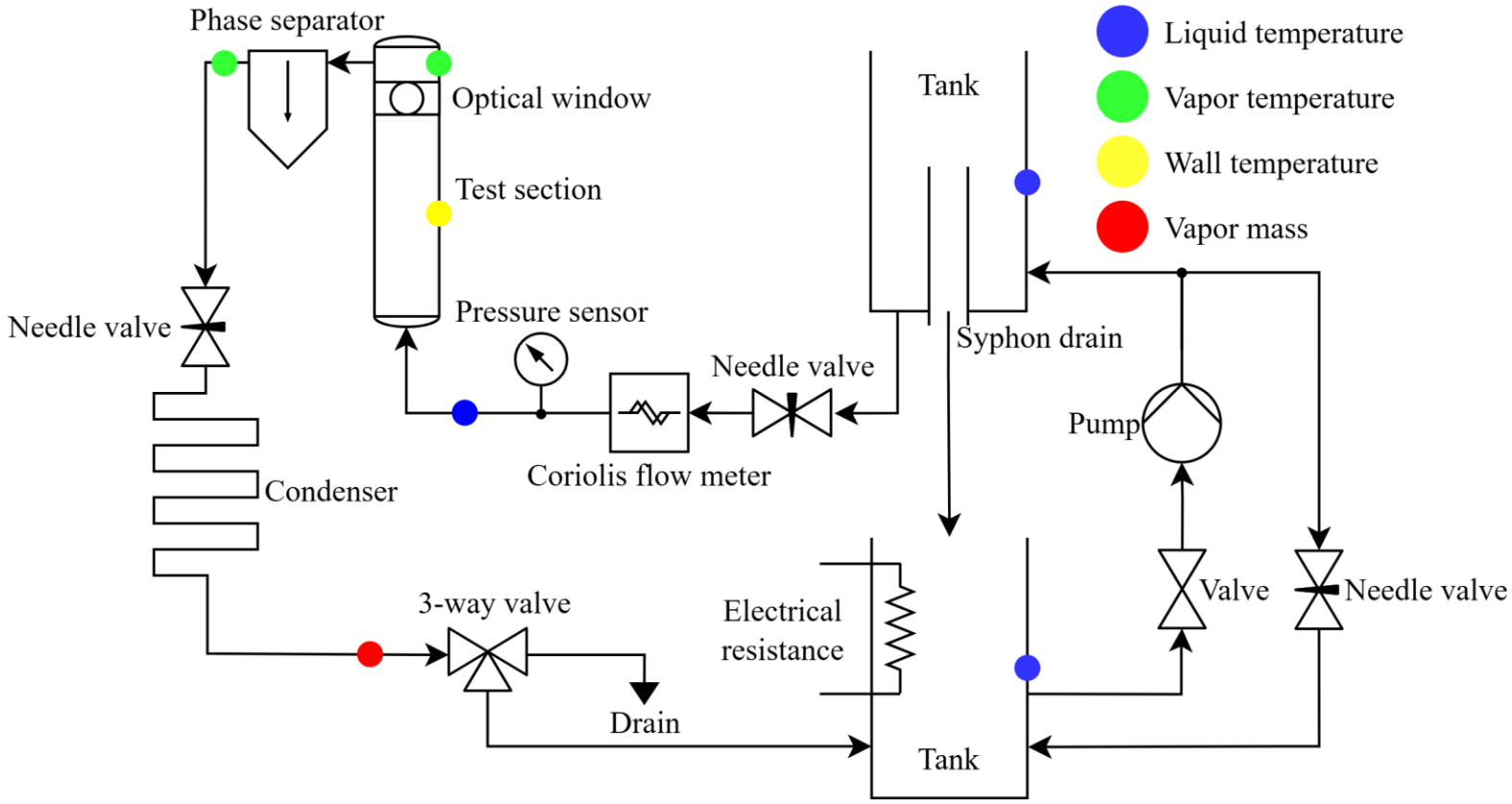
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Improved lumped methodology





Experimental apparatus



Experimental configurations

