

Méthodes inverses

par analyse nodale et modale à partir
d'images multiéchelles (μm au cm) de
champs de température

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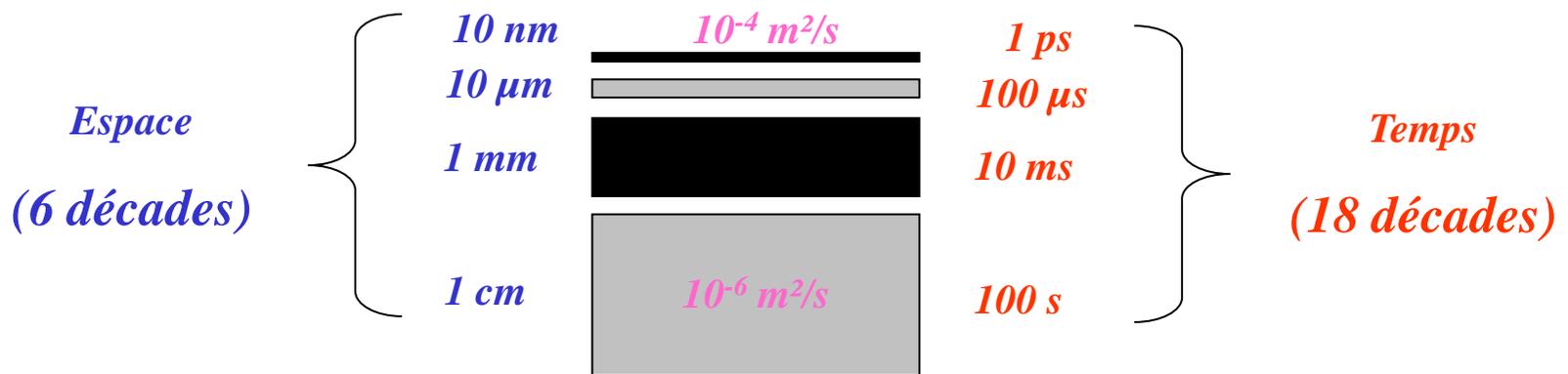
Objectifs: *Mesure de propriétés thermophysiques multiéchelles*

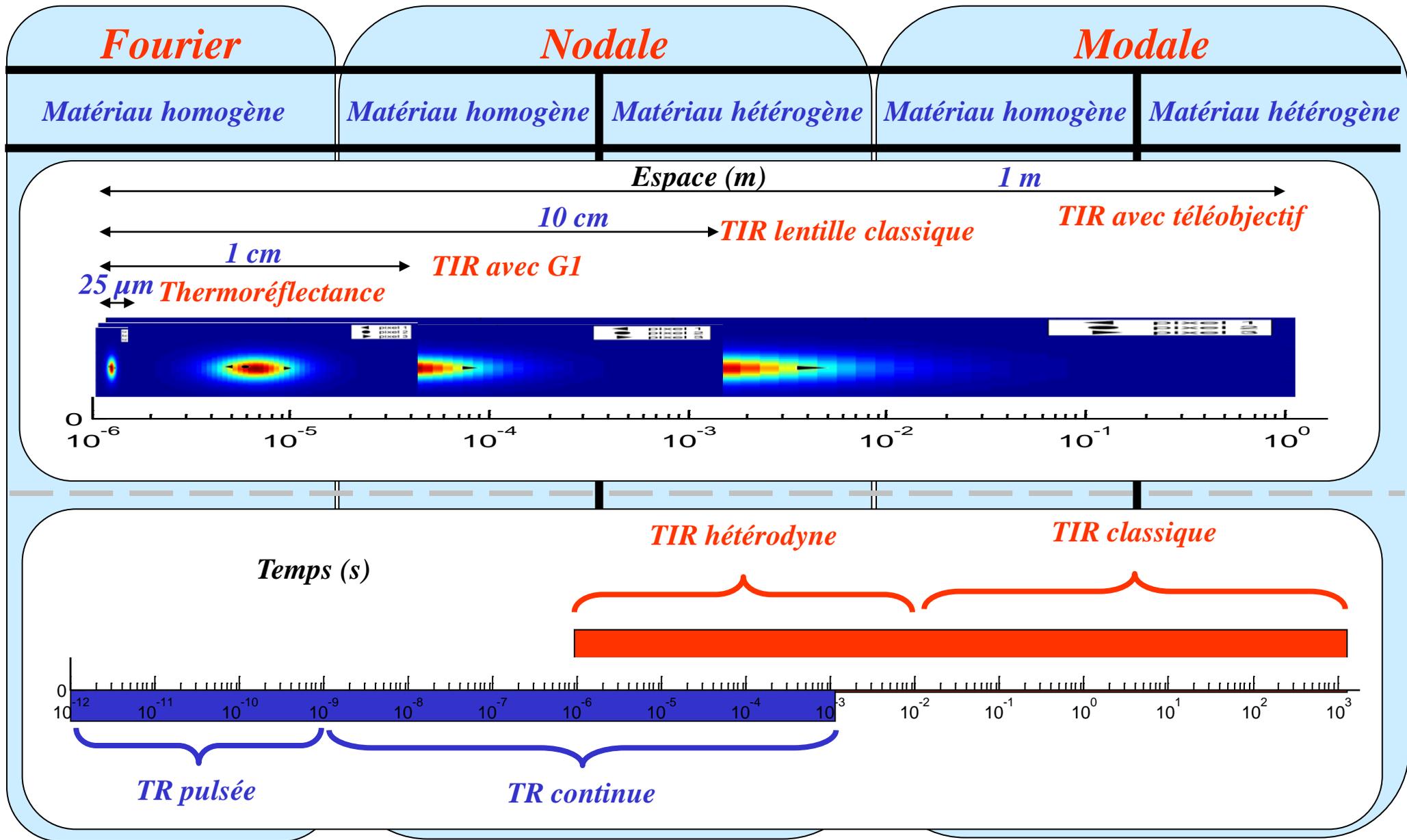
1 Problèmes : Estimation de *champs de propriétés thermophysiques* (diffusivité...) dans des matériaux *hétérogènes*

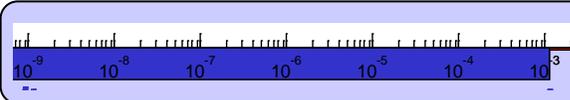
$$\text{div}(\lambda \cdot \vec{\text{grad}} T(x, y, t)) = \rho C_p \left(\frac{\partial T(x, y, t)}{\partial t} \right)$$

2 Outils : *Mesure de champs de températures* (thermoréflectance, camera IR), *modèles analytiques et numériques* (quadripôle, différences finies...) et *méthodes inverses* (Fourier, nodale : OLS, TLS et modale: SVD)

3 Exemple: Multicouche, multiéchelle

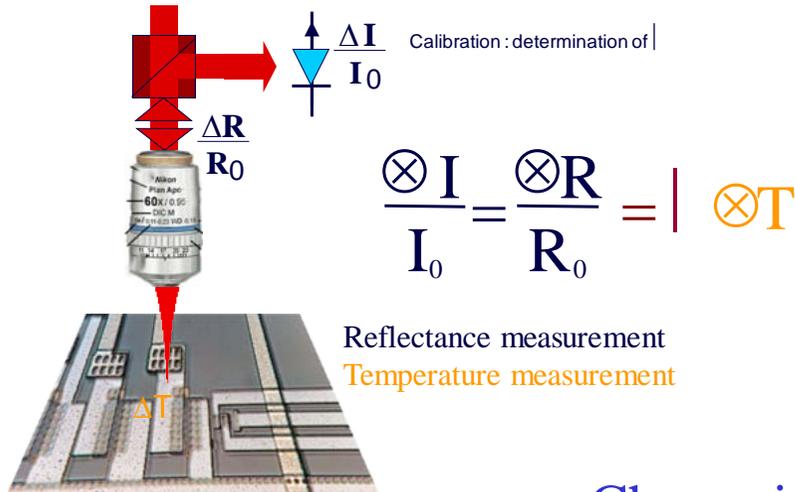




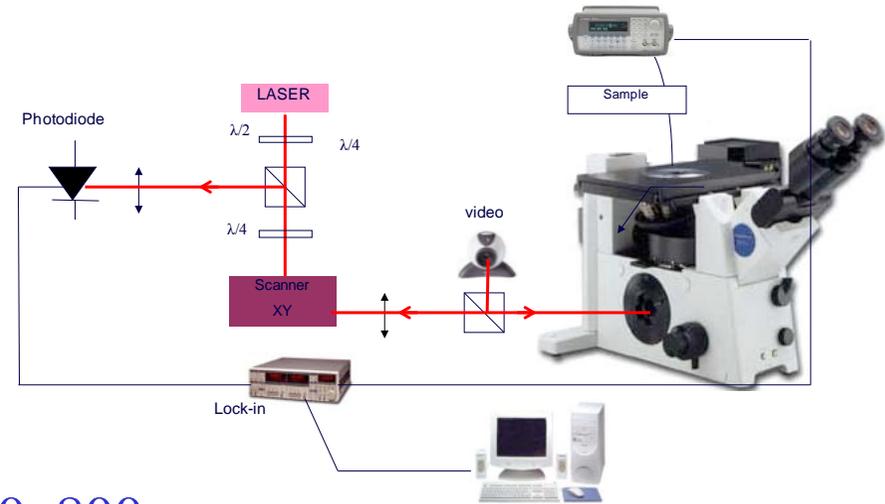


Thermoréflectance continue (S. Dilhaire)

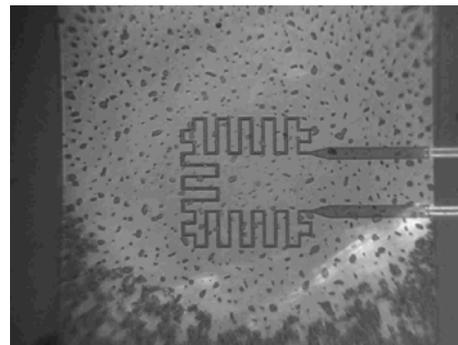
Principe de la thermoréflectance ponctuelle



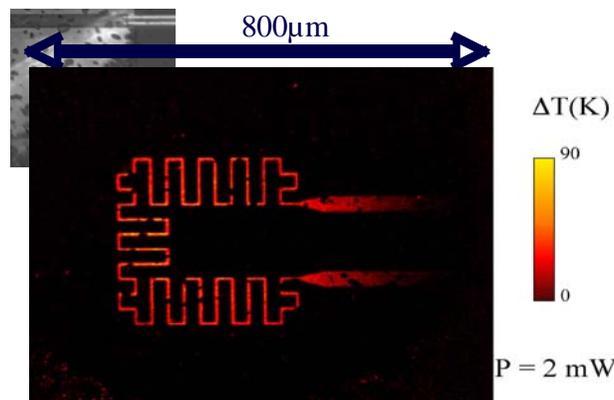
Principe de la thermoréflectance par imagerie



Champ imagé 800x800 μm

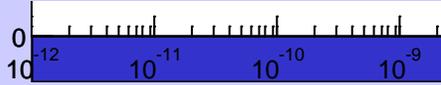


Film IR (x10) lors de la congélation



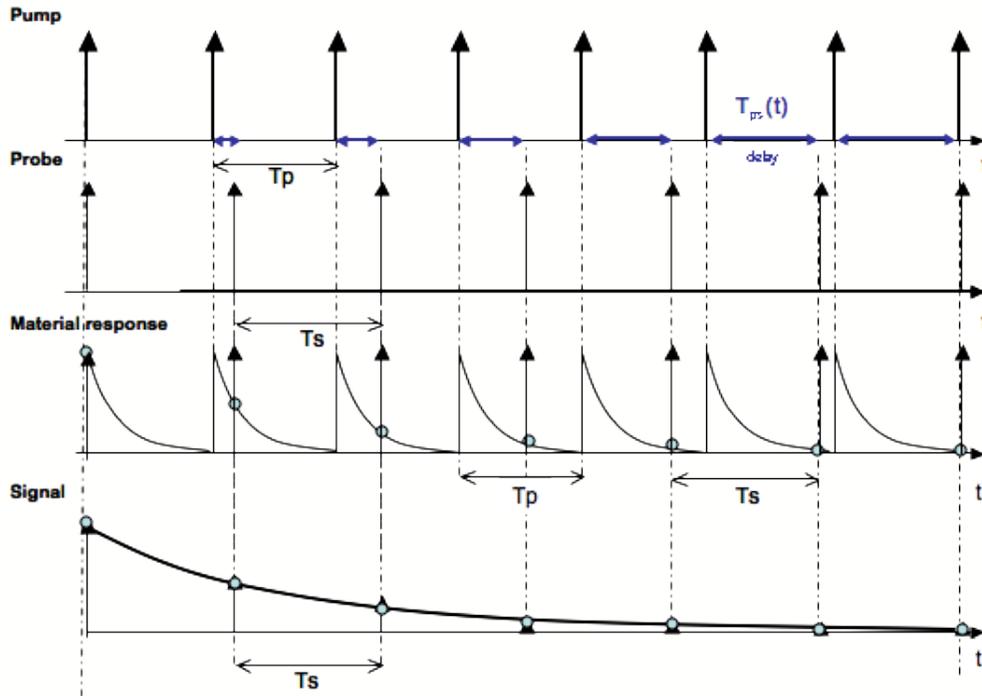
Film IR (x10) lors de la congélation

Thermoréfectance pulsée (S. Dilhaire)



Principe de l'hétérodyne (ou stroboscopie)

Réponse thermique à un impact laser de 100fs.

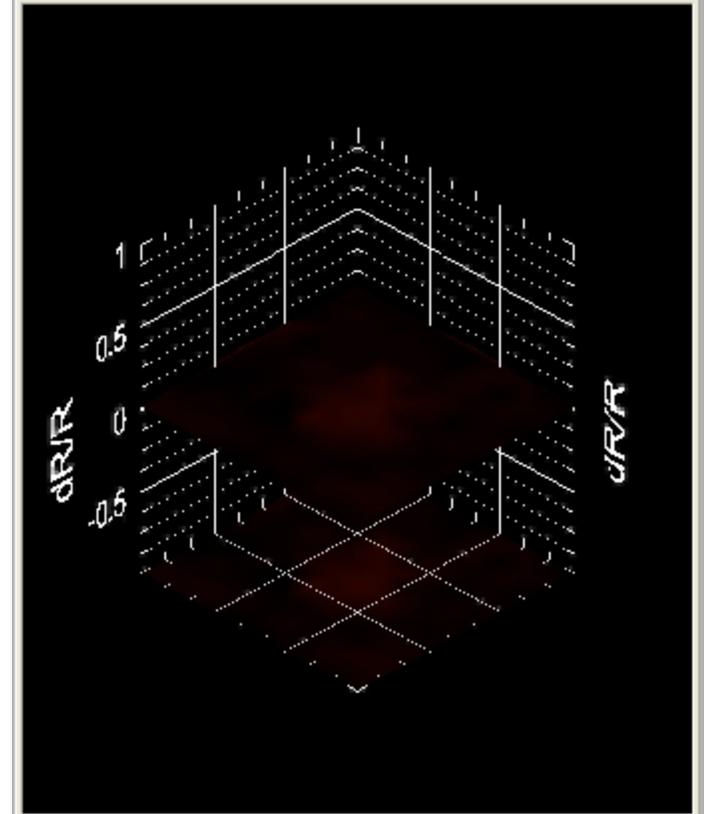


$T_p = 10\text{ns}$

$T_s = 10\text{ns} + 1\text{ps}$

100MHz - 1THz

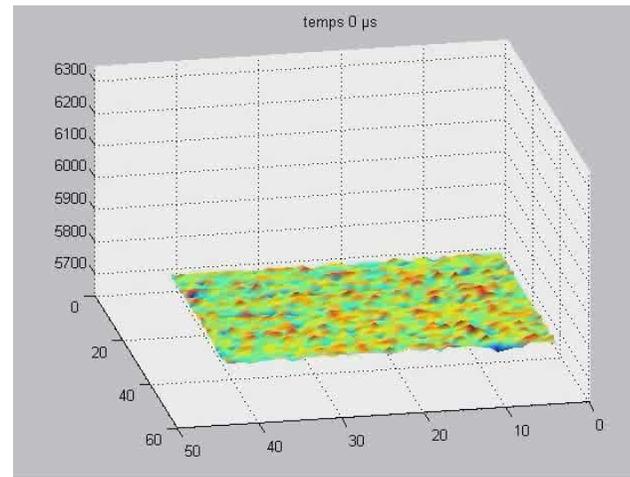
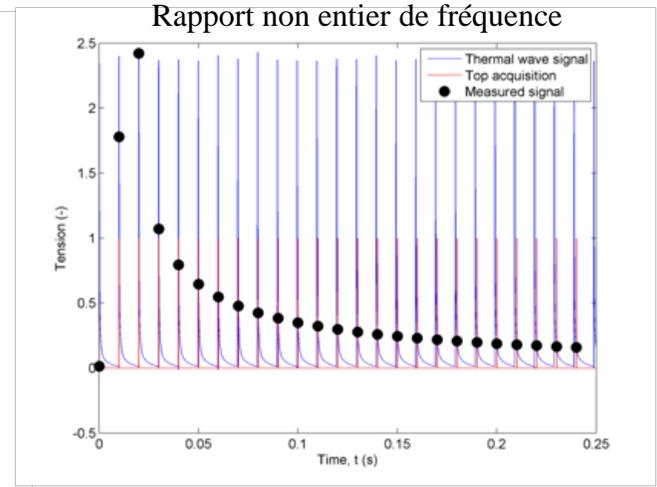
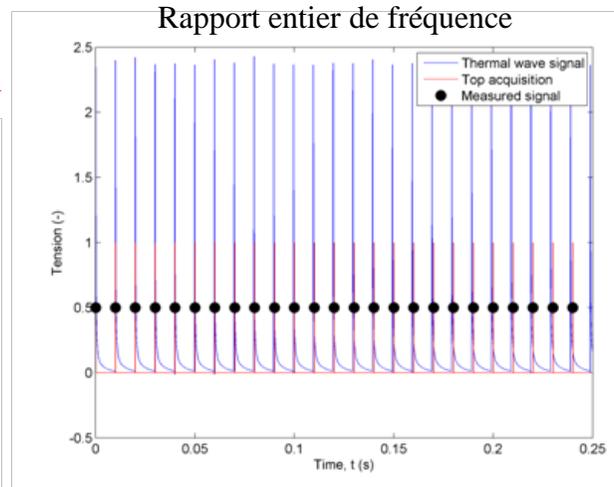
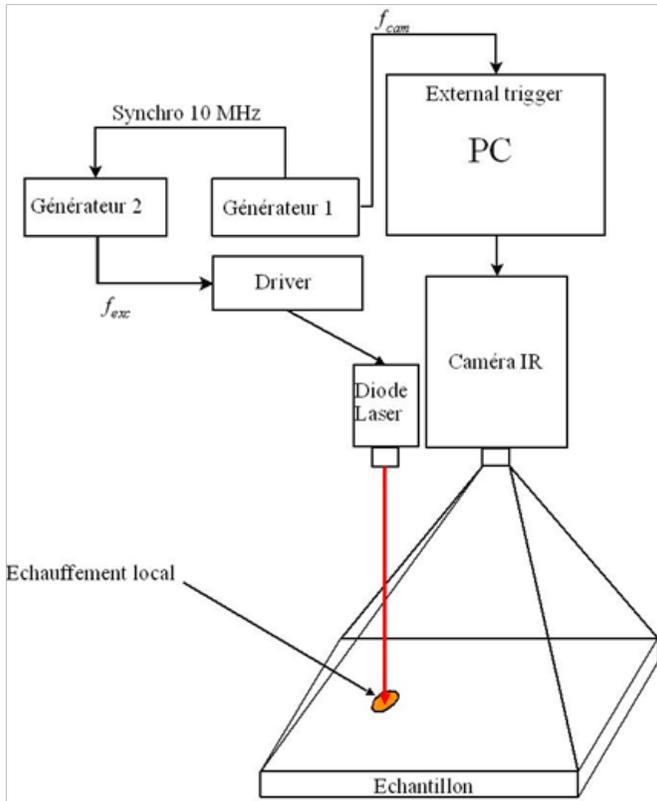
Delay = -10.0 ps



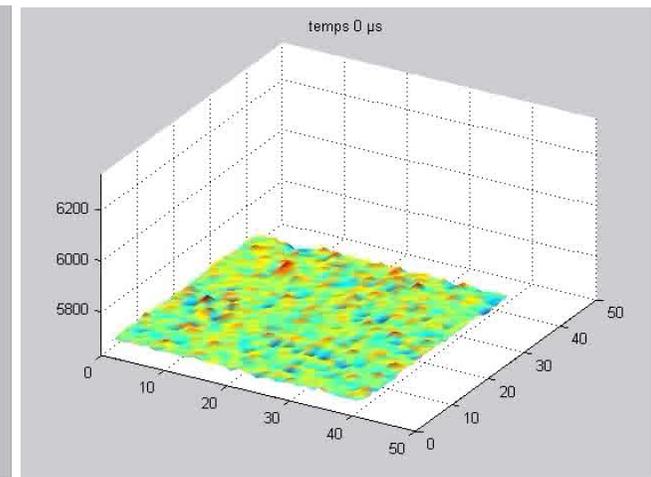
Champ imagé 5x5 microns

Thermographie IR hétérodyne

Principe de l'hétérodynage par TIR

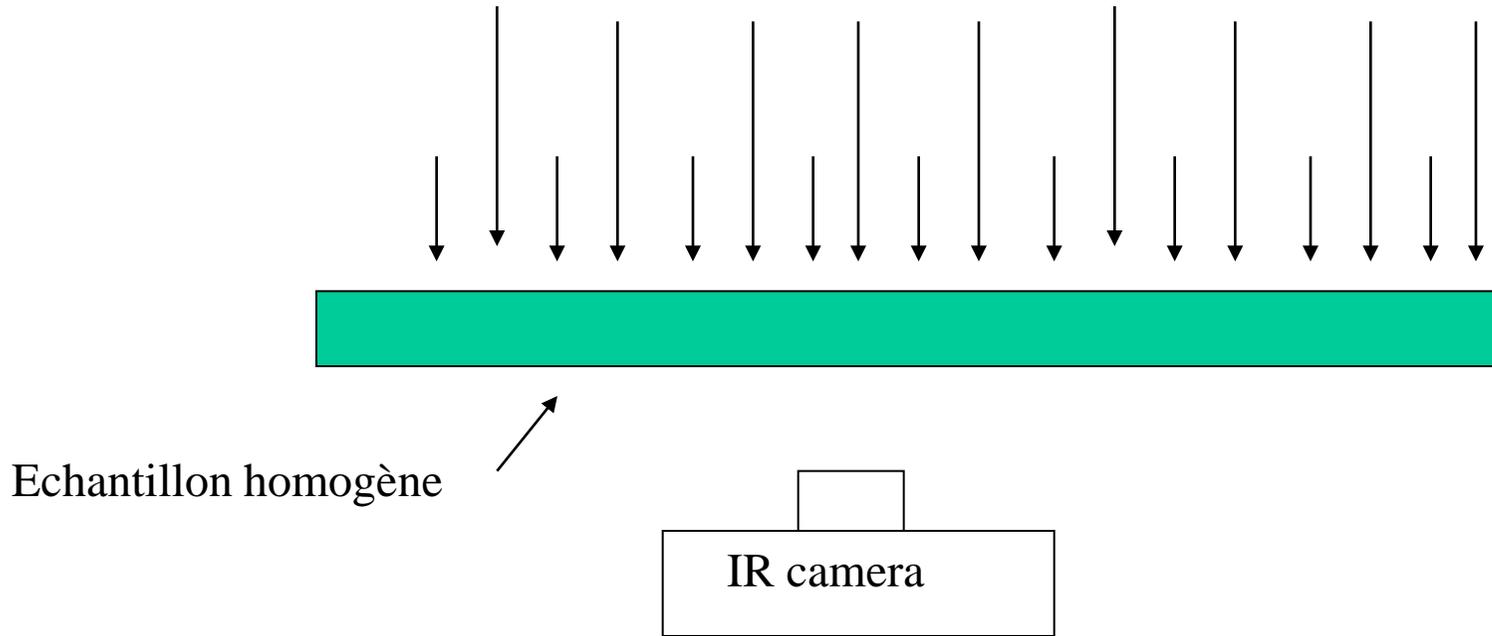


Film IR à 1 kHz sans hétérodynage



Film IR à 500 kHz avec hétérodynage

Application avec caméra IR au cas impulsionnel en milieu homogène

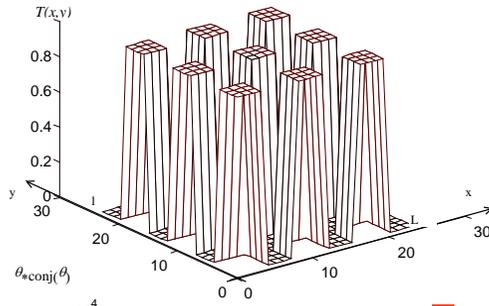


➔ Analyse de la transformation de Fourier spatiale du champ de température

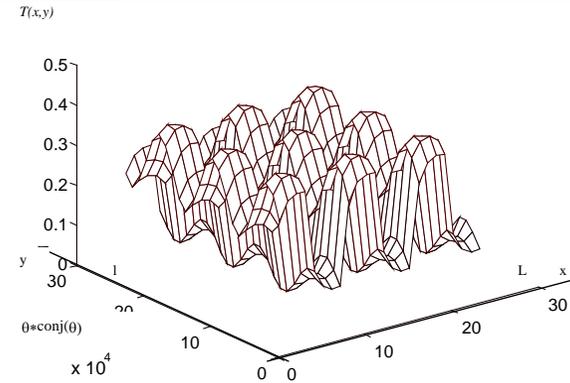
Philippi I., Batsale J.C., Maillet D. et Degiovanni A. : (1995) Measurement of thermal diffusivity through processing of infrared images processing, Rev. Sci. Instru., 66(1), pp182-192.
Krapez J.C., 1999 Mesure de diffusivité longitudinale de plaques minces par méthode de grille-
 Journée SFT: "Thermographie IR quantitative" ONERA Mars 1999

Application avec caméra IR au cas impulsionnel en milieu homogène

Images réelles

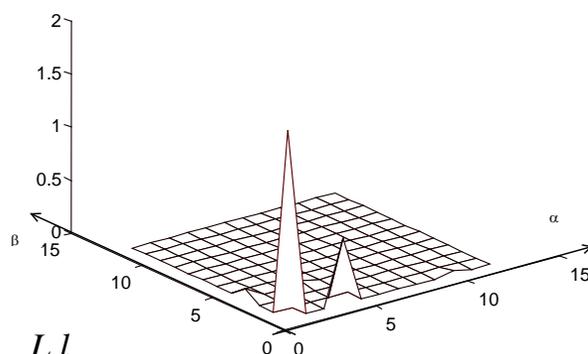
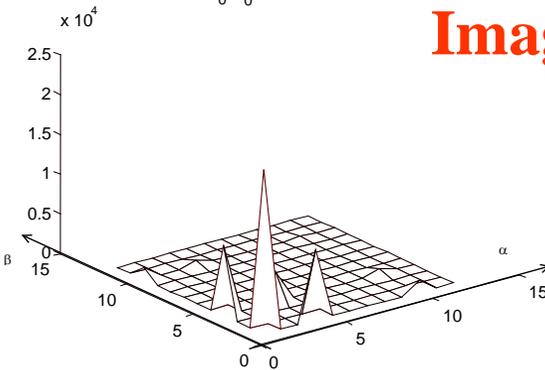


$t=0$



$t > 0$

Images transformées



Transformée de Fourier

$$\Theta(\alpha_n, \beta_m, z, t) = \int_0^L \int_0^L \cos(\alpha_n x) \cos(\beta_m y) \cdot T(x, y, z, t) \cdot dx dy$$

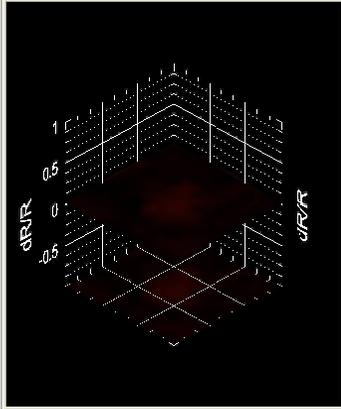
Expression 1D espace de Fourier $\theta(t, \alpha) = \theta(t = 0, \alpha) * \exp(-a\alpha^2 t - ht)$

Méthode d'estimation

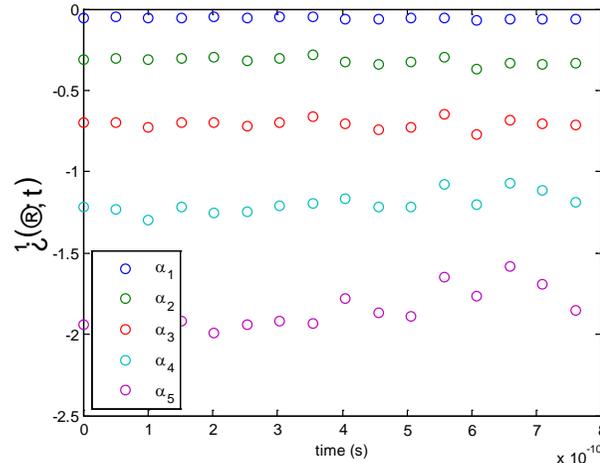
$$\ln \frac{\theta(\alpha, e, t_2)}{\theta(\alpha, e, t_1)} = \ln \frac{\theta(0, e, t_2)}{\theta(0, e, t_1)} - a_x \cdot \alpha^2 \cdot (t_2 - t_1)$$

Validation sur les mesures hétérodynes

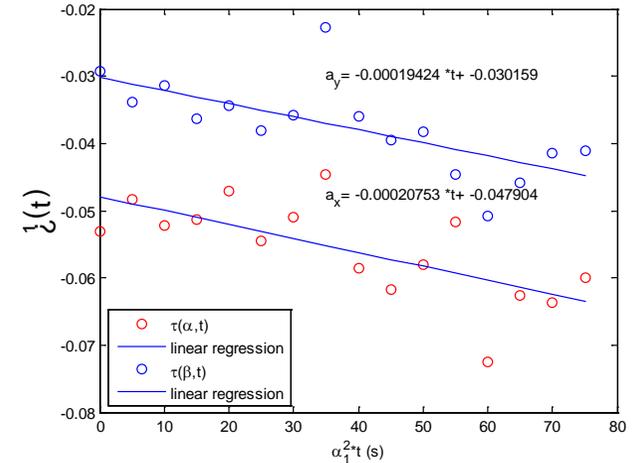
Delay = -10.0 ps



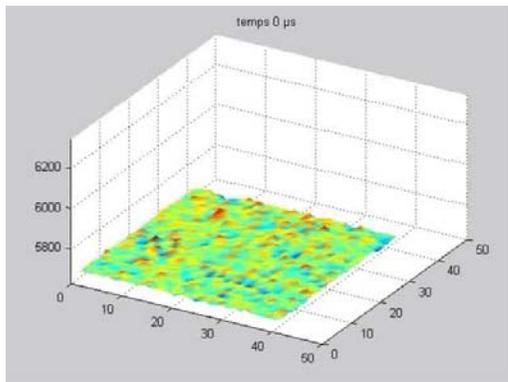
Film TR ps



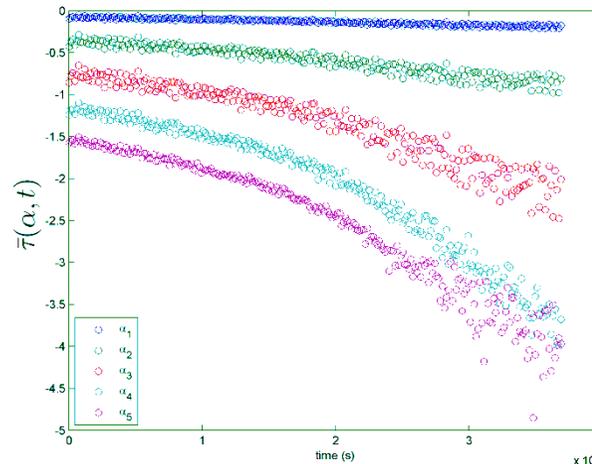
Transformée de Fourier selon x à différentes fréquences



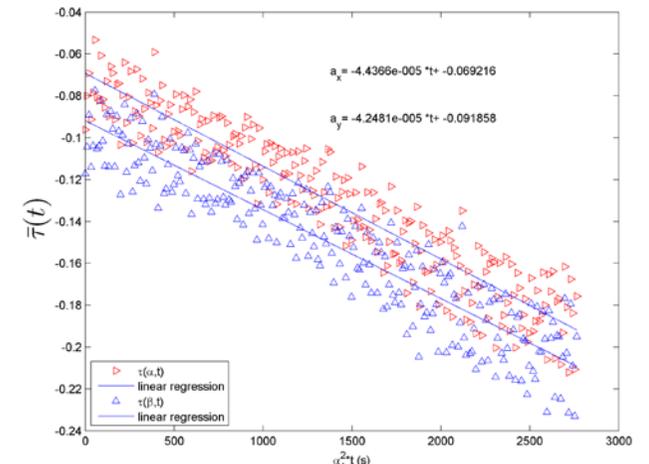
TF selon x et y, estimation de la diffusivité



Film TIR μs



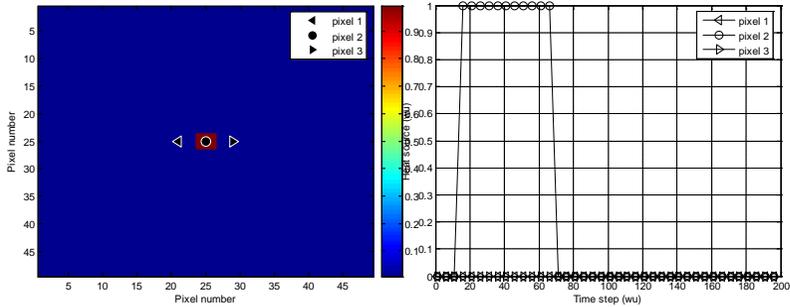
Transformée de Fourier selon x à différentes fréquences



TF selon x et y, estimation de la diffusivité

Estimation de diffusivité sur 5x5 μm par TR et sur 1x1 mm par TIR

Formulation du problème inverse sur les simulations



Evolution spatiale et temporelle du terme source

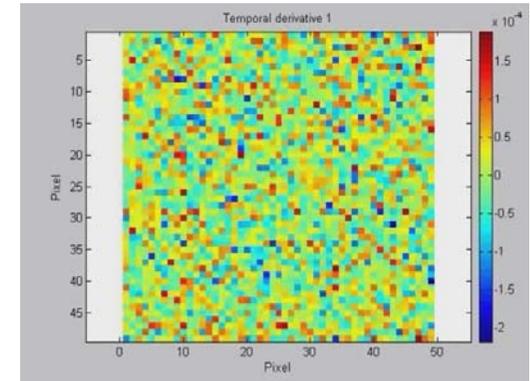
Comment estimer la diffusivité thermique et le terme source ?

$$Fo_{i,j} \Delta T_{i,j}^k + \Phi_{i,j}^k = \delta T_{i,j}^k$$

$$\Delta T_{i,j}^k = (T_{i+1,j}^k + T_{i-1,j}^k + T_{i,j+1}^k + T_{i,j-1}^k - 4T_{i,j}^k)$$

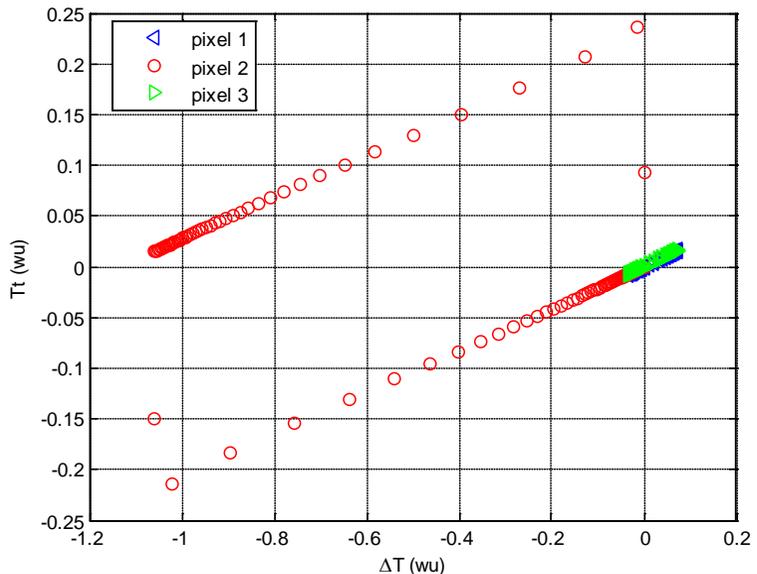
$$\delta T_{i,j}^k = T_{i,j}^{k+1} - T_{i,j}^k$$

$$Fo_{i,j} = \frac{a_{i,j} \Delta t}{\Delta x^2} \quad \Phi_{i,j}^k = \frac{\varphi_0 \Delta t}{\rho C_p}$$

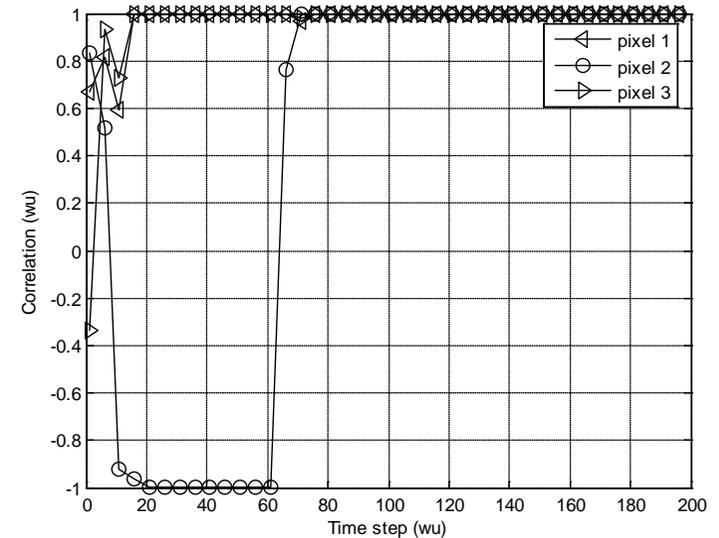


Champ de température simulé

Cherche relation linéaire = tracer le Laplacien en fonction de la dérivée temporelle



$$\rho_{i,j}^{F_t} = \frac{\sum \Delta T_{i,j}^k \delta T_{i,j}^k}{\|\Delta T_{i,j}^k\|_{F_t} \|\delta T_{i,j}^k\|_{F_t}} = \langle \Delta T, \delta T \rangle \rightarrow \pm 1$$



Calcul de la corrélation entre le Laplacien et la dérivée temporelle

$$\rho_{i,j}^{F_t} = \frac{\sum_{F_t} \Delta T_{i,j}^k \delta T_{i,j}^k}{\|\Delta T_{i,j}^k\|_{F_t} \|\delta T_{i,j}^k\|_{F_t}} = \langle \Delta T, \delta T \rangle \rightarrow \pm 1$$

Si corrélation $\Rightarrow 1$

Inversion à 1 paramètre + masque de corrélation

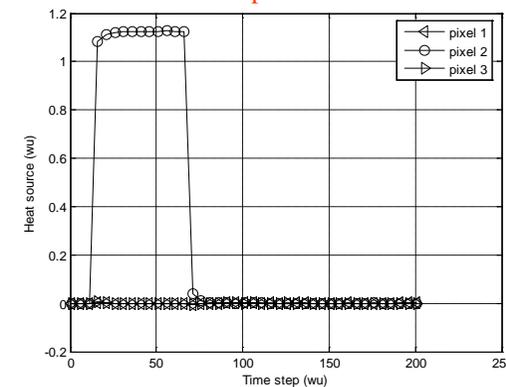
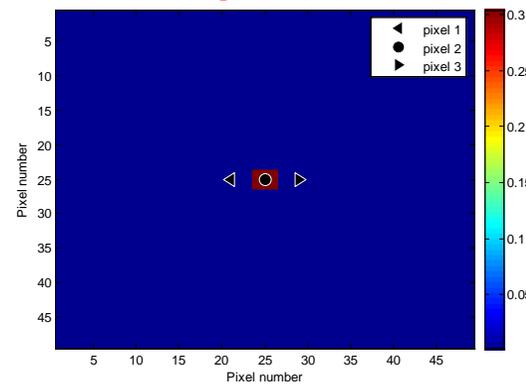
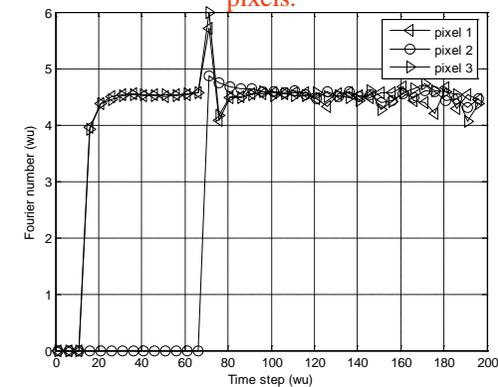
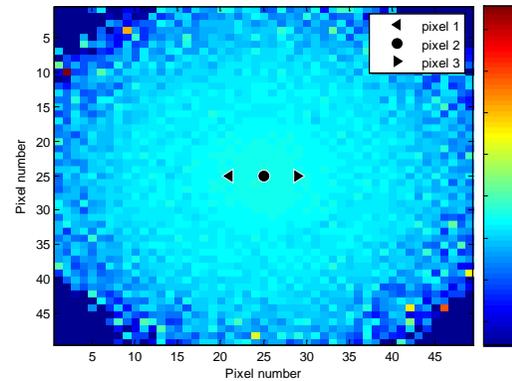
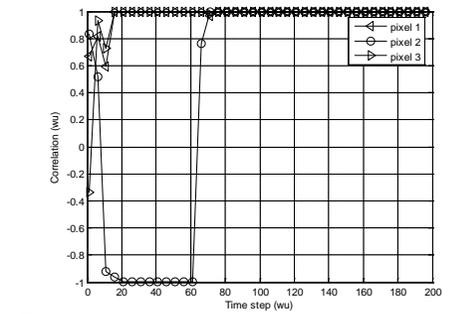
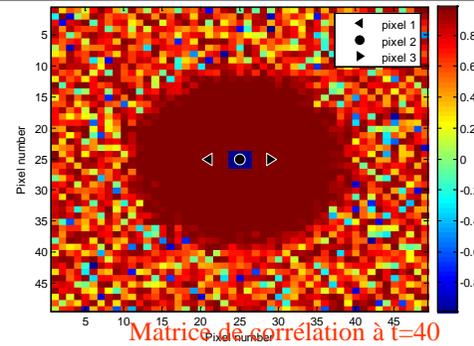
Estimation

$$\frac{1}{Fo_{i,j}^{F_t}} = \frac{\sum_{F_t} \Delta T_{i,j}^k \delta T_{i,j}^k}{\|\delta T_{i,j}^k\|_{F_t} \|\delta T_{i,j}^k\|_{F_t}} = \frac{\rho_{i,j}^{F_t} \|\Delta T_{i,j}^k\|_{F_t}}{\|\delta T_{i,j}^k\|_{F_t}^{est}} = \sum_{k=1}^{N-F_t} \frac{1}{Fo_{i,j}^k} \cdot (\rho_{i,j}^k \rightarrow 1) / \sum_{k=1}^{N-F_t} (\rho_{i,j}^k \rightarrow 1)$$

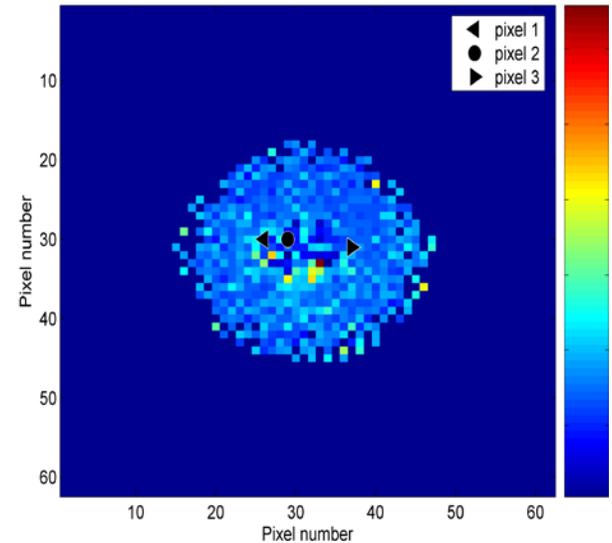
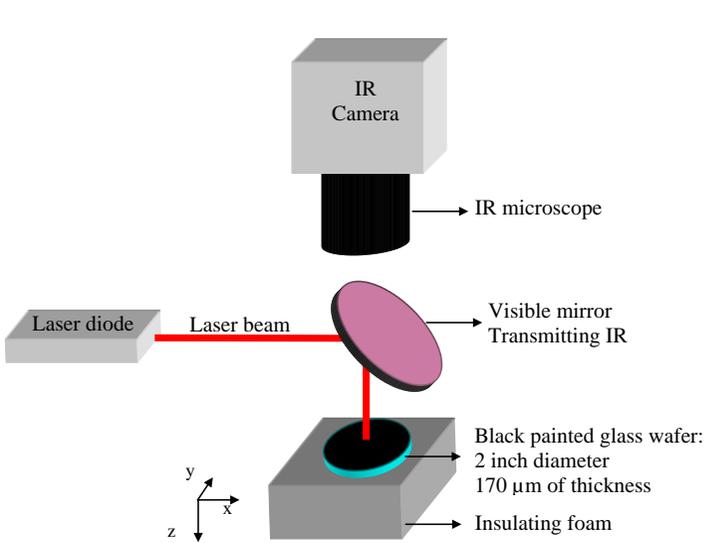
Calcul du terme source avec la diffusivité estimée

$$\Phi_{i,j}^k = (\delta T_{i,j}^k - Fo_{i,j}^{est} \Delta T_{i,j}^k) (\rho_{i,j}^k \rightarrow -1)$$

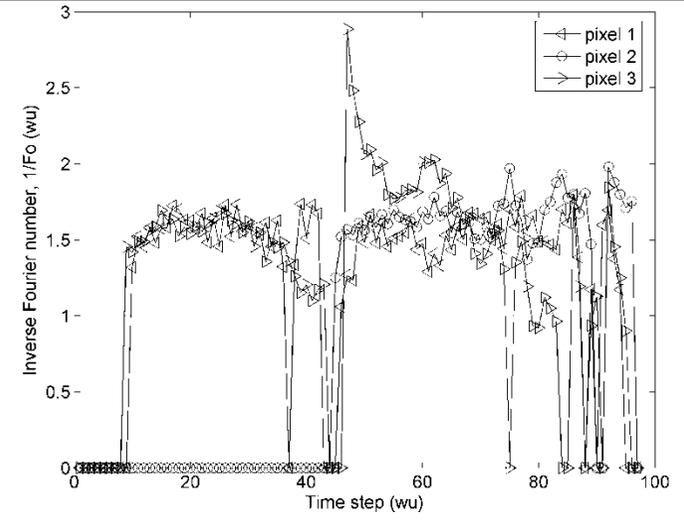
$$\Phi_{i,j}^{est} = \sum_{k=1}^{N-F_t} \Phi_{i,j}^k (\rho_{i,j}^k \rightarrow -1) / \sum_{k=1}^{N-F_t} (\rho_{i,j}^k \rightarrow -1)$$



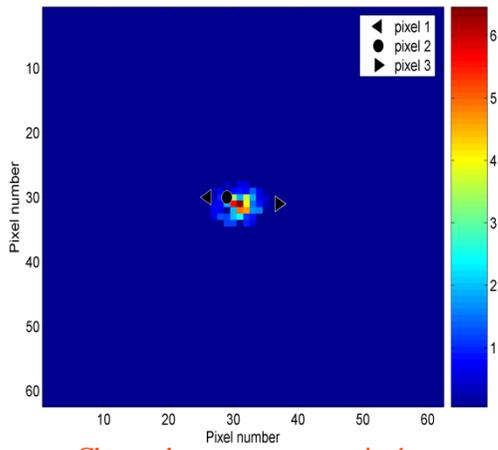
Application expérimentale à une lame de verre



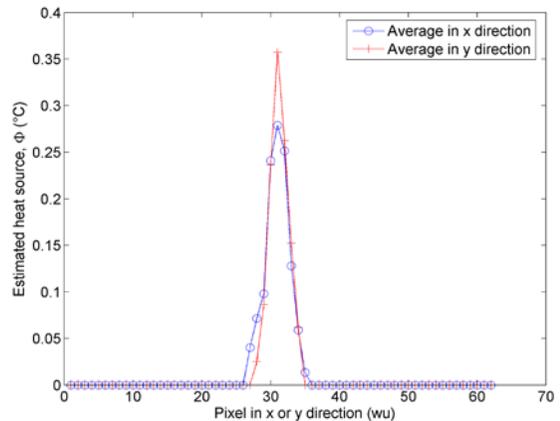
Champ de diffusivité estimé



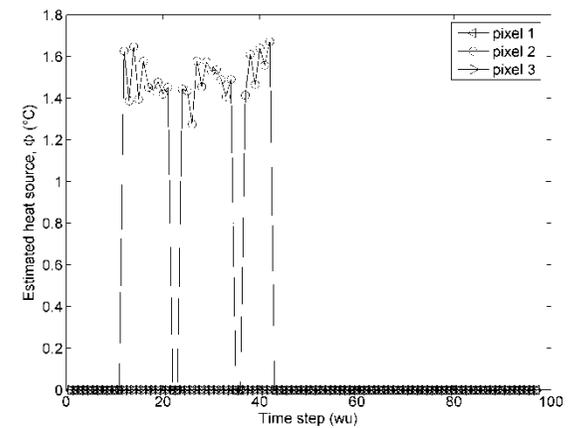
Evolution temporelle de la diffusivité



Champ de terme source estimé



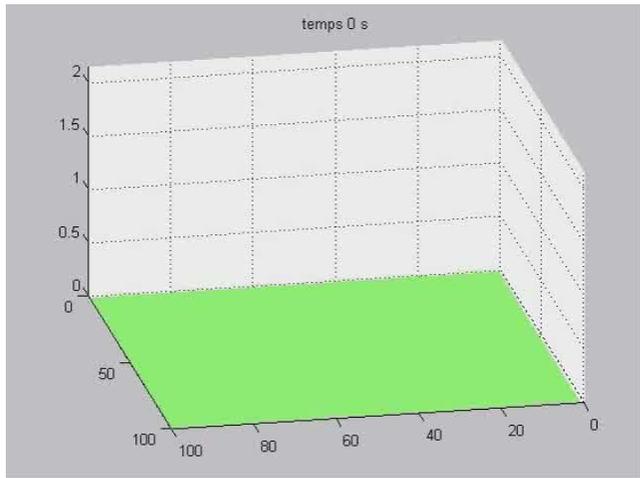
Répartition spatiale du terme source estimé



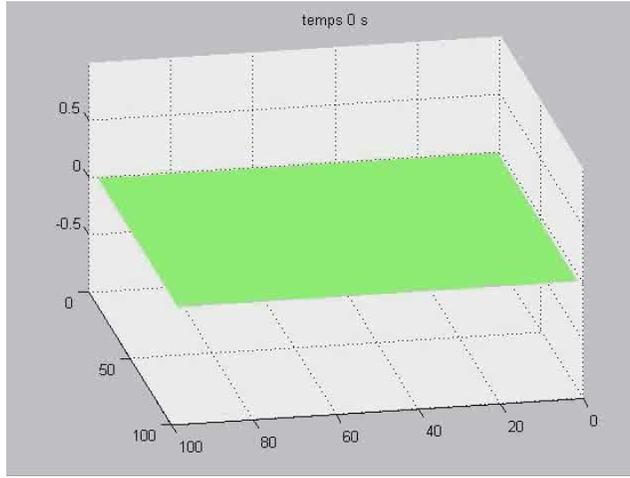
Evolution temporelle du terme source estimé

Diffusivité estimée $3.3 \cdot 10^{-7} \text{ m}^2/\text{s}$, Littérature $3.4 \cdot 10^{-7} \text{ m}^2/\text{s}$, Validation de la méthode

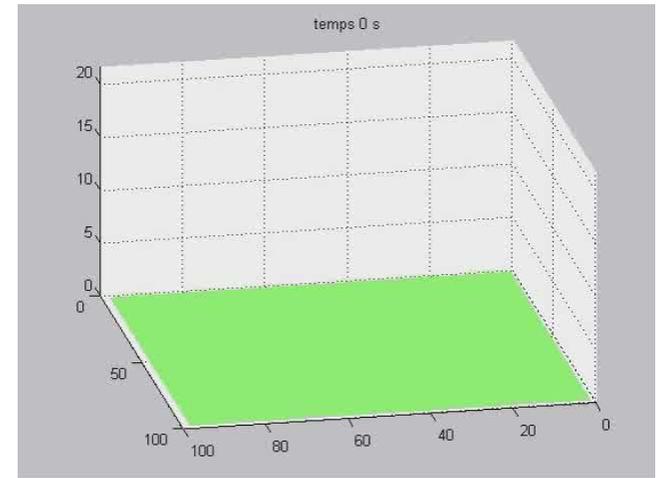
Exemple milieu hétérogène et excitation créneau aléatoire



Champ de température simulé



Champ de corrélation



Cartographie de Fourier estimé

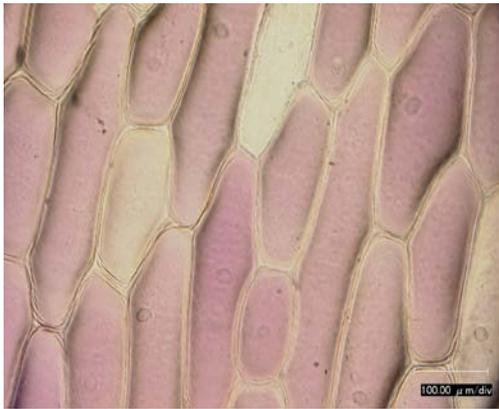


Image visible (x10) d'une pelure d'oignon

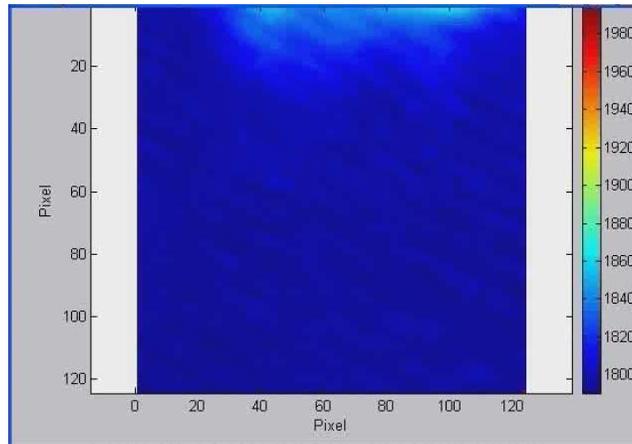
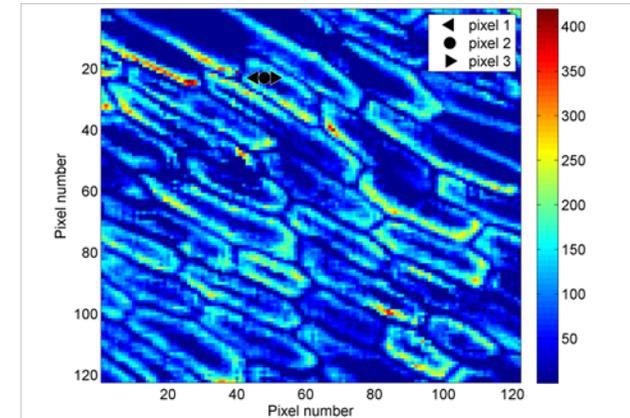


Image (x10) lors de la congélation



Cartographie de diffusivité obtenue

SVD/KLD techniques in association with lock-in thermography

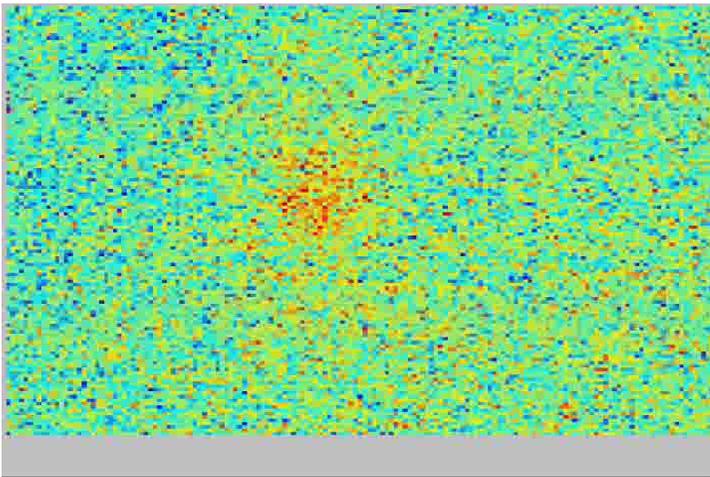
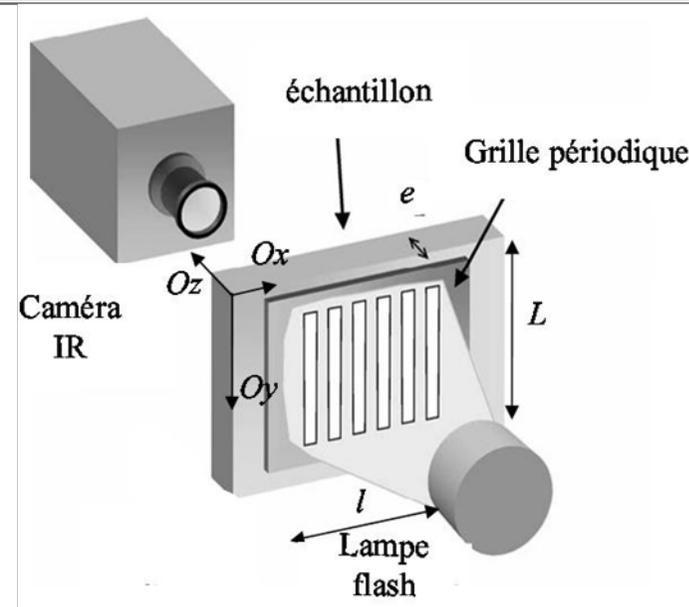
Huge amount of noisy data !!

Main assets

- No analytical solutions required. Few constraints concerning excitations (time & space patterns)
- Homogeneous & heterogeneous materials
- Optimum data and noise reduction
- Phases recognition and interfaces location
- Parsimonious estimates for intrinsic properties (diffusivities) and exchange coefficients
- Point-by-point and whole-domain approaches

Constraints

- Thin sample, 2D heat transfer models
- Spatially uncorrelated noise



SVD/KLD (PCA/POD/HT/...)
 A tool providing **optimal low-dimensional approximations** of high-dimensional sets of signals
 A tool providing **optimal** linear filtering of regular signals

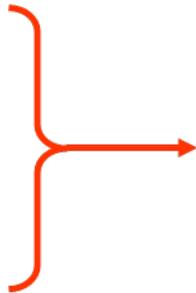
Energy function

$$W(x, x') \equiv \int_t T(x, t)T(x', t)dt$$

Spectral decomposition

$$W(x, x') = \sum_{m=1}^{\infty} \sigma_m^2 V_m(x)V_m(x')$$

$$\mathbf{x} = (x_1, x_2, x_3)$$



$$\forall t, T(x, t) = \sum_{m=1}^{\infty} V_m(x)z_m(t)$$

Eigenfunctions $V_m(x)$ forms a orthonormal basis

States $z_m(t)$ are orthogonal

Spatially-uncorrelated noise has no effect on eigenfunctions, the noise being entirely reported on states

Optimal low dimensional approximations
 (minimum norm of the approx. error)

$$\forall t, T_r(x, t) = \sum_{m=1}^r V_m(x)z_m(t)$$

Optimal linear filtering
 (minimum risk)

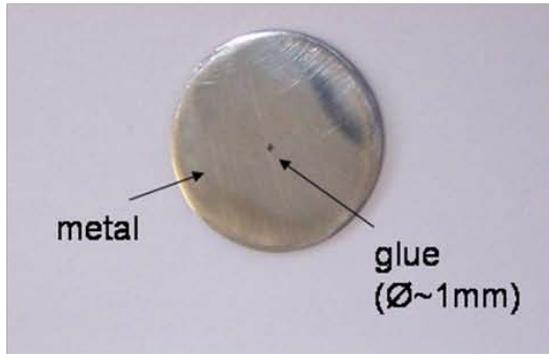
$$\hat{T}(x, t) = \sum_{m=1}^{\infty} V_m(x) \left(\frac{\sigma_m^2}{\sigma_m^2 + \sigma_{\epsilon, m}^2} \right) \tilde{z}_m(t)$$

- Thin samples
- Uniform heating
- Very short times (s)
- Very low noise sensitivity
- Many numerical tests
- Some experimental tests

SVD/KLD

An efficient tool for **phases discrimination and interfaces location** – Microstructure identification



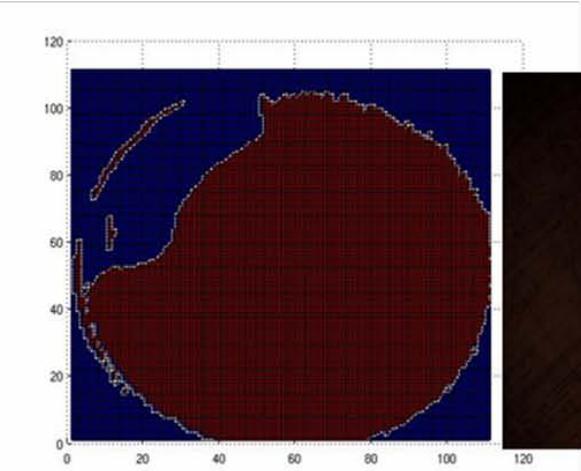
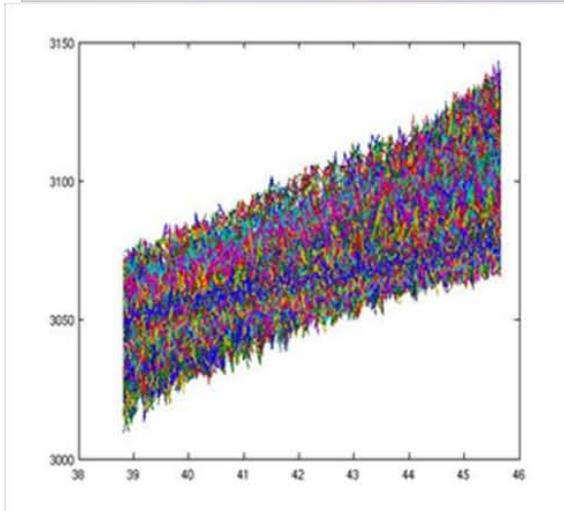


Number of phases

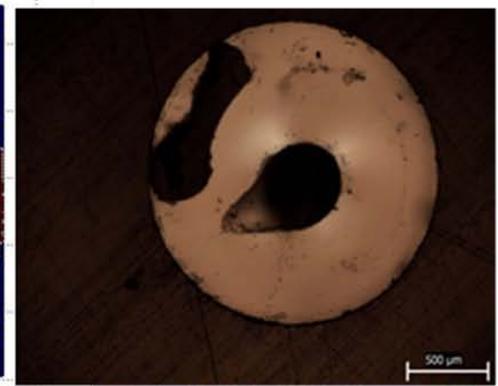
Rank of the energy matrix W = Number of phases

Eigenfunctions sign → Phases discrimination

$$\forall (x,y) \in \Omega_i \ \& \ (x',y') \in \Omega_j, \exists m / V_m(x,y)V_m(x',y') < 0$$

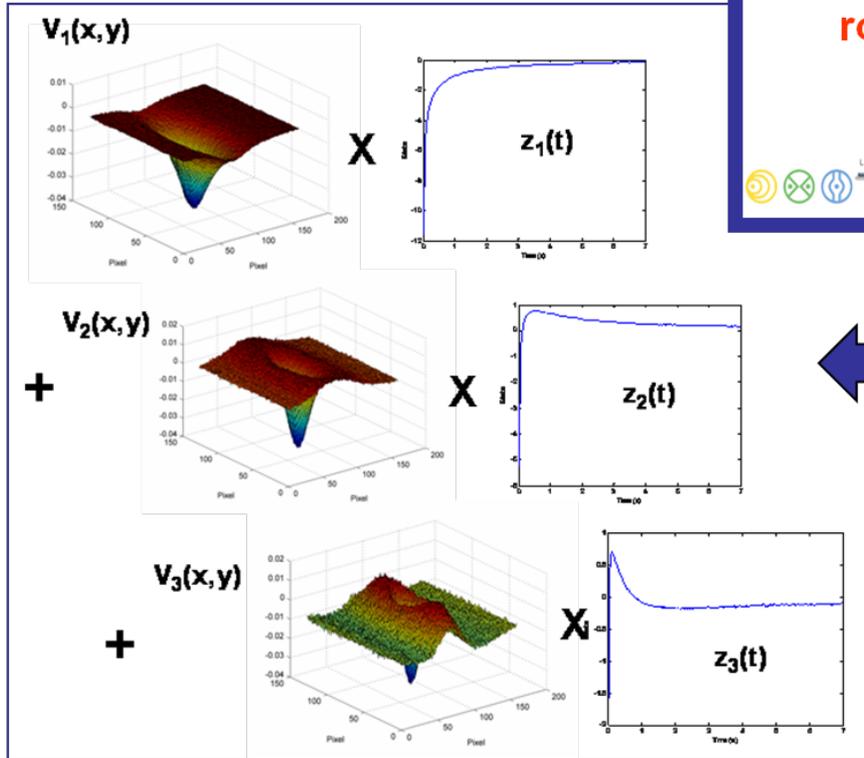


Phases and interfaces based on V2 sign



Optical Microscopic picture

Data required for estimations



Three surfaces

250 x 250 pixels
62500 data

almost noise free

Three time series

2000 sampling times
6000 data

optimal filtering

SVD/KLD

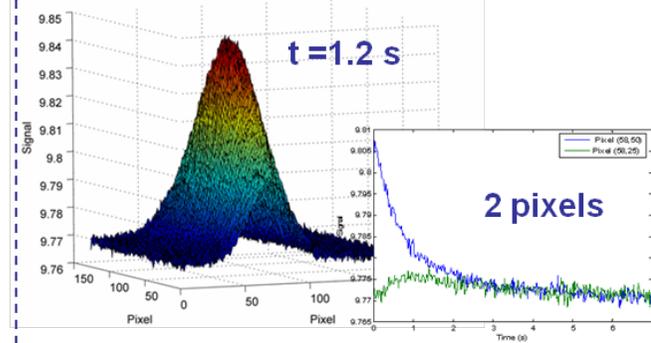
A tool providing **parsimonious & noise-robust estimates** for homogeneous and heterogeneous materials characterization



Raw $T(x,y,t)$ data

250 x 250 pixels
2000 sampling times

125 millions of noisy data



68500 data
Strong noise reduction

Fundamental equations

KLD of T - Heat transfer equations + Integral theorems + V&z Orthogonality

Homogeneous orthotropic material

$$\frac{z_i(t)z_k(t)}{\sigma_i^2 + \sigma_k^2} \Big|_{t=0}^{t_f} = \alpha_x \left\langle \frac{\partial^2 V_i(x,y)}{\partial x^2}, V_k(x,y) \right\rangle_{\Omega} + \alpha_y \left\langle \frac{\partial^2 V_i(x,y)}{\partial y^2}, V_k(x,y) \right\rangle_{\Omega} - \beta \delta_{ik}$$

Two-phases material with Rth at the interfaces

$$\frac{z_k^2(t)}{2\sigma_k^2} \Big|_{t=0}^{t_f} = \sum_{i=1,2} \alpha_i \langle \nabla^2 V_k, V_k(x) \rangle_{\Omega_i} - \sum_{i=1,2} \beta_i \langle V_k(x), V_k(x) \rangle_{\Omega_i}$$

Linear least squares method
 applied on some few equations
 which are automatically chosen



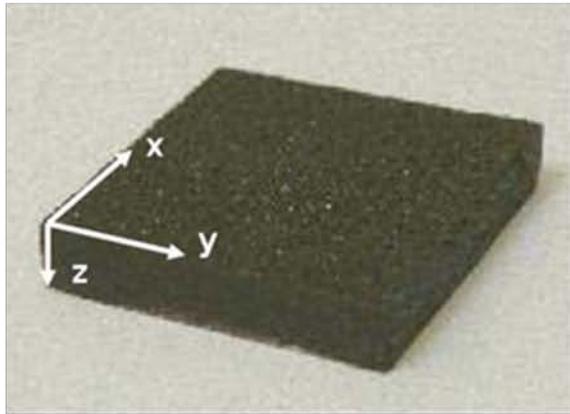
Equations selection
 Number of unknowns
 Noise propagation through KLD

Heterogeneous plate: Estimated values for thermal parameters.

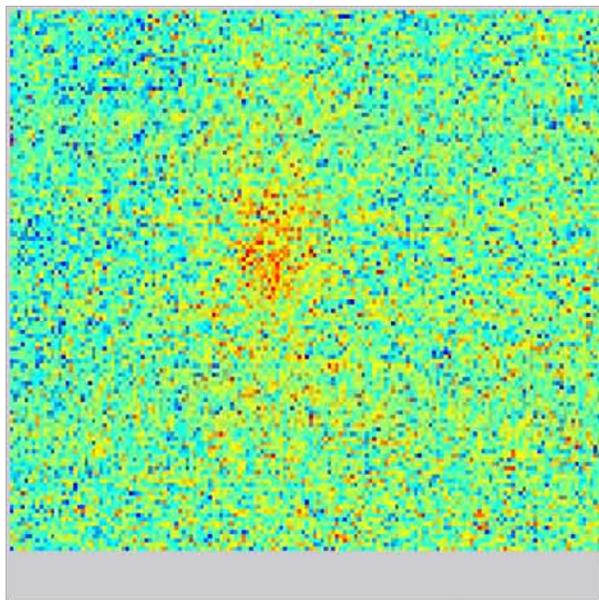
	Noise amplitude (°C)		
	± 0.50	± 0.10	± 0.02
$\hat{\alpha}_1 (\times 10^{-6} m^2 s^{-1})$	0.1524	0.1526	0.1511
$ (\alpha_1 - \hat{\alpha}_1) / \alpha_1 \times 100$	0.60	0.73	0.23
$\hat{\alpha}_2 (\times 10^{-6} m^2 s^{-1})$	0.5001	0.5013	0.5044
$ (\alpha_2 - \hat{\alpha}_2) / \alpha_2 \times 100$	1.00	0.74	0.12

Numerical example – 2 phase plate – Laser spot

CARBON/EPOXY
 composite material



No VACUUM



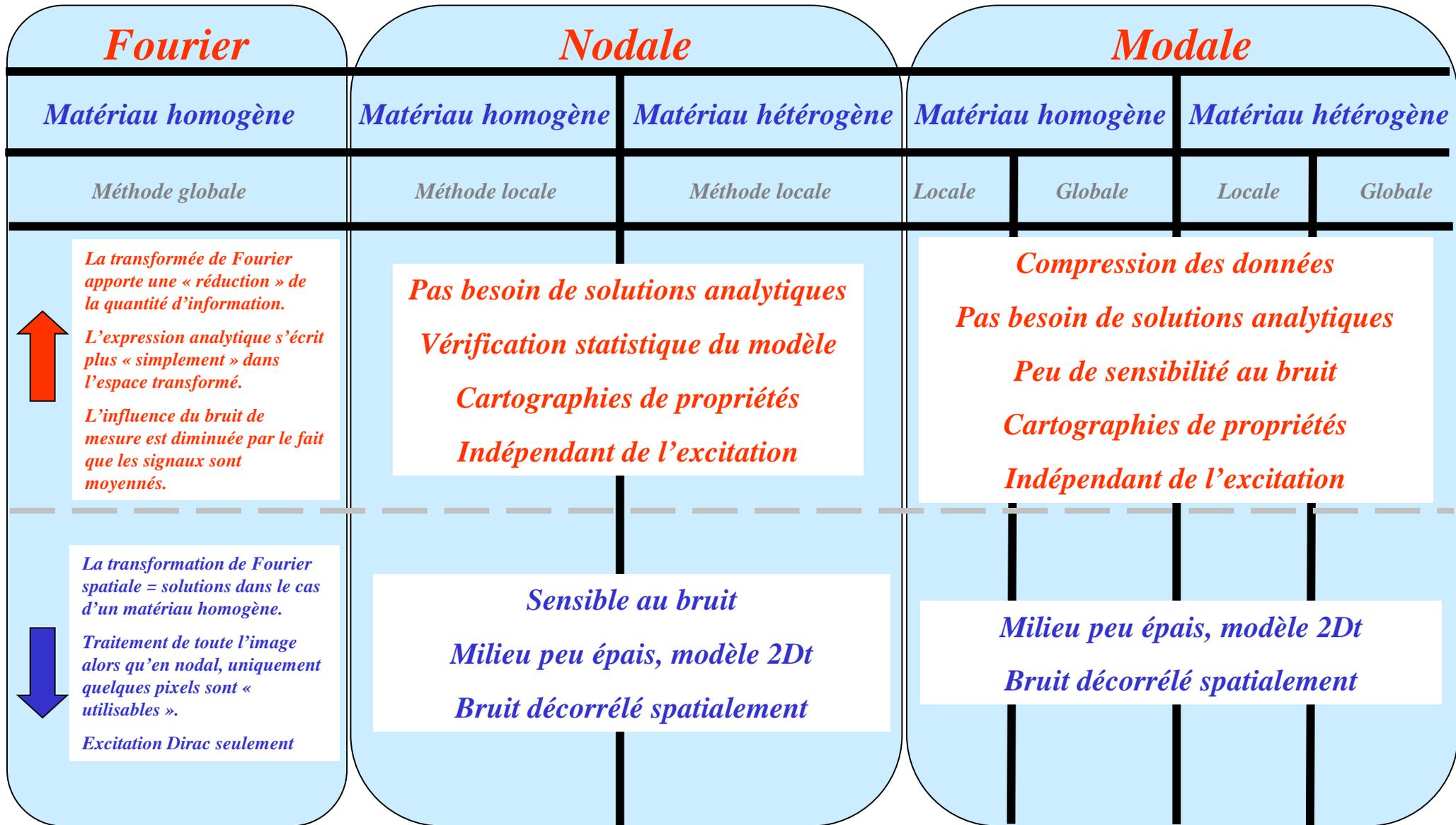
Experimental test

$$\left. \frac{z_i(t)z_k(t)}{\sigma_i^2 + \sigma_k^2} \right|_{t=0}^{t_f} = \alpha_x \left\langle \frac{\partial^2 V_i(x,y)}{\partial x^2}, V_k(x,y) \right\rangle_{\Omega} + \alpha_y \left\langle \frac{\partial^2 V_i(x,y)}{\partial y^2}, V_k(x,y) \right\rangle_{\Omega} - \beta \delta_{ik}$$

3 equations
 M (3 x 3)

$$\begin{bmatrix} \alpha_x \\ \alpha_y \end{bmatrix} = (M'M)^{-1}(M'Y)$$

RESULTS (~100°C)	ICAM CETHIL	TREFLE
α_x (m ² /s)	4×10^{-7}	3.80×10^{-7}
α_y (m ² /s)	3.5×10^{-6}	3.54×10^{-6}



Dispositifs de mesure

Mesure de champs de température de $0.5 \mu\text{m}$ au m

Mesure de champs de température de 10^{-12} à 10^3 s

Tous types d'excitation périodiques ou pas

Méthodes inverses

Permettent des caractérisations multiéchelles (locale et globale)

Méthodes nodale et modale permettent tout type d'excitation (spatiale et temporelle)

Caractériser matériau hétérogènes (nodales et modales)

Repérage des interfaces (modales), modélisation analytique par domaine homogènes

Validations

Confrontation des 3 méthodes (Fourier, nodale et modale) sur matériau homogène

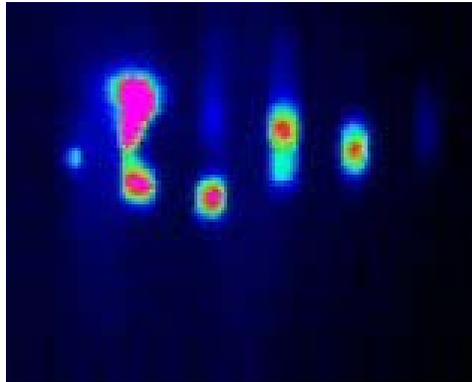
Confrontation des 3 méthodes aux différentes résolutions (spatiales et temporelles)

Application à des matériaux hétérogènes

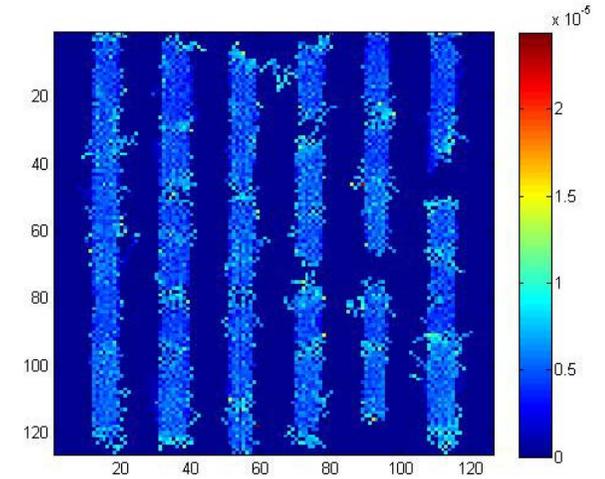
Estimation simultanée et aléatoire de diffusivités et de termes sources



Image visible de l'échantillon



Film IR, balayage faisceau laser



Cartographie de diffusivité obtenue

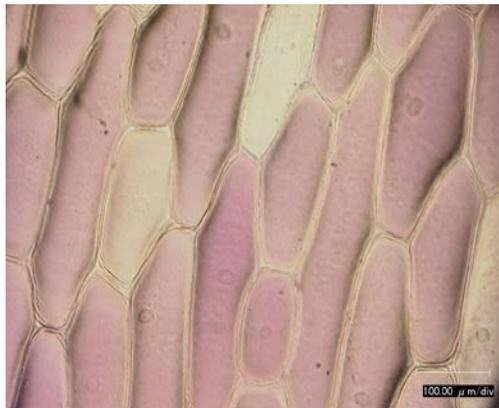
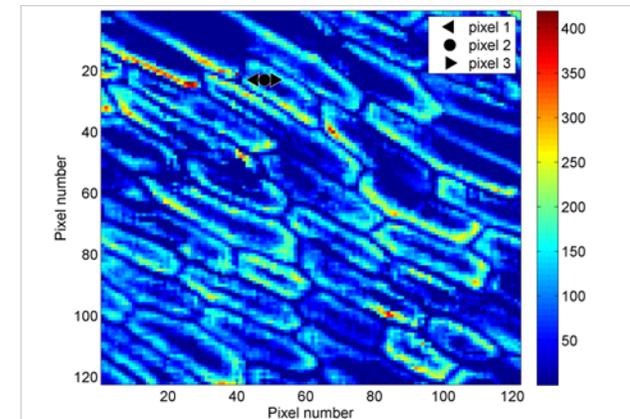
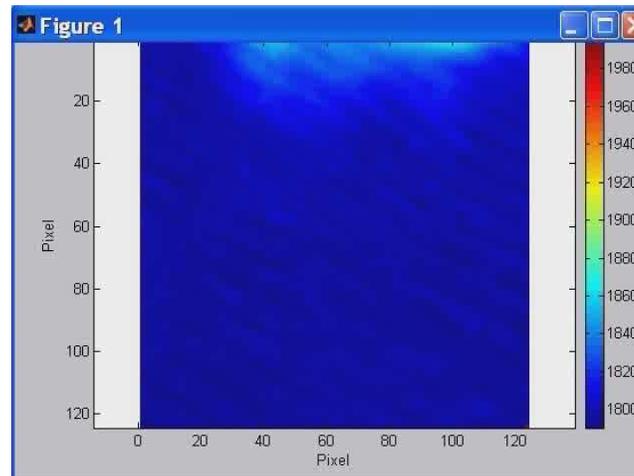


Image visible (x10) d'une pelure d'oignon



Cartographie de diffusivité obtenue