"Caractérisation dans le plan de matériaux isolants par thermographie infrarouge et méthode convolutive"

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" Measurement of the In-Plane Thermal Diffusivity of Materials by Infrared Thermography " (DOI: 10.1007/s10765-005-4511-z)
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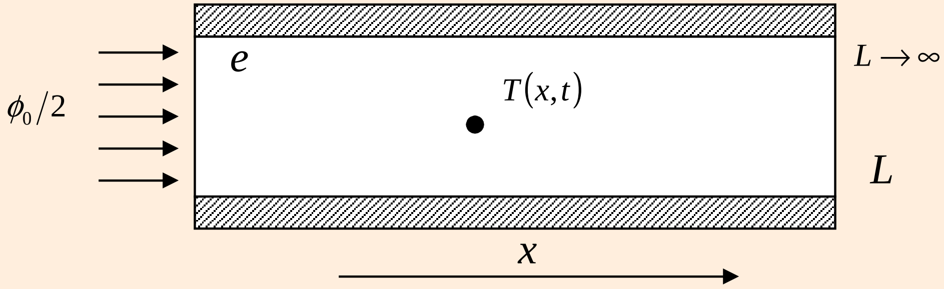
The in-plane diffusivity measurement techniques ①

- Solution in the case of a semi-infinite medium submitted to a heat step stimulation :



Assumptions:
- Semi-infinite medium
- One directional heat transfer
- Insulated

$$T(x,t) = \frac{\phi_0 x}{\lambda}\sqrt{\frac{at}{x^2}}\left(\frac{1}{\sqrt{\pi}}\exp\left(-\frac{x^2}{4at}\right) - \sqrt{\frac{x^2}{4at}}\,erfc\left(\sqrt{\frac{x^2}{4at}}\right)\right)$$

a: thermal diffusivity

λ: thermal conductivity

Method ① : *1 position and 2 different times (**Steere, 1966** & **Harmathy, 1964**)*

$$\frac{T(x,2t)}{T(x,t)} = \frac{\sqrt{2}\,ierfc\left(\frac{1}{2\sqrt{2}}\sqrt{\frac{x^2}{at}}\right)}{ierfc\left(\frac{1}{2}\sqrt{\frac{x^2}{at}}\right)}$$

Another technique:

$$\frac{at_m}{e^2} = \frac{0{,}5\,\tau/t_m}{(1-\tau/t_m)\ln(1-\tau/t_m)^{-1}}$$

Steere, 1967

Square pulse (duration τ)

The in-plane diffusivity measurement techniques ②

Method ② : *2 different positions at a given time (**Katayama, 1969**)*

$$\frac{T_L}{T_0} = \exp\left(\frac{-L^2}{4at}\right) - \sqrt{\frac{\pi e^2}{4at}}\,erfc\left(\sqrt{\frac{L^2}{4at}}\right)$$

Drawbacks:

- Heat Flux Distribution must be perfectly known
- Heating element is ideal (no capacity) and in perfect contact with the sample
- Medium is assumed to be perfectly insulated
- *1D* Heat transfer

Method ③ : *Laplace transform (**Kavianipour & Beck, 1977**)* $\theta(p) = \mathcal{L}\left(T(t)\right) = \int_0^\infty T(t)\exp^{(-pt)}\,dt$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{a_x}\frac{\partial T}{\partial t} \xrightarrow{\mathcal{L}} \frac{\partial^2 \theta}{\partial x^2} = \frac{p}{a_x}\theta$$

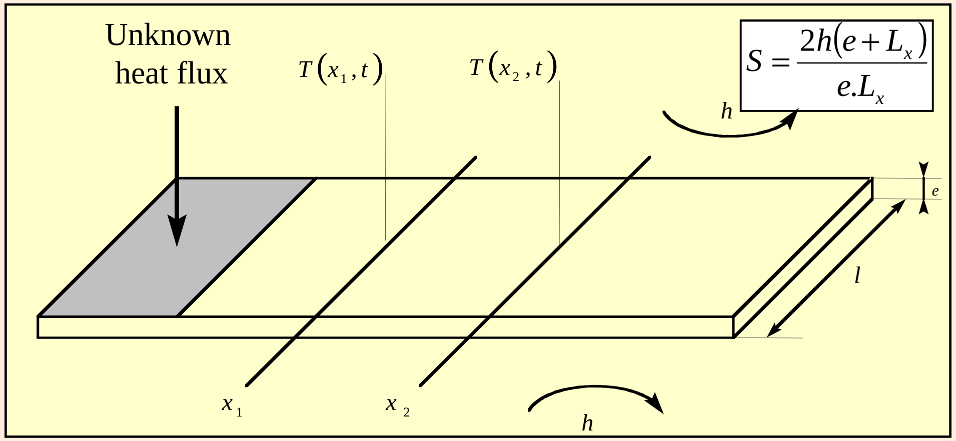
$$\theta(x,p) = \frac{\varphi_0(p)}{\lambda\sqrt{p/a_x}}\exp\left(-\sqrt{p/a_x}\,x\right)$$

$$\ln^2\left(\frac{\theta(x_2,p)}{\theta(x_1,p)}\right) = (x_2 - x_1)^2\,\frac{p}{a_x}$$

The solution is independent of the in-time heat flux distribution

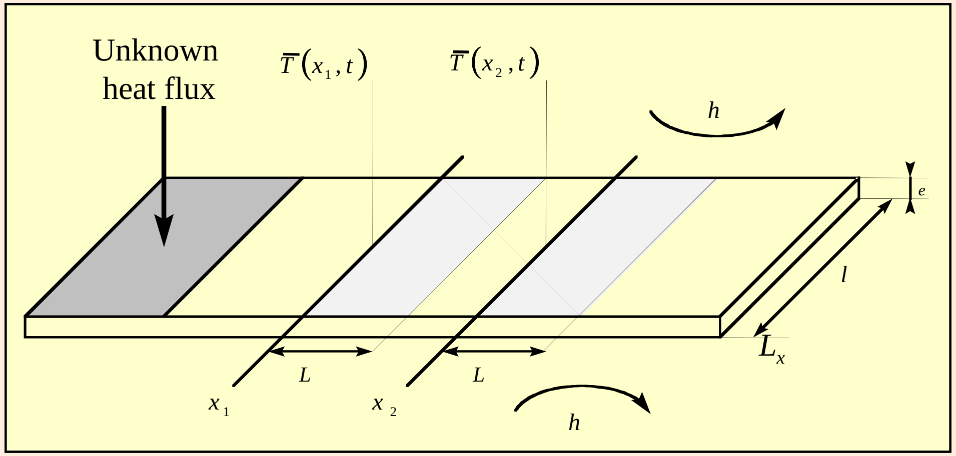
The in-plane diffusivity measurement techniques ③

Method ④ : *Fin's method (**Hadisaroyo, 1992**)*



$$\frac{\partial^2 T}{\partial x^2} - \frac{S}{\lambda_x}\left(T - T_{ext}\right) = \frac{1}{a_x}\frac{\partial T}{\partial t} \quad \xrightarrow{L} \quad \frac{\partial^2 \theta}{\partial x^2} = \left(\frac{p}{a_x} + \frac{S}{\lambda_x}\right)\theta$$

$$\ln^2\left(\frac{\theta(x_2, p)}{\theta(x_1, p)}\right) = (x_2 - x_1)^2\left(\frac{p}{a_x} + \frac{S}{\lambda_x}\right)$$

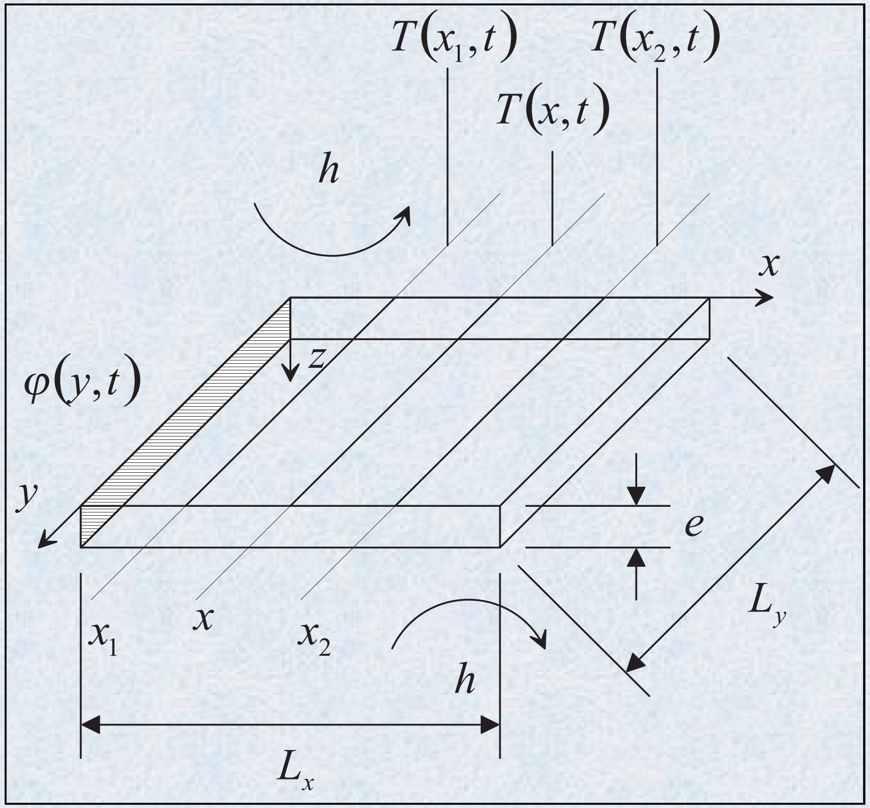


*Both diffusivity **a*** and heat losses ***h***
are taken into account
(semi-infinite medium is assumed)

Interest of the infrared camera:
- Non-necessary in space uniform stimulation
- *1D* transfer in the case of insulating materials

The *Fin*'s method in Transitory Regime

To measure λ_x, the sample is stimulated in x=0 by a non necessary uniform and constant heat flux $\varphi(y,t)$ or temperature step $T_0(y,t)$. The averaged temperature is then considered:



$e << L_x$ or L_y *Fin's Approximation*

$$\Delta\overline{T}(x,t) = \overline{T}(x,t) - T_{ext}$$

- Heat Transfer Equation :

$$\frac{\partial^2 \Delta\overline{T}}{\partial x^2} - \frac{2h(e+L)}{eL}\frac{\Delta\overline{T}}{\lambda_x} = \frac{1}{a_x}\frac{\partial \Delta\overline{T}}{\partial t}$$

- Boundaries Conditions :

in $x=0$ $\quad -\lambda_x \left.\frac{\partial \Delta\overline{T}}{\partial x}\right|_{x=0} = \overline{\varphi}_0(t)$ or $\left.\Delta\overline{T}\right|_{x=0} = \overline{T}_0(t) - T_{ext}$

in $x=L_x$ $\quad -\lambda_x \left.\frac{\partial \Delta\overline{T}}{\partial x}\right|_{x=L} = \overline{\varphi}_L(t)$ or $\left.\Delta\overline{T}\right|_{x=L_x} = \overline{T}_L(t) - T_{ext}$

- Initial Condition :

$\Delta\overline{T} = 0$ at $t=0$

Theoretical Model: *Solution in the Laplace domain*

The solution is obtained in the Laplace domain (integral transform)

$$F(p) = \mathcal{L}\left(f(t)\right) = \int_0^\infty f(t)\exp^{(-pt)}\,dt$$

The Quadrupole formulation allows to linearly links the Laplace transforms of the inner and outer temperatures and fluxes:

$$\theta = \mathcal{L}(T) \qquad \Phi = \mathcal{L}(\varphi)$$

$$\begin{bmatrix}\theta_0\\ \Phi_0\end{bmatrix} = \begin{bmatrix}A_x & B_x\\ C_x & D_x\end{bmatrix}\begin{bmatrix}\theta_x\\ \Phi_x\end{bmatrix} = [M_x]\begin{bmatrix}\theta_x\\ \Phi_x\end{bmatrix} \quad \rightarrow \quad \begin{bmatrix}\theta_0\\ \Phi_0\end{bmatrix} = [M_e]\begin{bmatrix}\theta_e\\ \Phi_e\end{bmatrix}$$

$$A_x = D_x = \cosh(kx)$$
$$B_x = \sinh(kx)/\lambda k$$
$$C_x = \lambda k \cdot \sinh(kx)$$
$$\left(k = \sqrt{p/a_x}\right)$$

Two reference temperature profiles are chosen in $x=x_1$ and $x=x_2$

$$\theta(x,p) = \theta_1(p)\cdot F_1(x,p) + \theta_2(p)\cdot F_2(x,p)$$

with: $F_1(x,p) = \dfrac{\sinh(\alpha(x_2 - x))}{\sinh(\alpha(x_2 - x_1))}$ and $F_2(x,p) = \dfrac{\sinh(\alpha(x - x_1))}{\sinh(\alpha(x_2 - x_1))}$

$$\left(\alpha = \sqrt{p/a_x + 2h/e\lambda_x}\right)$$

Theoretical Model: *Solution in the Time domain*

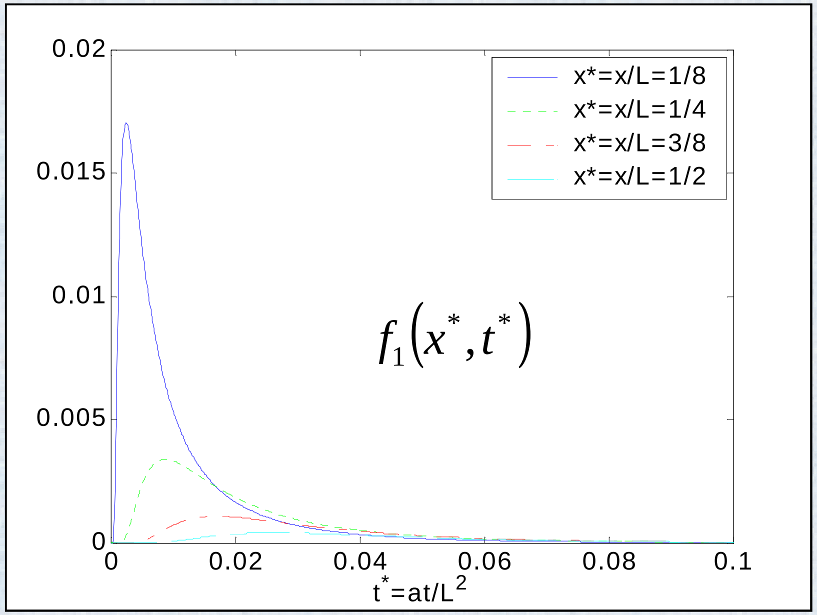
In the time domain, the solution can be written as a sum of two convolution products:

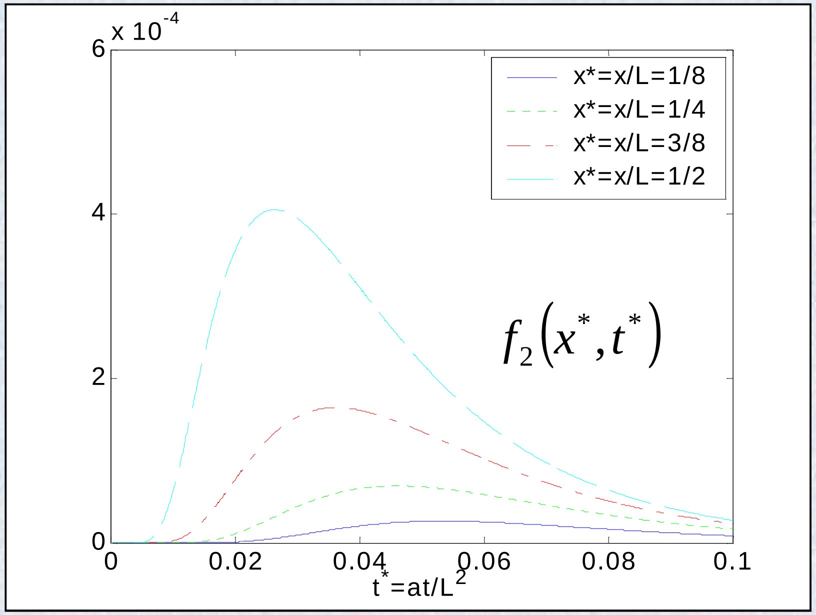
$$T(x,t) = T_1(t) \otimes f_1(x,t) + T_2(t) \otimes f_2(x,t)$$

with: $f_i(x,t) = \mathcal{L}^{-1}(F_i(x,p))$

measured by infrared camera

f_1 and f_2 are functions of the unknown parameters: ***a*** *and* ***H***





Direct Model: *Test Case*

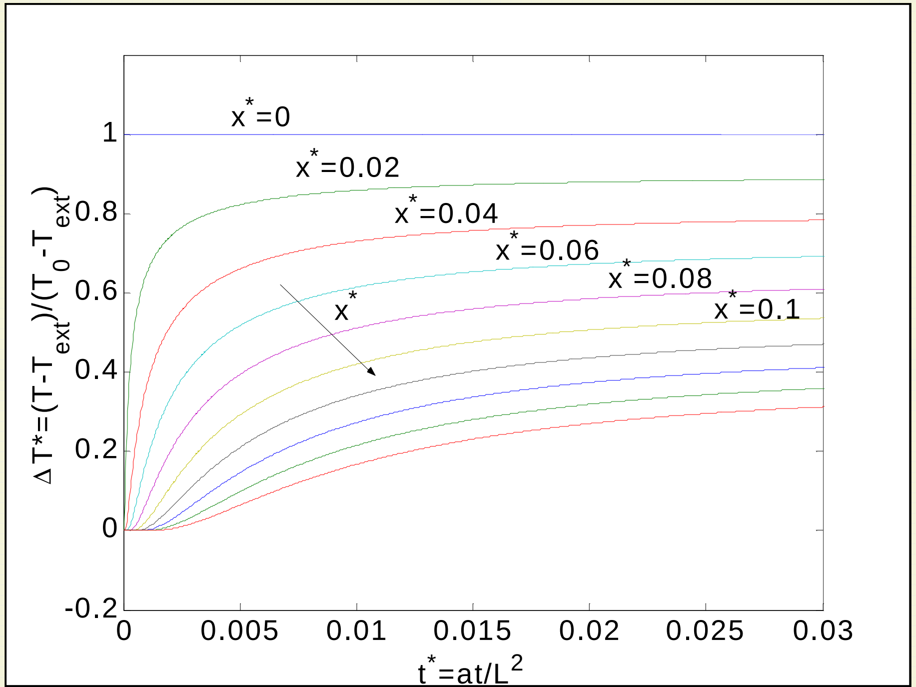
Test Case: *square sample L=40mm and e=1mm* $a = 5.10^{-7}\ m^2.s^{-1}$ $\lambda = 1W.m^{-1}.K^{-1}$ $h = 10W.m^{-2}.K^{-1}$

Temperature step in *x=0* $\theta_0(p) = (T_0 - T_{ext})/p$ Insulated in *x=L*

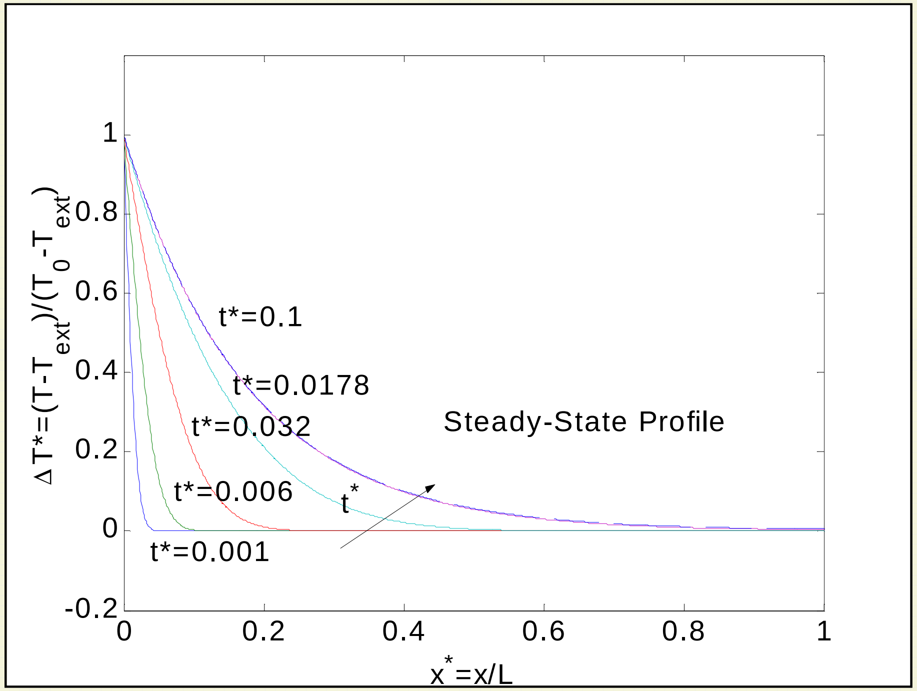
Temperature $$\theta^*(x,p) = \frac{\theta(x,p)}{T_0 - T_{ext}} = \frac{1}{p}\cdot\frac{\cosh(\alpha(x_2 - x))}{\cosh(\alpha(x_2 - x_1))}$$

Profile $$\Delta T_\infty^{\ *}(x) = \lim_{p\to\infty} p.\theta^*(p) = \frac{\cosh(\alpha_0(x_2 - x))}{\cosh(\alpha_0(x_2 - x_1))}$$

- In-time Variations:



- In-space Variations:



Inverse Model: *Parameter Estimation*

Ordinary Least Squares Method:

$$S = \sum_{i=1}^{n_t} \left(T_{the}(t_i,\beta) - T_{exp}(t_i) \right)^2 \qquad \beta = \begin{bmatrix} a \\ H \end{bmatrix}$$

Unknown parameters β_j

$$T_{the}(t_i,\beta) = \begin{bmatrix} T_{the}(0, t_i,\beta) \\ \vdots \\ T_{the}(x_k,t_i,\beta) \\ \vdots \\ T_{the}(L_x,t_i,\beta) \end{bmatrix} \Big\} (n_x+1) \text{ profiles} \quad \text{and: } T_{exp}(t_i) = \begin{bmatrix} T_{exp}(0, t_i) \\ \vdots \\ T_{exp}(x_k,t_i) \\ \vdots \\ T_{exp}(L_x,t_i) \end{bmatrix} \quad \text{with: } x_k = \frac{k}{n_x} L_x \quad (k = 0,\ldots,n_x)$$

$$\frac{\partial S}{\partial \beta_j} = 0 \rightarrow \sum_{i=1}^{n_t} \frac{\partial T_{the}(t_i,\beta)}{\partial \beta_j} \left(T_{the}(t_i,\beta) - T_{exp}(t_i) \right) = 0 \quad (\forall j)$$

$$X_{ij} = \beta_j \frac{\partial T_{the}(t_i,\beta)}{\partial \beta_j}$$ *Sensitivity Coefficient*

Ideal case $T_{exp}(t_i) = T_{the}(t_i,\beta) + \varepsilon(t)$

$E(\varepsilon) = 0$ and $V(\varepsilon) = \sigma_b^{\,2}$

$$\hat{\beta} = \beta + \left(X^t X \right)^{-1} X^t \varepsilon(t)$$

$$E(\hat{\beta}) = \beta$$

$$V(\hat{\beta}) = \sigma_b^{\,2} \left(X^t X \right)^{-1} = \sigma_b^{\,2} \begin{bmatrix} Var(a) & Cov(a,H) \\ Cov(a,H) & Var(H) \end{bmatrix}$$

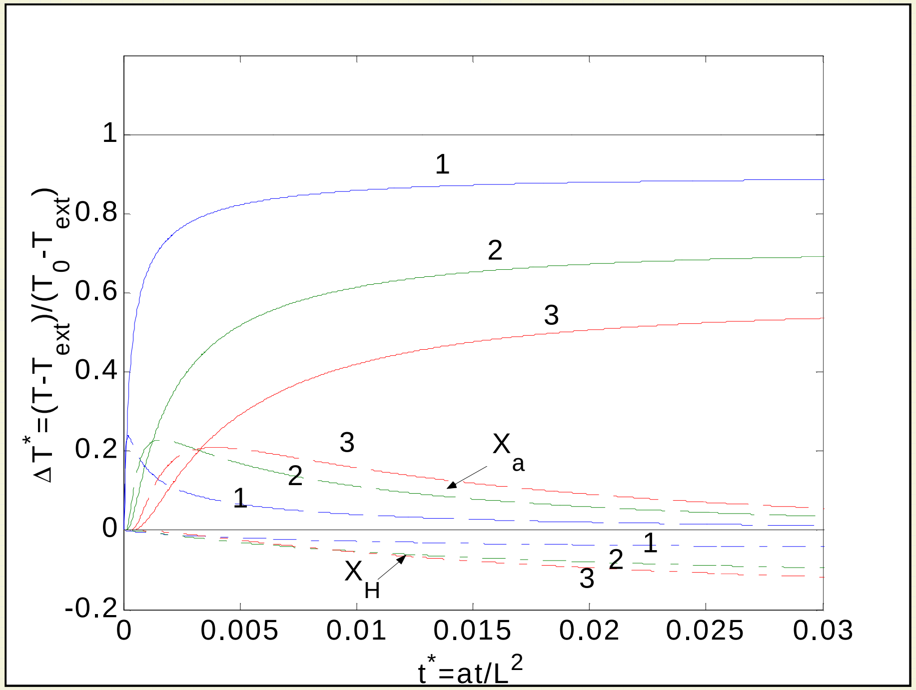
Non-ideal case

$$T_{exp}(x_k,t_i,\beta) = T_{the}(x_k,t_i,\beta)\Big|_{\varepsilon_1=\varepsilon_2=0} + \varepsilon_1(t_i) \otimes f_1(x_k,t_i) + \varepsilon_2(t_i) \otimes f_2(x_k,t_i)$$

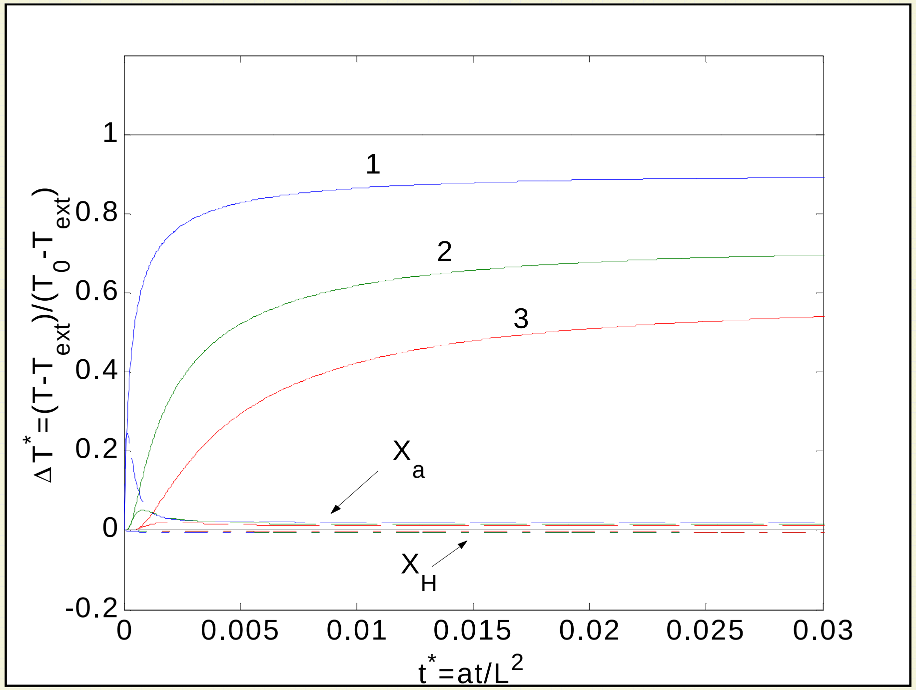
$$\sigma_a^{\,2} = \sigma_b^{\,2}.Var(a) \qquad \sigma_H^{\,2} = \sigma_b^{\,2}.Var(H)$$

Optimal choice of the reference profiles

- 2 static profiles:



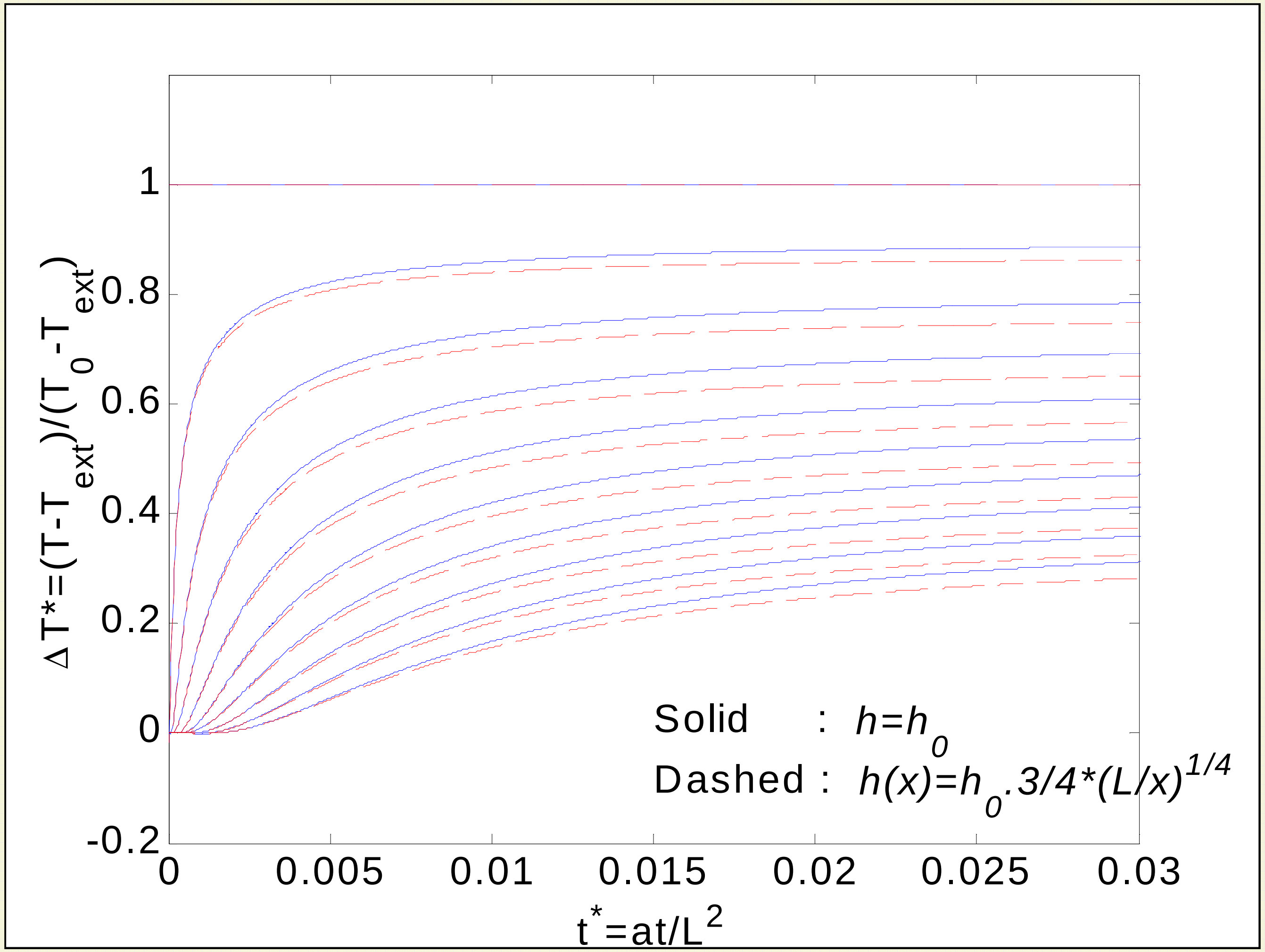
- 2 moving profiles:



	$Var(a)$	$Var(H)$	$Cov(a,H)$	$\rho(a,H)$
Direct Model	0.011	0.036	0.015	0.743
General Model				
$x_1=0 \quad x_2=L$	0.011	0.036	0.015	0.743
$x_1=0 \quad x_2=x_{k+1}$	0.090	0.397	0.106	0.562
$x_1=x_{k-1} \quad x_2=L$	0.135	0.529	0.180	0.676
$x_1=x_{k-1} \quad x_2=x_{k+1}$	0.882	24.389	3.652	0.787

Natural & Optimal Choice: $x_1=0$ and $x_2=L_x$

Effect of a non-uniform heat transfer coefficient ①



In-space varying heat transfer coefficient

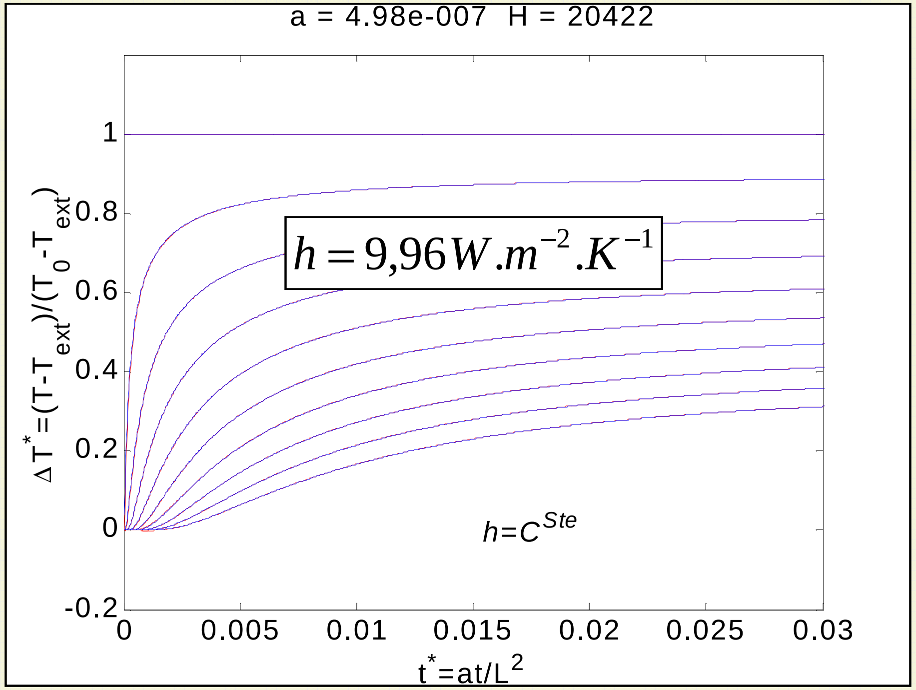
$$h(x) = \frac{3}{4} h_0 . \left(\frac{L}{x}\right)^{1/4}$$

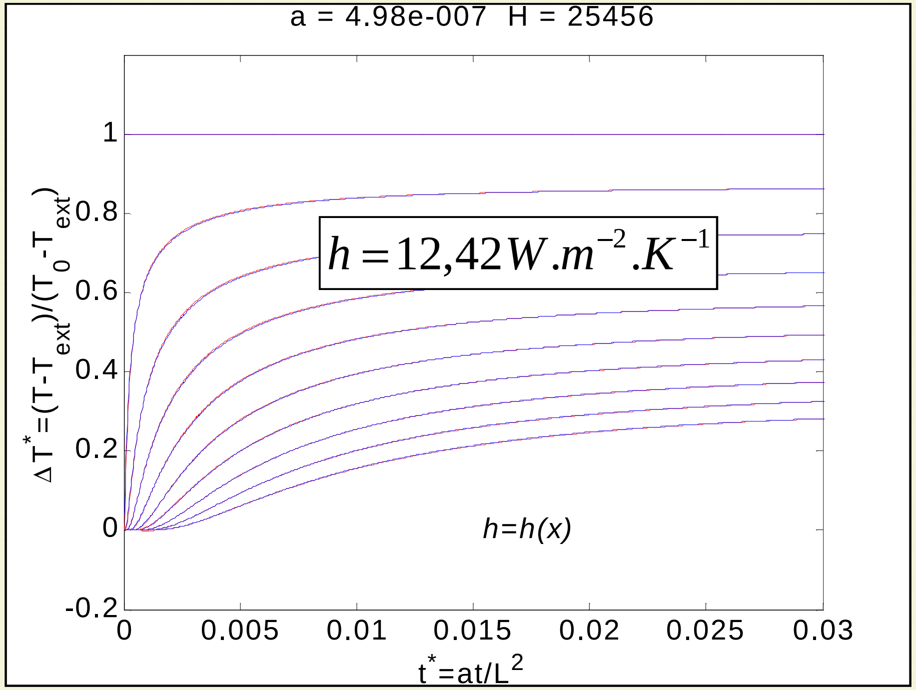
with:

$$\bar{h} = \frac{1}{L}\int_0^L h(x)dx = h_0$$

(FEM Software - FlexPDE)

Effect of a non-uniform heat transfer coefficient ②





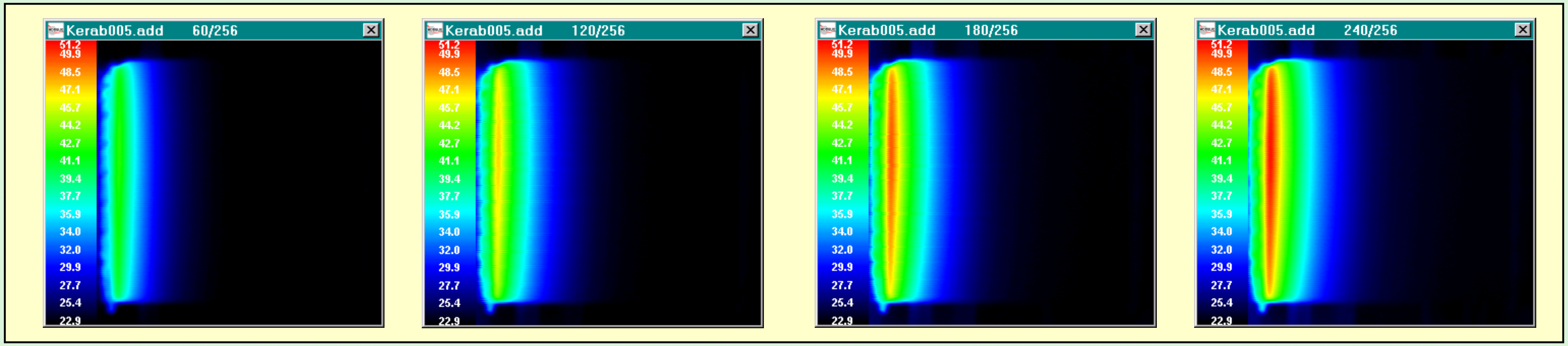
Nominal values:

$a = 5.10^{-7}\,m^2.s^{-1}$ and $H = 20\,500$

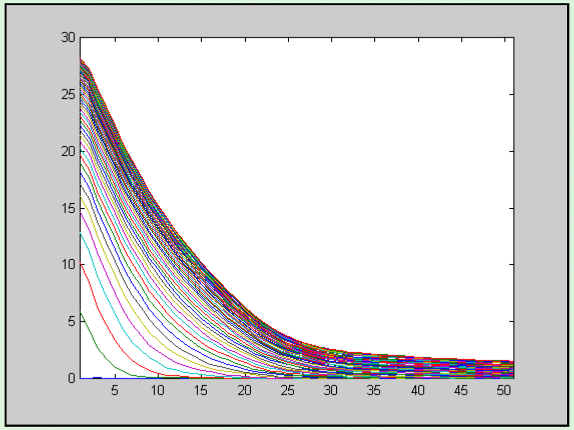
$$\begin{cases} h = 10\,W.m^{-2}.K^{-1} \\ \lambda = 1\,W.m^{-1}.K^{-1} \end{cases}$$

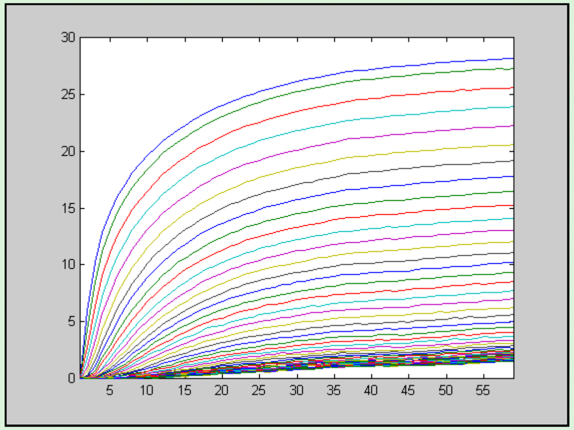
Experimental Results

- *Thermograms (line source stimulation)*:

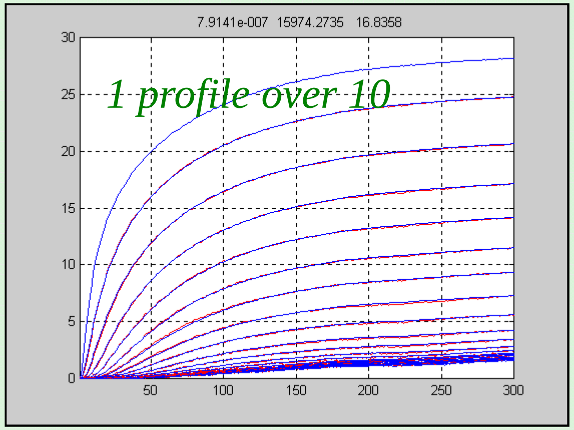


- *Temperature profiles and thermograms*:





- *Estimation*:



Conclusions

An in-plane diffusivity estimation method by convolution method for low conductive materials has been presented.

It is a very ***simple*** and ***semi-analytical*** method that allows us using two reference measured temperatures as boundary conditions

- To measure the in-plane diffusivity of low conductive materials
- Whatever the In-Space and In-Time stimulation heat flux,
- And heat transfer coefficient variations are.

Thank you for your attention …