Modélisation multiphysique de la cuisson du pain

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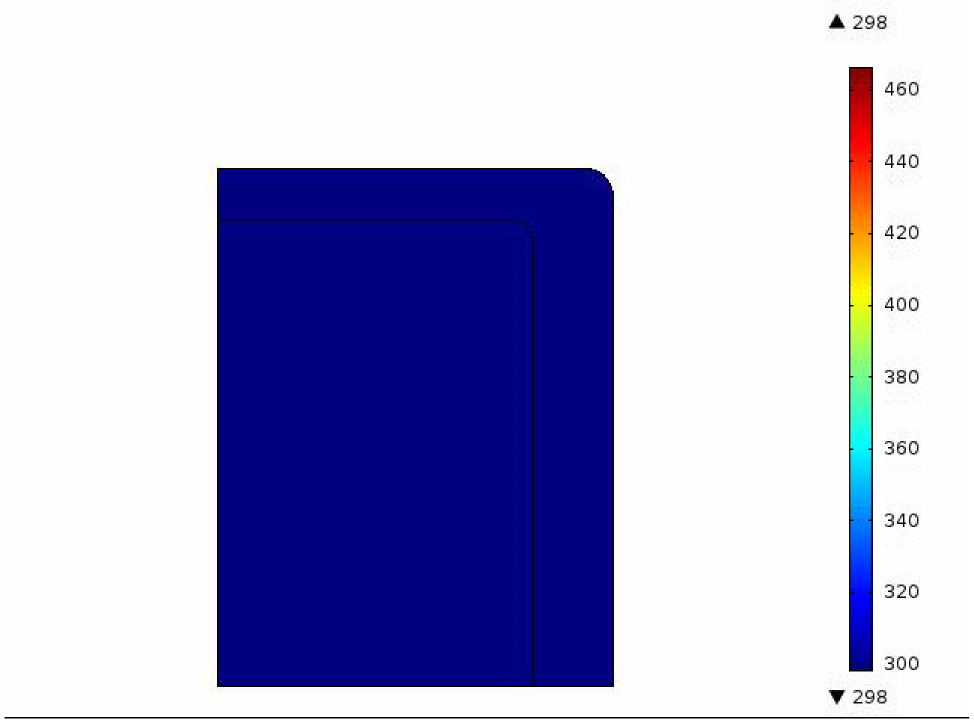


Société Française de Thermique
Groupe « Thermique Appliquée »
4 novembre 2016

Introduction

Principaux mécanismes (1)

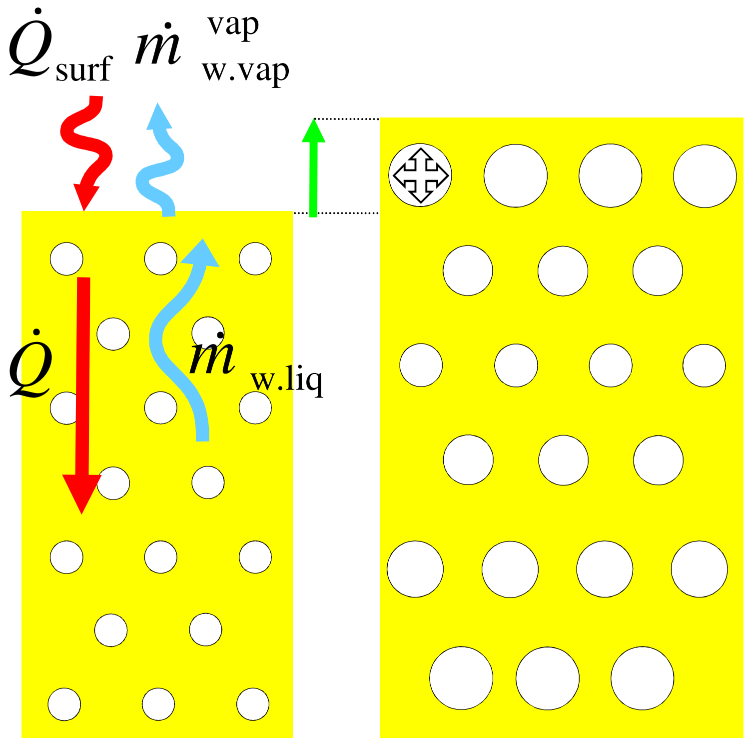
a) Échauffement de la surface et évaporation de l'eau à la surface

b) Transport de la chaleur par diffusion de la surface au cœur

c) Transport de l'eau liquide du cœur à la surface

d) Augmentation de la pression

e) Déformation du produit



Introduction

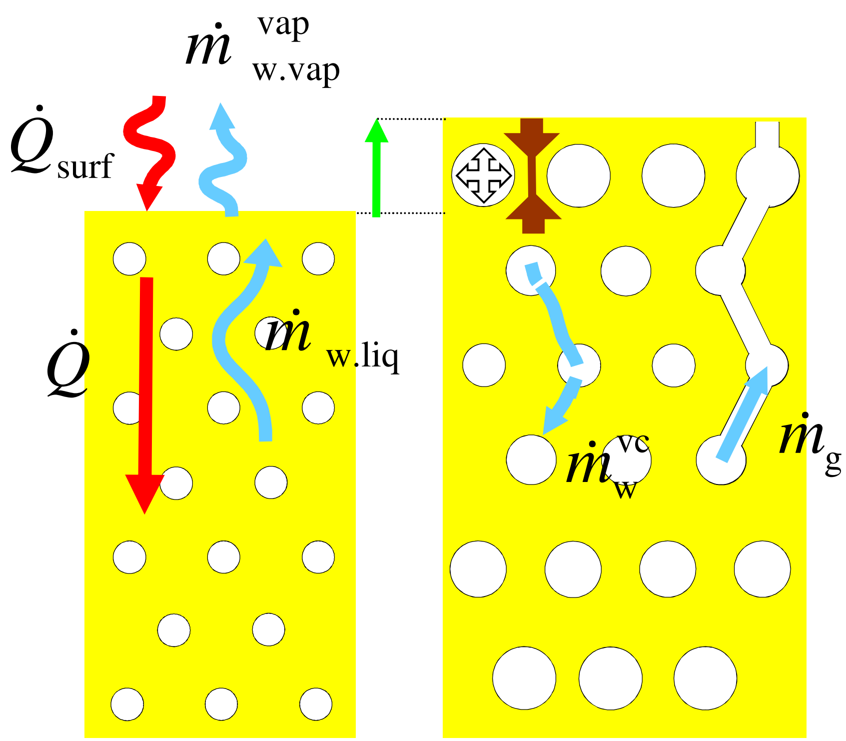
Principaux mécanismes (2)

f) Contraintes dans la pâte freinent l'expansion

g) Transport de l'eau par évapo-condensation-diffusion

h) Ouverture de la porosité

i) Migration de la phase gazeuse



Plan

1/ Modèle

2/ Validation expérimentale

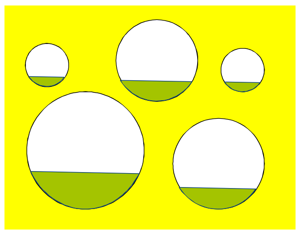
3/ Interprétations

1/ Modèle

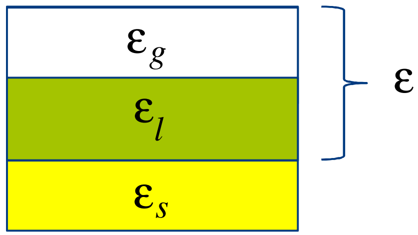
1.1/ Principales hypothèses et notations (1)

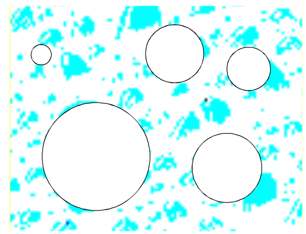
Régime transitoire

Approche des milieux continus :



Volume élémentaire représentatif





→ milieu continu diphasique

→ toutes les variables sont continues dans le milieu poreux

Références: Lucas, T ; Doursat, C. ; Grenier, D. ; Wagner, M.; Trystram, G., Flick, D. **2015**. Modeling of bread baking with a new, multi-scale formulation of evaporation-condensation-diffusion and evidence of compression in the outskirts of the crumb. **J. Food Engineering.**

Nicolas, V., Vanin, F., Doursat, C., Grenier, D., Lucas, T., Flick, D., 2014. Modelling bread baking with focus on global and local deformationom: . AIChE J.

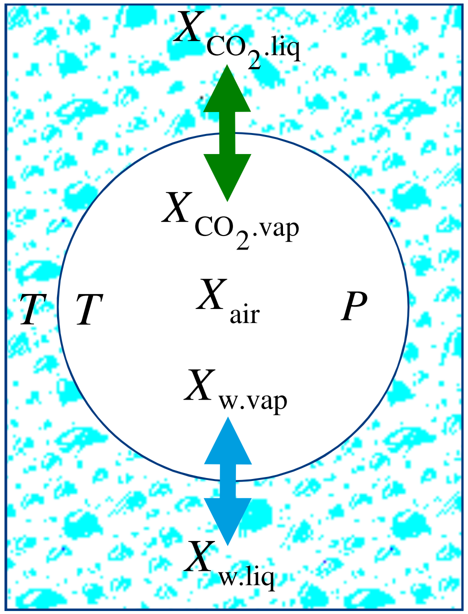
1/ Modèle

1.1/ Principales hypothèses et notations (2)

Notations :

β	kg matière sèche / m^3 pâte
ε	porosité
X_α	kg α / matière sèche

Équilibre local et équations d'état :



$$P_w\left(X_{w.liq},T\right)=a_w\left(X_{w.liq},T\right)P_{v.sat}(T)=\frac{\beta RT}{\varepsilon}\frac{X_{w.vap}}{M_w}$$

Solubilisation du CO_2 : loi de Henry

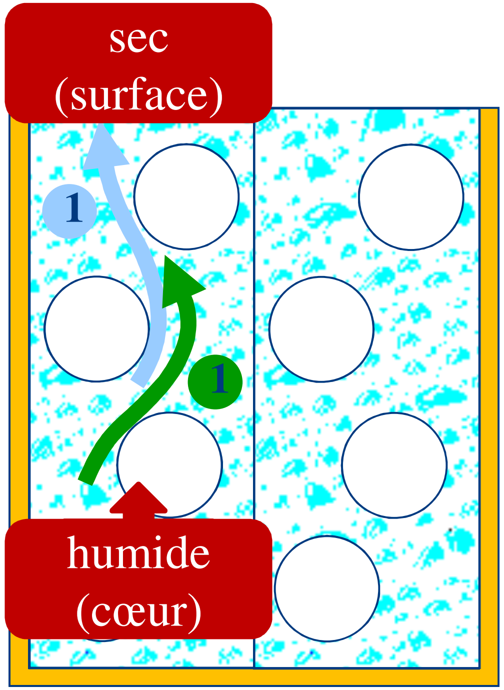
$$P_{CO_2}=\frac{X_{CO_2.liq}}{M_{CO_2}S_{CO_2}X_{w.liq}}T=\frac{1-\varepsilon}{\varepsilon}\frac{\beta X_{CO_2.vap}}{M_{CO_2}}RT$$

$$\hat{H}=\left(C_{p_{ms}}+\sum_\alpha X_\alpha C_{p_\alpha}\right)\left(T-T_{ref}\right)+X_{w.vap}\,L_w+X_{CO_2.vap}\,L_{CO_2}$$

1/ Modèle

1.2/ Transferts de matière (1)

Diffusion dans la phase liquide



$$\vec{m}_{wl} = -D_{wl}\vec{\nabla}X_{wl}$$ (1)

$$\vec{m}_{cl} = -D_{cl/wl}\vec{\nabla}X_{cl}$$ (1)

$$D_{wl} = 10^{-6}\exp\left(\left(-2{,}8 + 2X_{wl}\right)\varepsilon_g\right)$$

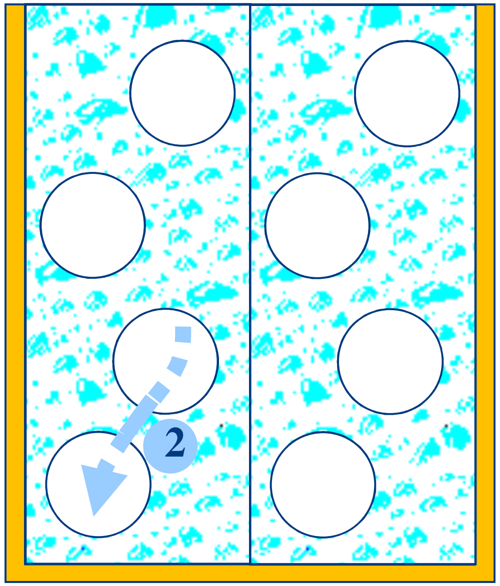
(Zhang and Datta, 2006)

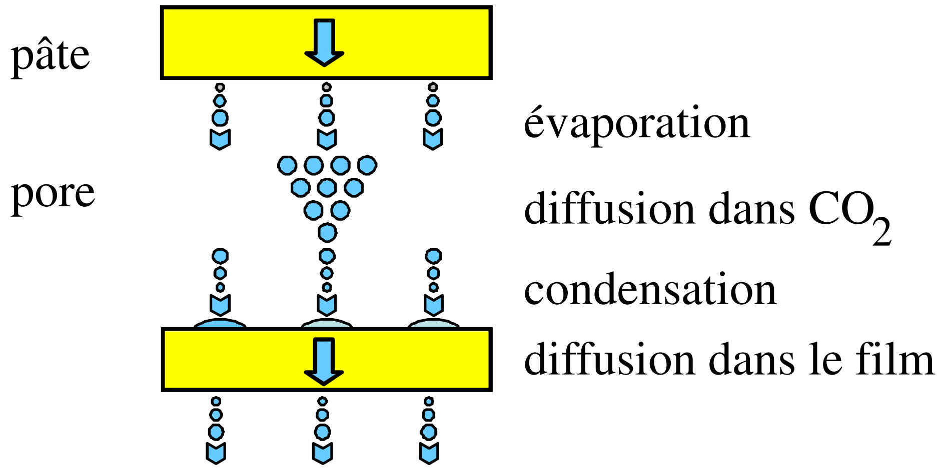
$$D_{cl/wl} = 2.8\ 10^{-9}\frac{\varepsilon_l}{\tau}$$

1/ Modèle

1.2/ Transferts de matière (2)

Porosité fermée : évapo-condensation-diffusion





Hypothèses :

- on se place à l'échelle d'une bulle
- en régime permanent
- transferts 1D

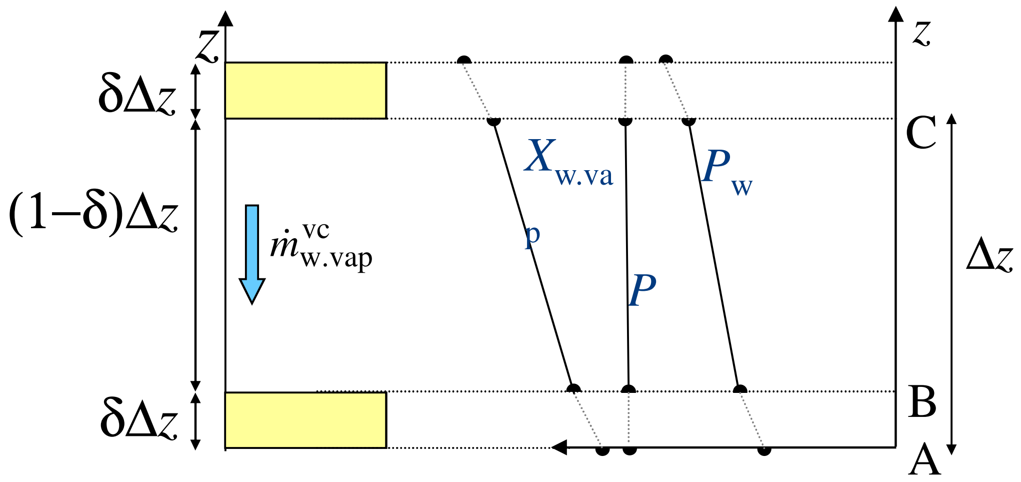
1/ Modèle

1.2/ Transferts de matière (2)

Porosité fermée : évapo-condensation-diffusion

Dans la bulle : diffusion mutuelle des deux espèces

$$\begin{cases} \dfrac{\dot m_{w.vap}}{M_w} = x_{w.vap} \quad C v^* \quad - D_{vap} C \dfrac{\partial x_{w.vap}}{\partial z} \\ \dfrac{\dot m_{CO_2.vap}}{M_{CO_2}} = x_{CO_2.vap} \quad C v^* \quad - D_{vap} C \dfrac{\partial x_{CO_2.vap}}{\partial z} \end{cases}$$



1/ Modèle

1.2/ Transferts de matière (2)

Porosité fermée : évapo-condensation-diffusion

Dans la bulle : diffusion mutuelle des deux espèces

Le CO_2 étant immobile

$$\dot{m}_{\mathrm{w.vap}} = -\frac{M_{\mathrm{w}}}{1-x_{\mathrm{w.vap}}} D_{\mathrm{vap}}\, C \frac{\partial x_{\mathrm{w.vap}}}{\partial z}$$

ou

$$\dot{m}^{\mathrm{vc}}_{\mathrm{w.vap}} = -\frac{M_{\mathrm{w}}}{RT} \frac{D_{\mathrm{vap}}}{1-x_{\mathrm{w.vap}}} \frac{\partial P_{\mathrm{w}}}{\partial z}$$

$$\dot{H}^{\mathrm{vc}} = \dot{m}^{\mathrm{vc}}_{\mathrm{w}} \left(C_{p_{\mathrm{w.vap}}} \left(T - T_{\mathrm{ref}} \right) + L_{\mathrm{w.ref}} \right)$$

1/ Modèle

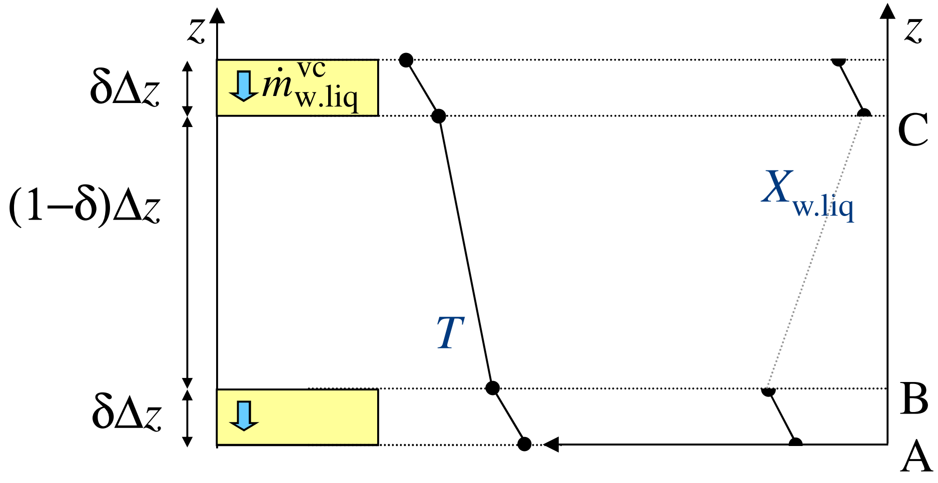
1.2/ Transferts de matière (2)

Porosité fermée : évapo-condensation-diffusion

Dans le film de pâte :

$$\dot{m}^{vc}_{w.liq} = -\rho_{w.liq} D_{liq} \frac{\partial X_{w.liq}}{\partial z}$$

$$\dot{Q} + \dot{H}^{vc} = -\lambda \frac{\partial T}{\partial z} + \dot{m}^{vc}_{w.liq} Cp_{w.liq} (T - T_{ref})$$

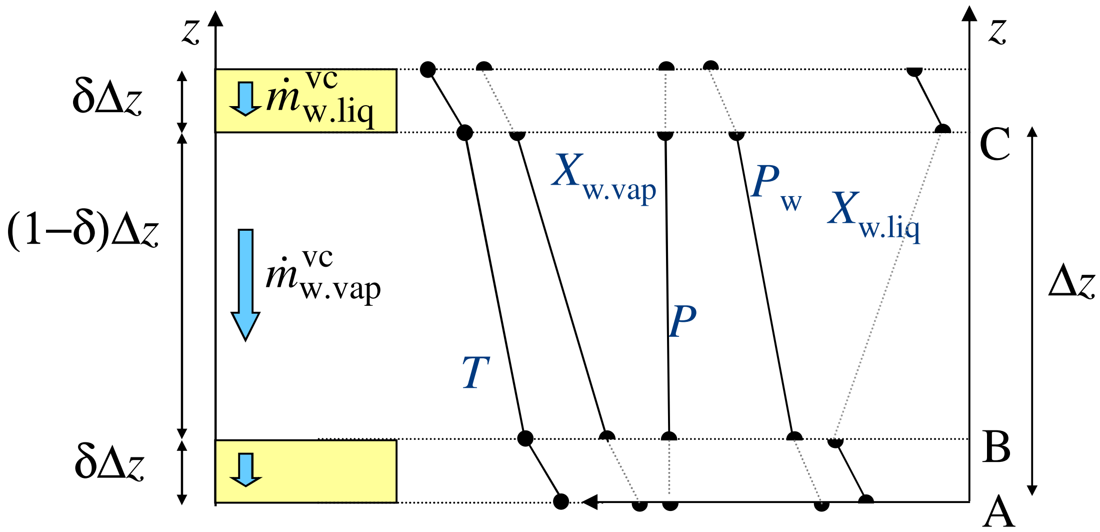


1/ Modèle

1.2/ Transferts de matière (2)

Porosité fermée : évapo-condensation-diffusion

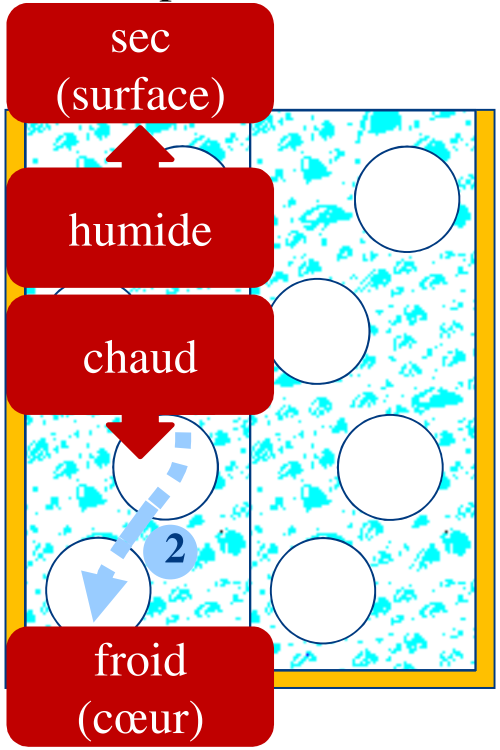
Continuité à l'interface (Point B) :

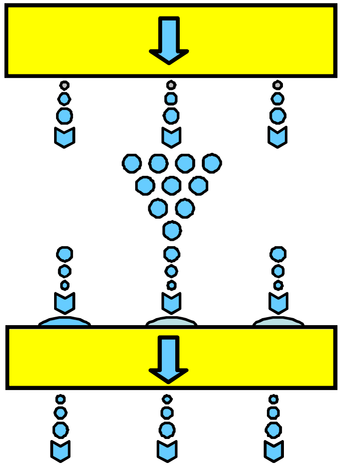


1/ Modèle

1.2/ Transferts de matière (2)

Évapo-condensation





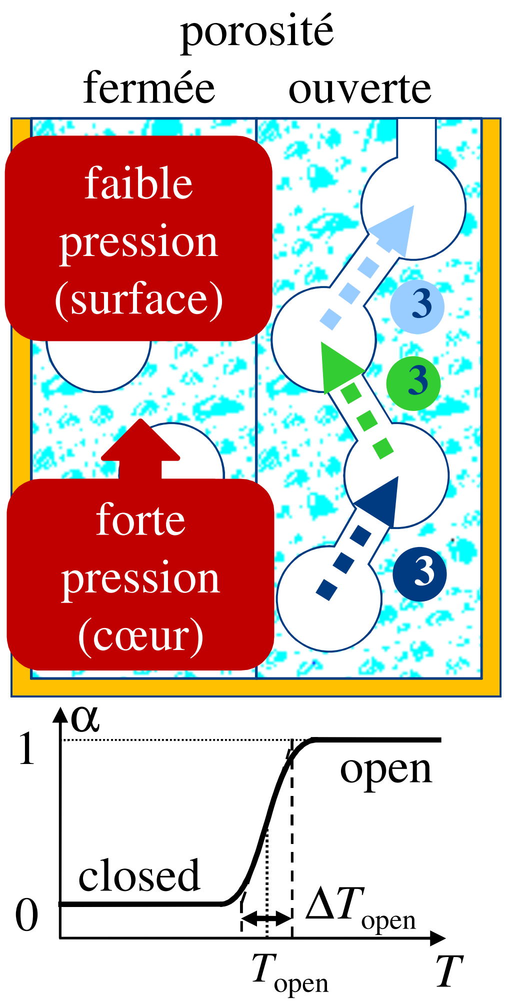
$$\dot{m}_{wv}^{vc} = -\frac{\varepsilon}{(1-\delta)\frac{RT}{M_w}\frac{1-X_{w.vap}}{D_{vap}} + \delta\left(\frac{1}{\rho_{w.liq}D_{liq}}\frac{\partial P_v}{\partial X_{w.liq}} + \frac{L_w}{\lambda}\frac{\partial P_v}{\partial T}\right)}\vec{\nabla} P_{wv} \quad (2)$$

pore **film**

1/ Modèle

1.2/ Transferts de matière (3)

Ouverture des pores et flux Darciens et diffusifs



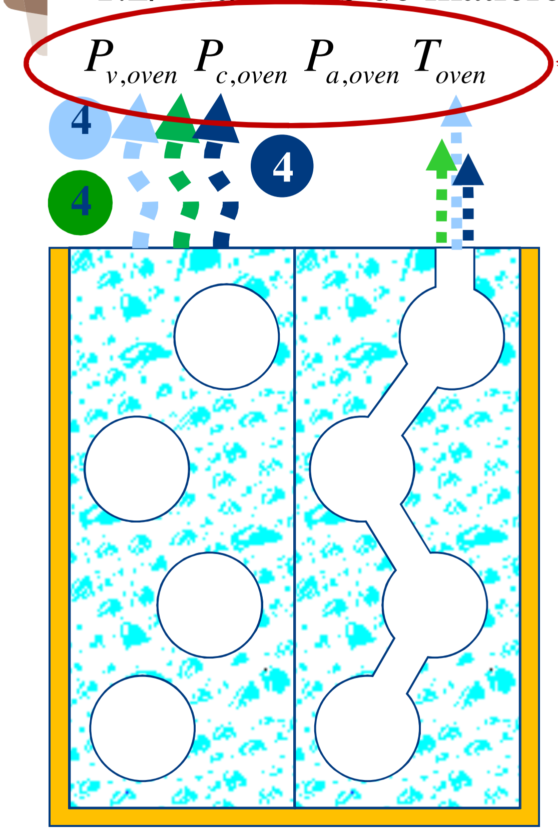
$$\vec{\dot{m}}_{wv} = \alpha\left(-\omega_{wv}\frac{K_g}{\nu_g}\vec{\nabla}P - \rho_g^i D^{eff}\vec{\nabla}\omega_{wv}\right) - (1-\alpha)A_{wvvc}\vec{\nabla}P_{wv}$$ 3

$$\vec{\dot{m}}_{cv} = \alpha\left(-\omega_{cv}\frac{K_g}{\nu_g}\vec{\nabla}P - \rho_g^i D^{eff}\vec{\nabla}\omega_{cv}\right)$$ 3

$$\vec{\dot{m}}_{a} = \alpha\left(-\omega_{a}\frac{K_g}{\nu_g}\vec{\nabla}P - \rho_g^i D^{eff}\vec{\nabla}\omega_{a}\right)$$ 3

1/ Modèle

1.2/ Transferts de matière (4)



Conditions aux limites :

- surface en contact avec l'ambiance du four

$$\dot{m}_{wv}\Big|_{oven} = k_c \frac{M_w}{R}\left(\frac{P_v}{T} - \frac{P_{v,oven}}{T_{oven}}\right)$$ 4

$$\dot{m}_{cv}\Big|_{oven} = k_c \frac{M_c}{R}\left(\frac{P_v}{T} - \frac{P_{c,oven}}{T_{oven}}\right)$$ 4

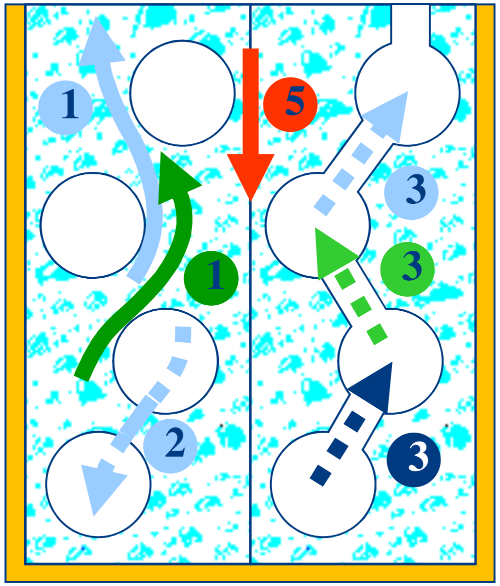
$$\dot{m}_{a}\Big|_{oven} = k_c \frac{M_a}{R}\left(\frac{P_v}{T} - \frac{P_{a,oven}}{T_{oven}}\right)$$ 4

- surface en contact avec le moule

$$\dot{m}_j\Big|_{mould} = 0$$

1/ Modèle

1.2/ Transferts de chaleur (1)



Flux diffusifs (conduction)

$$\dot{\vec{Q}} = -\lambda \vec{\nabla} T$$ 5

$$\lambda = \lambda_{dgh} \left(\frac{2\lambda_{dgh} + \lambda_g - 2\varepsilon_g (\lambda_{dgh} - \lambda_g)}{2\lambda_{dgh} + \lambda_g - \varepsilon_g (\lambda_{dgh} - \lambda_g)} \right) \quad \text{(Maxwell)}$$

$$\lambda_{dgh} = \lambda_s \frac{\varepsilon_s}{\varepsilon_s + \varepsilon_l} + \lambda_l \frac{\varepsilon_l}{\varepsilon_s + \varepsilon_l}$$

$$\lambda_s = 0.20141 + 1.3874\,10^{-3} (T - 273.15) + 4.3312\,10^{-6} (T - 273.15)^2$$

$$\lambda_g = \omega_{cv}\lambda_{cv} + \omega_a \lambda_a + \omega_{wv}\lambda_{wv}$$

Flux convectifs

$$\vec{H} = \vec{\dot{m}}_{wl} c_{p;wl} (T - T_{ref}) + \vec{\dot{m}}_{cl} c_{p;cl} (T - T_{ref})$$ 1 1

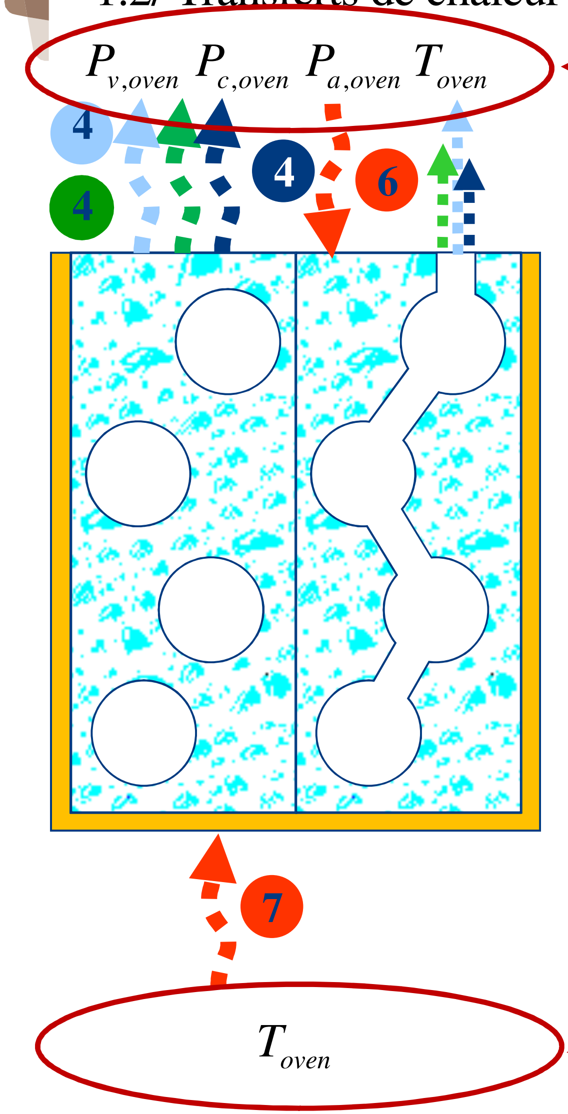
$$+ \vec{\dot{m}}_{wv} \left(c_{p;wv} (T - T_{ref}) + L_w \right)$$ 2 3

$$+ \vec{\dot{m}}_{cv} \left(c_{p;cv} (T - T_{ref}) + L_{CO_2} \right)$$ 3

$$+ \vec{\dot{m}}_a c_{p;a} (T - T_{ref})$$ 3

1/ Modèle

1.2/ Transferts de chaleur (2)



Conditions aux limites :

- surface en contact avec l'ambiance du four

$$\dot{H}\big|_{oven} = h_{surf}\left(T_s - T_{oven}\right) + \sigma\left(T_s^4 - T_{oven}^4\right) \qquad 6$$

$$\dot{H}_j\big|_{oven} = \dot{m}_j\big|_{oven}\left(c_{p;j}\left(T_s - T_{ref}\right) + L_j\right) \qquad 4 \quad 4 \quad 4$$

- surface en contact avec le moule

$$\dot{H}\big|_{mould} = h_{mould}\left(T_{oven} - T_{mould}\right) \qquad 7$$

1/ Modèle

1.3/ Équations de conservation de la masse et de l'énergie

- eau

$$\frac{\partial\left(\rho_{wl}^{a}+\rho_{wv}^{a}\right)}{\partial t}+\vec{\nabla}\cdot\left(\vec{\dot{m}}_{wl}+\vec{\dot{m}}_{wv}+\left(\rho_{wl}^{a}+\rho_{wv}^{a}\right)\vec{u}\right)=0$$

- dioxyde de carbone

$$\frac{\partial\left(\rho_{cl}^{a}+\rho_{cv}^{a}\right)}{\partial t}+\vec{\nabla}\cdot\left(\vec{\dot{m}}_{cl}+\vec{\dot{m}}_{cv}+\left(\rho_{cl}^{a}+\rho_{cv}^{a}\right)\vec{u}\right)=I_{CO_2}$$

- air

$$\frac{\partial\rho_{a}^{a}}{\partial t}+\vec{\nabla}\cdot\left(\vec{\dot{m}}_{a}+\rho_{a}^{a}\vec{u}\right)=0$$

- énergie

$$\frac{\partial\hat{H}}{\partial t}+\vec{\nabla}\cdot\left(\vec{\dot{Q}}+\vec{\dot{H}}+\hat{H}\vec{u}\right)=0$$

1/ Modèle

1.4/ Comportement mécanique (1)

- conservation de la matière sèche

$$\frac{\partial \rho_s^a}{\partial t} + \vec{\nabla} \cdot \left(\rho_s^a \vec{u} \right) = 0$$

- conservation de la quantité de mouvement

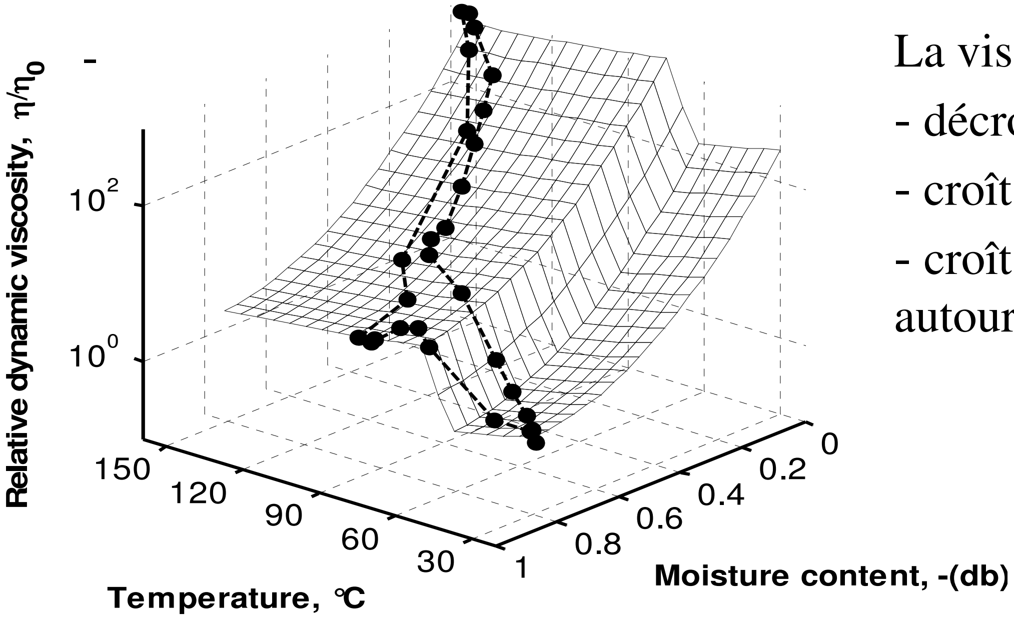
$$\frac{\partial (\rho \vec{u})}{\partial t} + \vec{\nabla} \cdot \left(\bar{\bar{\tau}} + (P - P_{atm}) \bar{\bar{I}} + \rho \vec{u} \vec{u} + \bar{\bar{\sigma}}_{sh} \right) = \rho \vec{g} \qquad P = P_{wv} + P_{cv} + P_a$$

La pâte est considérée comme un fluide visqueux :

$$\bar{\bar{\tau}} = \eta(T_{\max}, X_{wl}) \left(-\left(\vec{\nabla} \vec{u} + \vec{\nabla} \vec{u}^t \right) + \frac{2}{3} \left(\vec{\nabla} \cdot \vec{u} \right) \bar{\bar{I}} \right)$$

1/ Modèle

1.4/ Comportement mécanique (2)



La viscosité :

- décroit avec la température
- croît avec le séchage (x100)
- croît avec la gélatinisation (x10) autour de 65°C.

$$\eta = \eta_{ref} 10^{(G_T + G_{Xwl})}$$

$$G_T = 247.8\left(\frac{1}{T_{\max} - 140} - \frac{1}{T_{ref} - 140}\right) + p_3\left(\beta(T_{\max}) - \beta(T_{ref})\right)$$

$$G_{Xwl} = -p_1\left(X_{wl} - X_{wl;ref}\right) + p_2\left(X_{wl} - X_{wl;ref}\right)^2$$

Results from:

Vanin, F.M., Michon, C., Trystram, G., & Lucas, T. (**2010**). Simulating the formation of bread crust in the rheometer. ***Journal of Cereal Science***. *51: 277-283.*

Vanin, F. M., Michon, C., & Lucas, T. (**2013**). Effect of the drying rate on the complex viscosity of wheat flour dough transforming into crust and crumb during baking. ***Food Research International*** . 58(2): 290-297

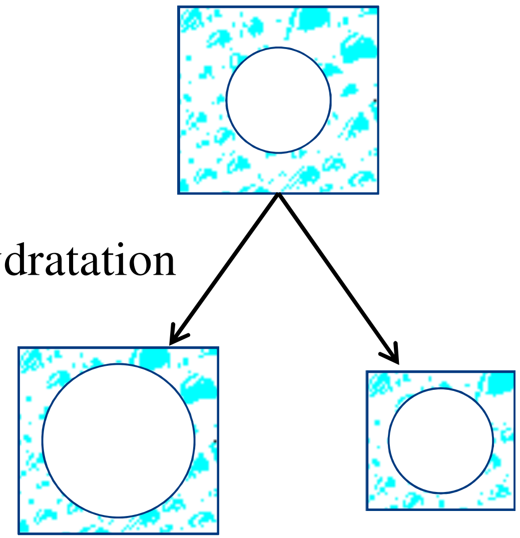
1/ Modèle

1.4/ Comportement mécanique (4)

Retrait hydrique

$$\frac{\partial(\rho\vec{u})}{\partial t} + \vec{\nabla}\cdot\left(\overline{\overline{\tau}} + (P - P_{atm})\overline{\overline{I}} + \rho\vec{u}\vec{u} + \overline{\overline{\sigma}}_{sh}\right) = \rho\vec{g}$$

déshydratation

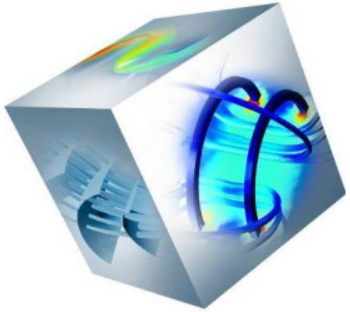


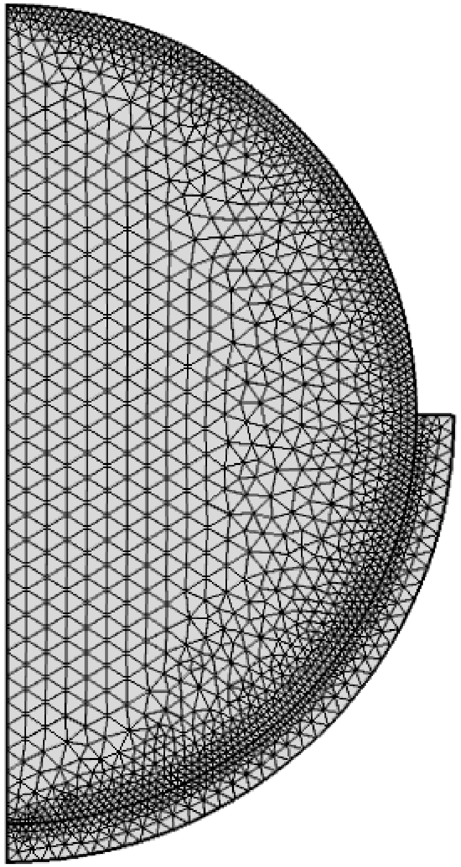
$$\overline{\overline{\sigma}}_{sh} = 0\,\overline{\overline{I}}$$

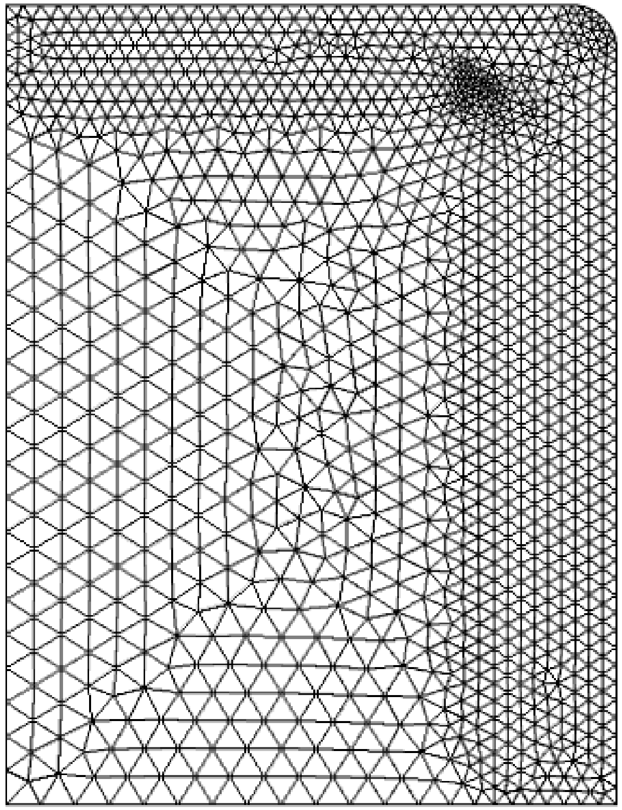
$$\overline{\overline{\sigma}}_{sh} = \frac{1}{3}\eta\,\varepsilon_s\frac{\rho_s^i}{\rho_{wl}^i}\frac{\partial X_{wl}}{\partial t}\overline{\overline{I}}$$

1/ Modèle

1.5/ Simulation (1)

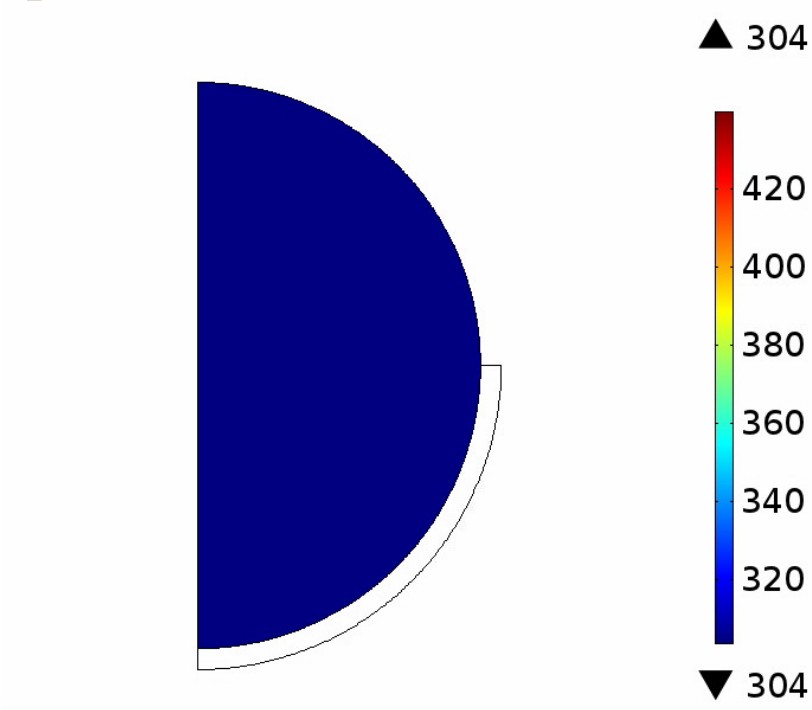




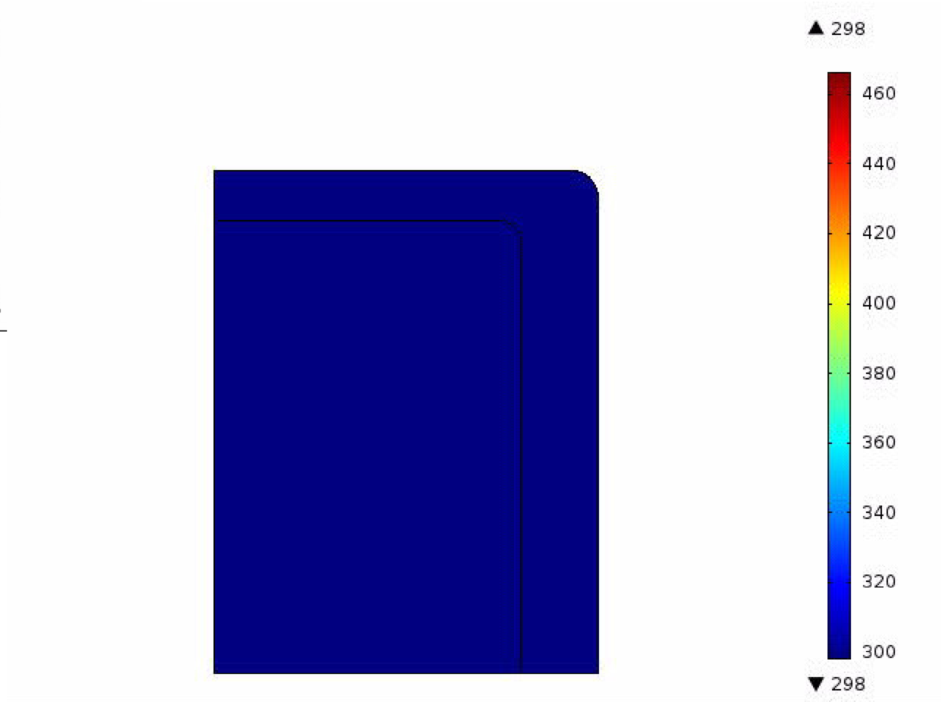


1/ Modèle

1.5/ Simulation (2)

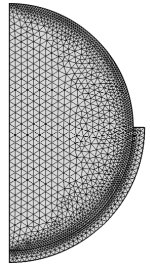


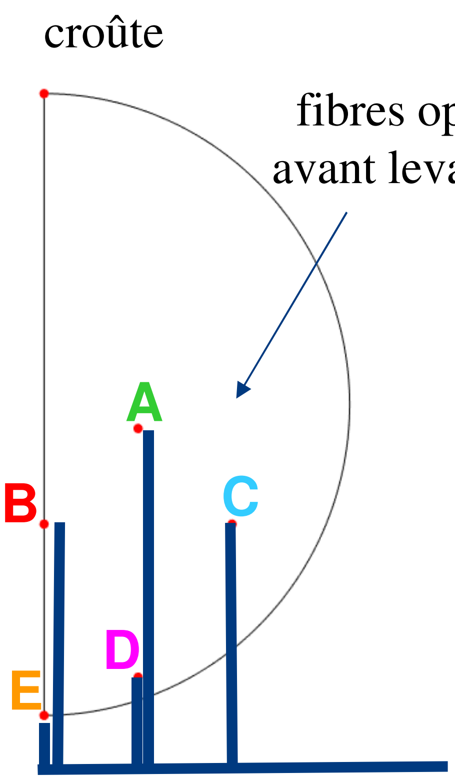
Température

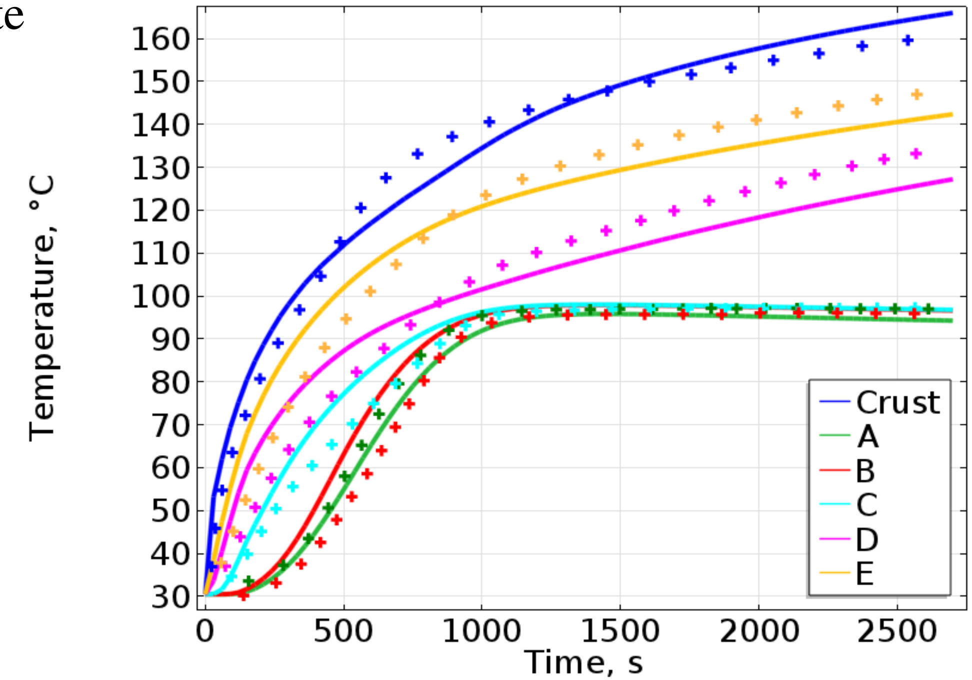


2/ Validation expérimentale

2.1/ Température







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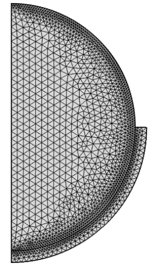
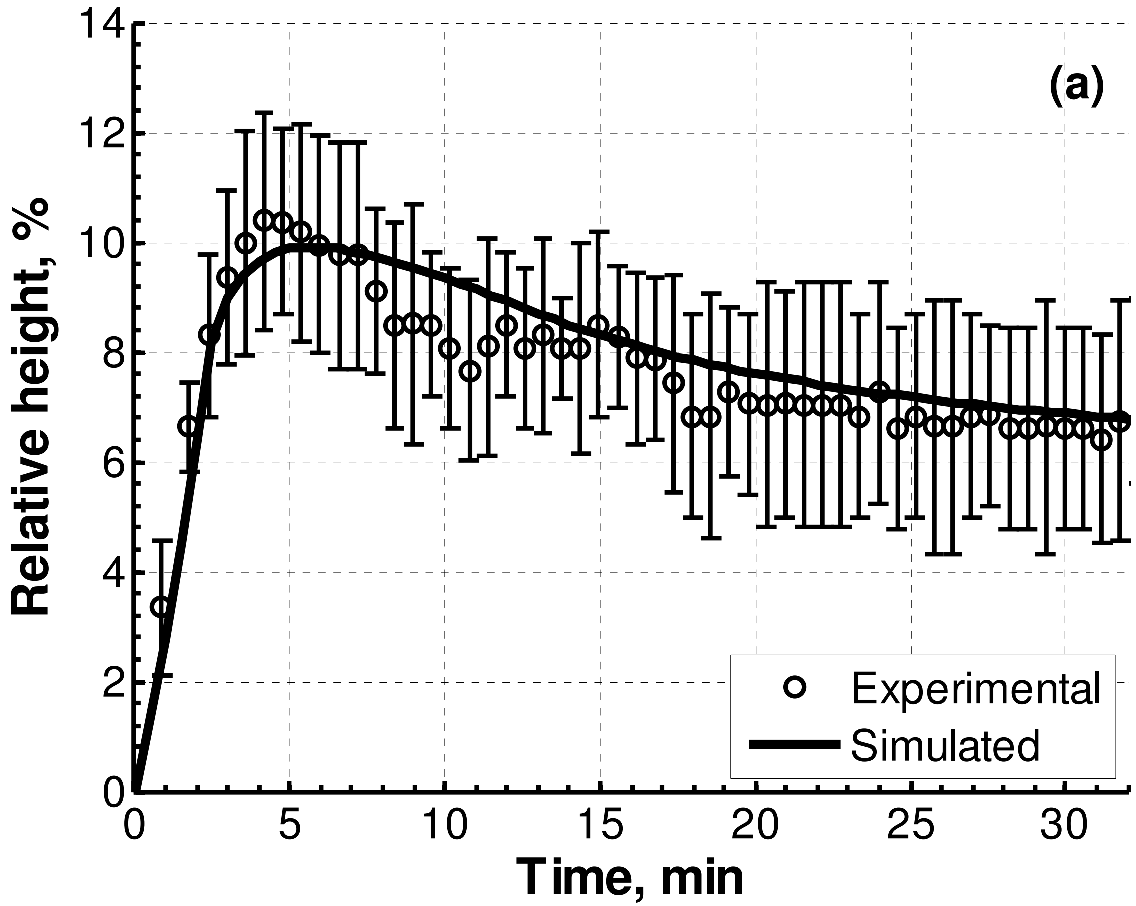


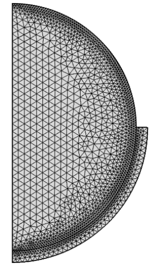
2/ Validation expérimentale

2.2/ Expansion : hauteur du produit (1)



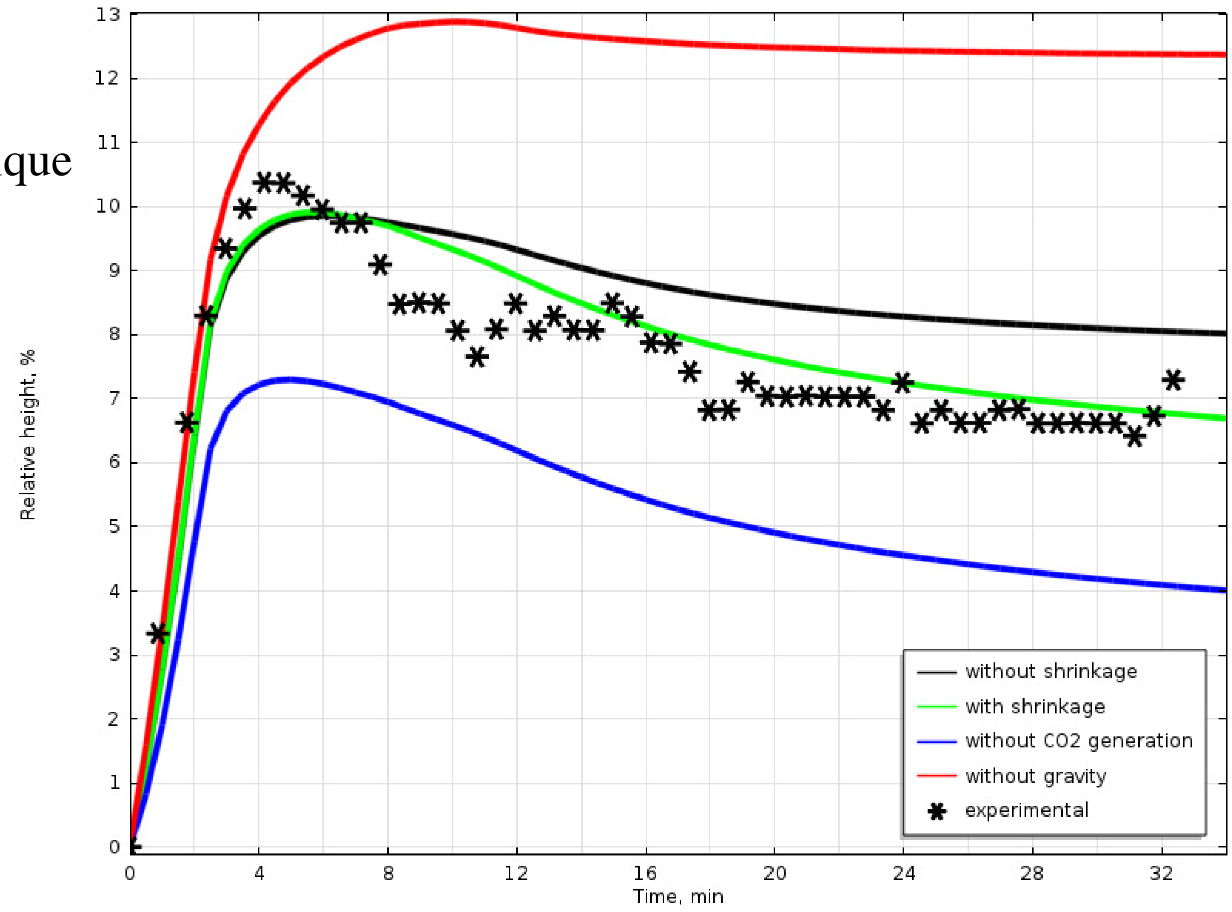
2/ Validation expérimentale

2.2/ Expansion : hauteur du produit (2)



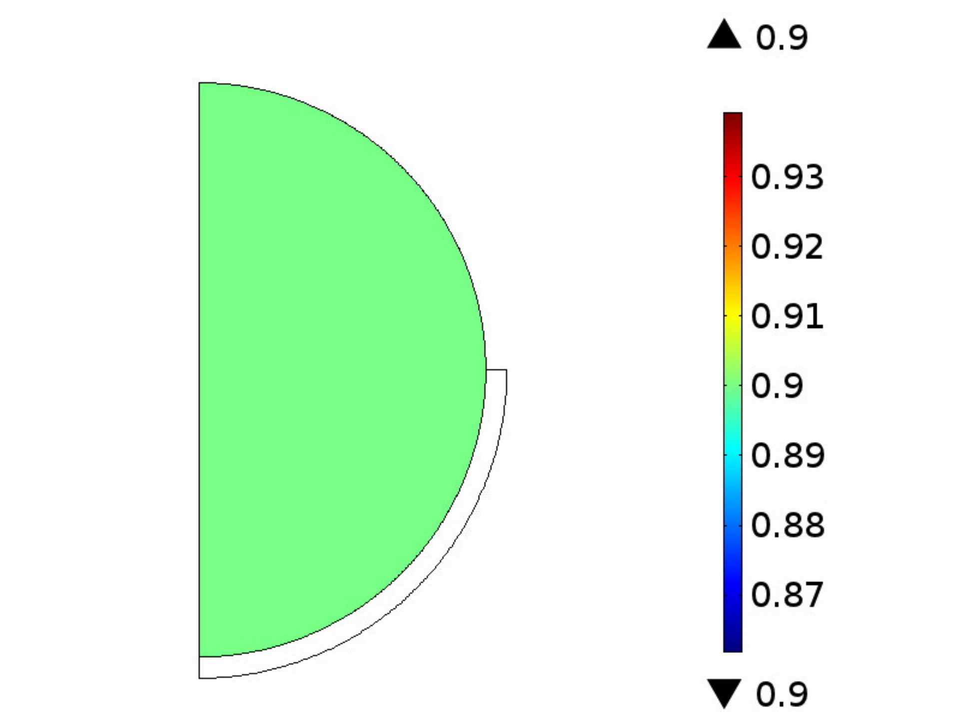
Il est nécessaire de prendre en compte :

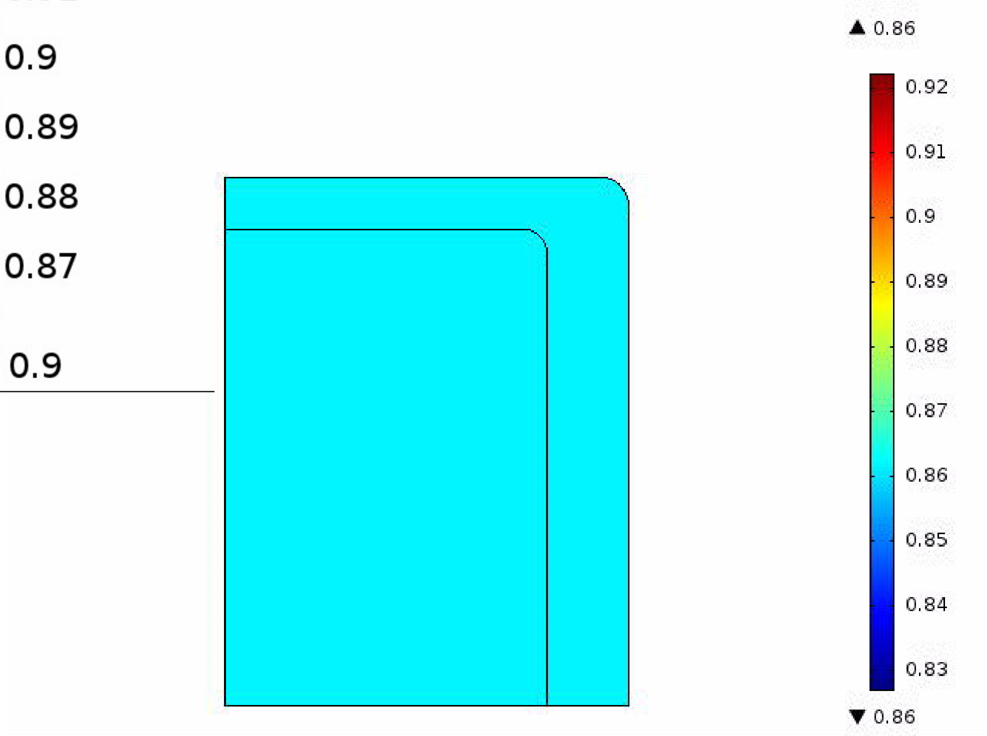
- la production de CO_2
- la gravité
- le retrait hydrique
- le changement de comportement mécanique



2/ Validation expérimentale

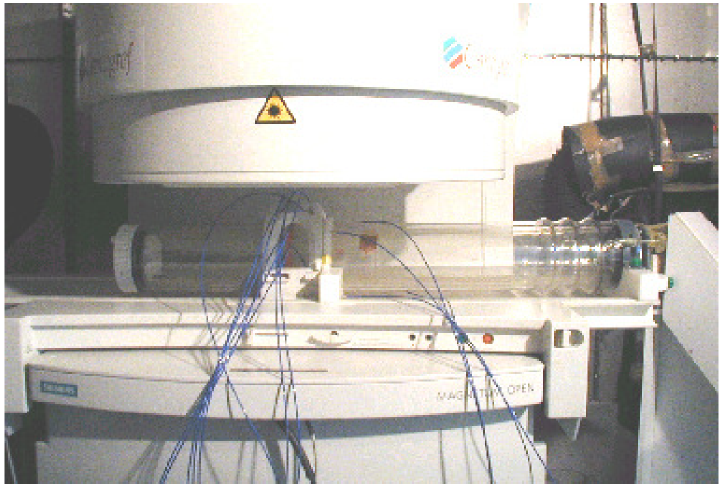
2.2/ Expansion : expansion locale (1)

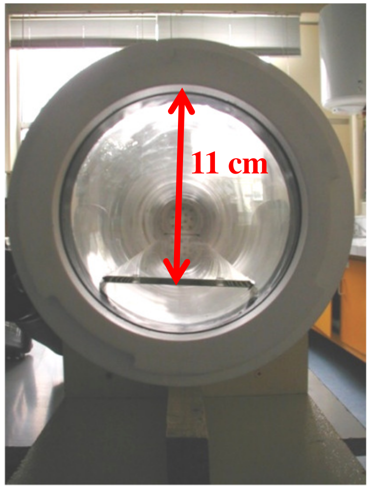




2/ Validation expérimentale

2.2/ Expansion : expansion locale (2)





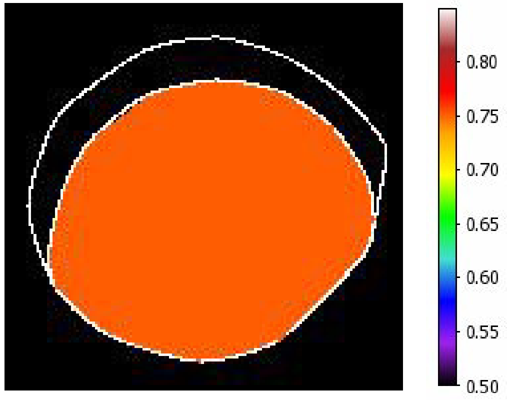
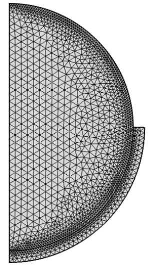


image IRM traitée
calcul de la fraction de gaz

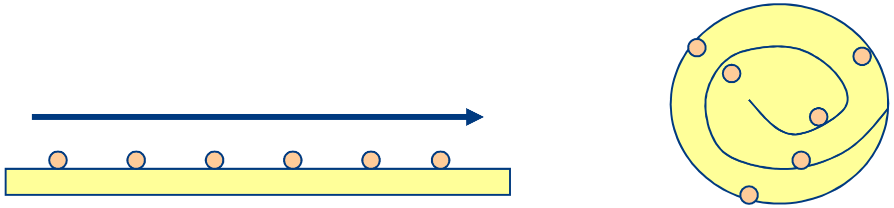
Results from: Wagner, M.J. ; Loubat, M. ; Sommier, A. ; Le Ray, D. ; Collewet, G. ; Broyart, B. ; Quintard, H. ; Davenel, A. ; Trystram, G. ; Lucas, T. **2008**. MRI study of bread baking: experimental device and MRI signal analysis. **Int. J. Food Sci.** Technol. 22: 331-339
Wagner, M. ; Quellec, S. ; Trystram, G. ; Lucas, T. **2008**. MRI evaluation of local expansion in bread crumb during baking. **J. Cereal Sci.** 27(4): 577-585



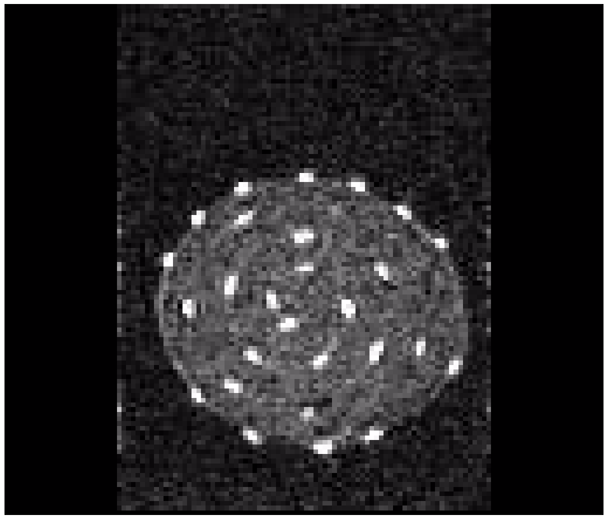
2/ Validation expérimentale

2.2/ Expansion : expansion locale (3)

Insertion des microcapsules le long de la ligne de façonnage



Coupe virtuelle en IRM avec microcapsules surbrillantes

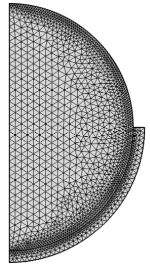


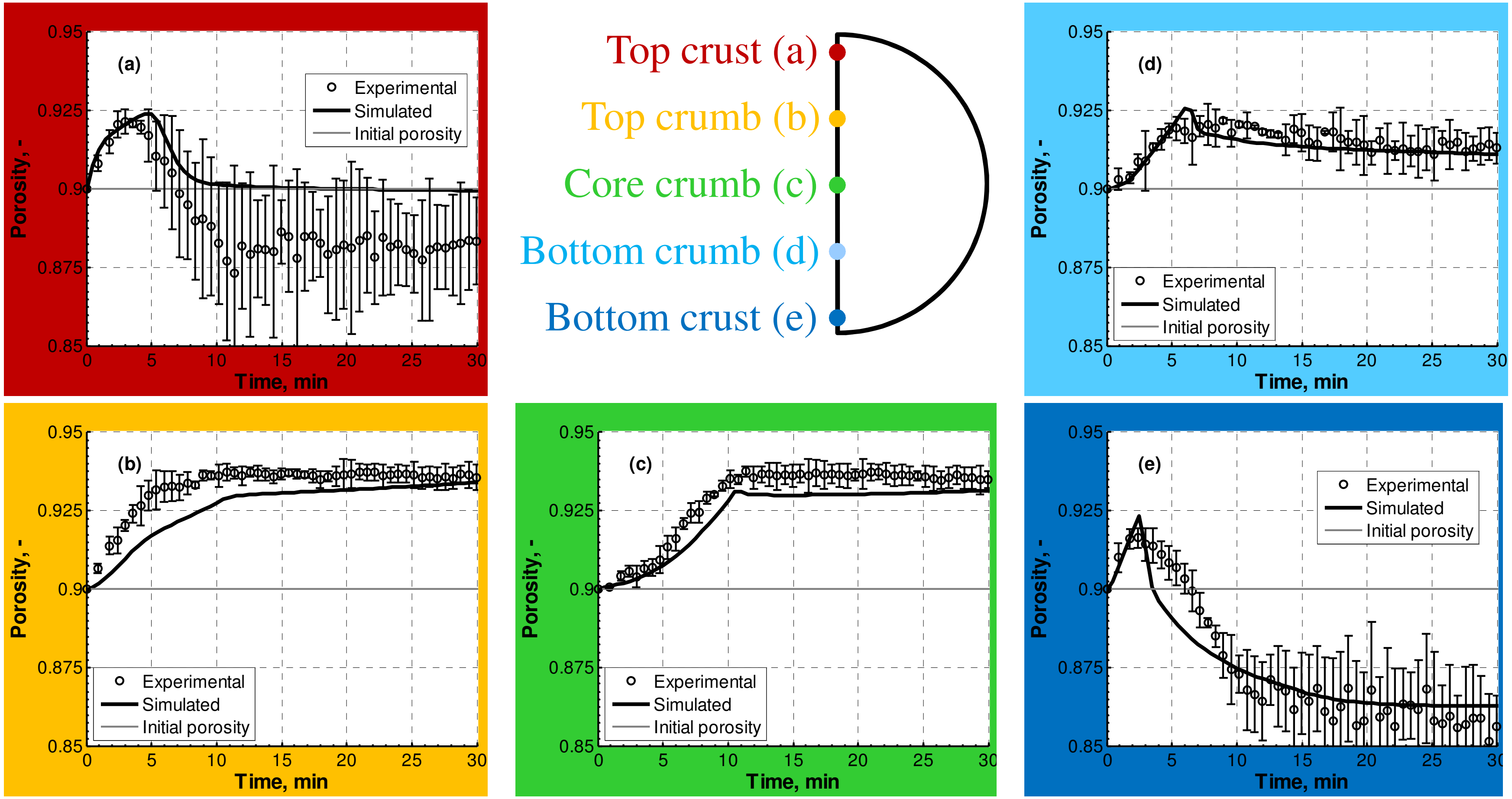
$$\varepsilon(t) = 1 - \frac{\left(1-\varepsilon(t_0)\right)}{\frac{S(t)}{S(t_0)}}$$

Results from: Wagner, M.J. ; Loubat, M. ; Sommier, A. ; Le Ray, D. ; Collewet, G. ; Broyart, B. ; Quintard, H. ; Davenel, A. ; Trystram, G. ; Lucas, T. **2008**. MRI study of bread baking: experimental device and MRI signal analysis**. Int. J. Food Sci.** Technol. 22: 331-339
Wagner, M. ; Quellec, S. ; Trystram, G. ; Lucas, T. **2008**. MRI evaluation of local expansion in bread crumb during baking. **J. Cereal Sci.** 27(4): 577-585

2/ Validation expérimentale

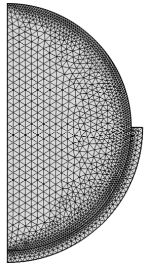
2.2/ Expansion : expansion locale (4)





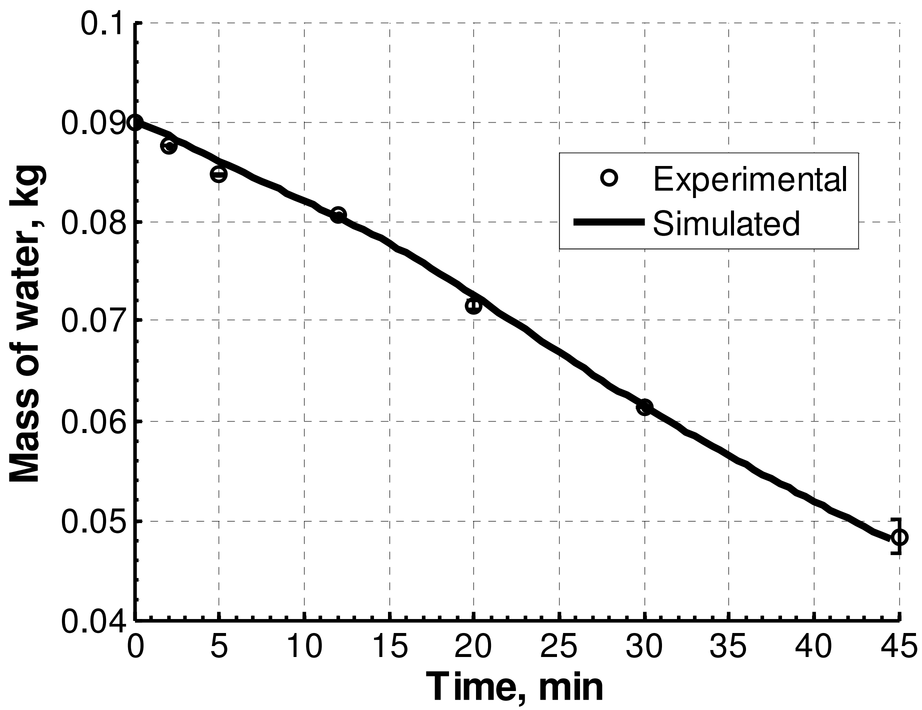
Results from:

Nicolas, V., Vanin, F., Doursat, C., Grenier, D., Lucas, T., Flick, D., 2014. Modelling bread baking with focus on global and local deformation. Submitted to AIChE J.



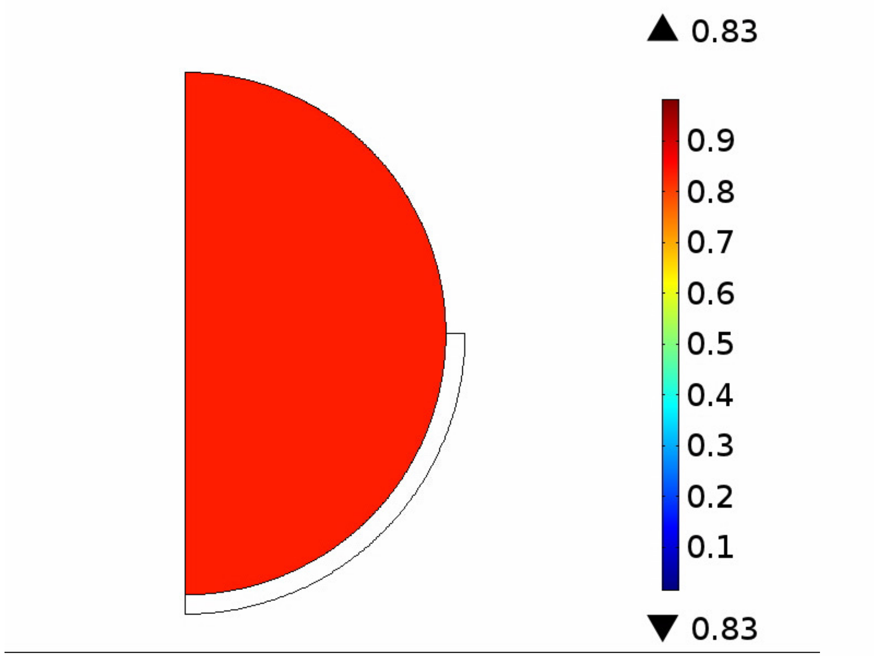
2/ Validation expérimentale

2.3/ Teneur en eau : pertes en eau

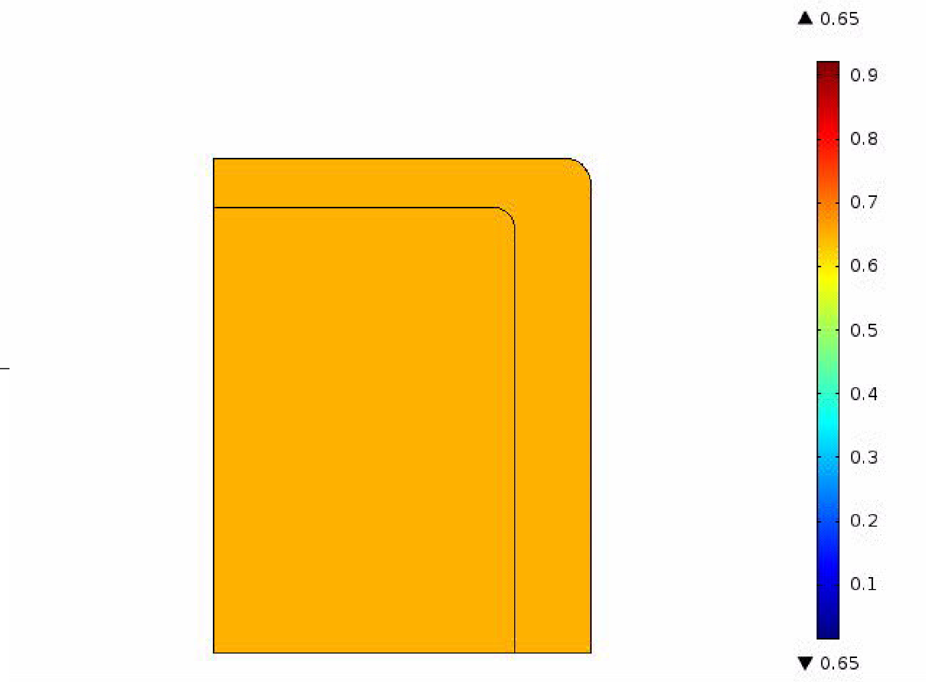


2/ Validation expérimentale

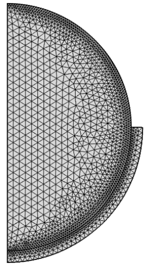
2.3/ Teneur en eau : profil d'humidité (1)



Teneur en eau



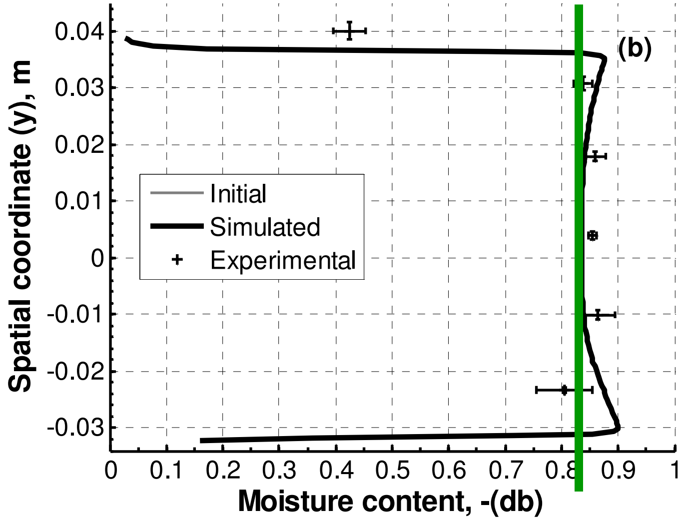
Results from:
Nicolas, V., Vanin, F., Doursat, C., Grenier, D., Lucas, T., Flick, D., 2014. Modelling bread baking with focus on global and local deformation. AIChE J.



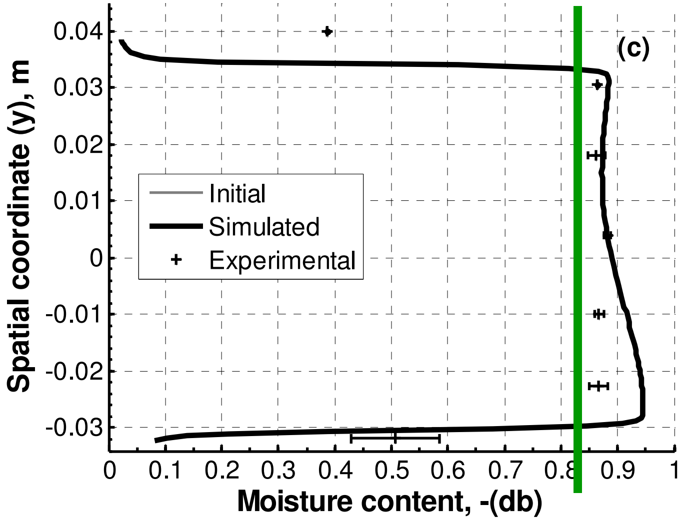
2/ Validation expérimentale

2.4/ Teneur en eau : profil d'humidité (2)

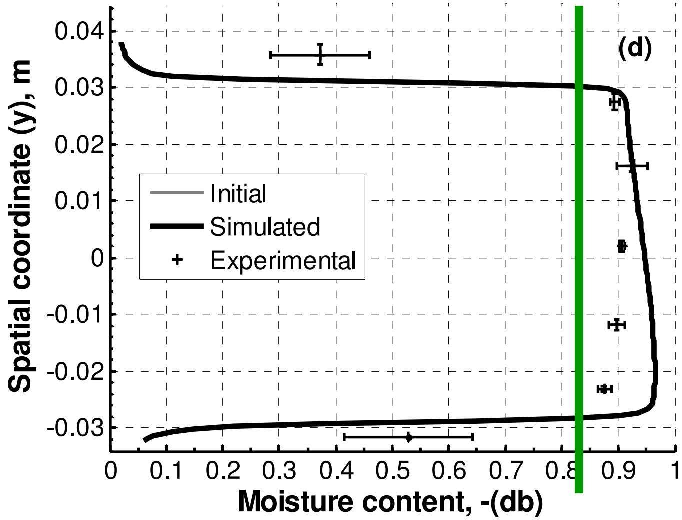
5 min



12 min

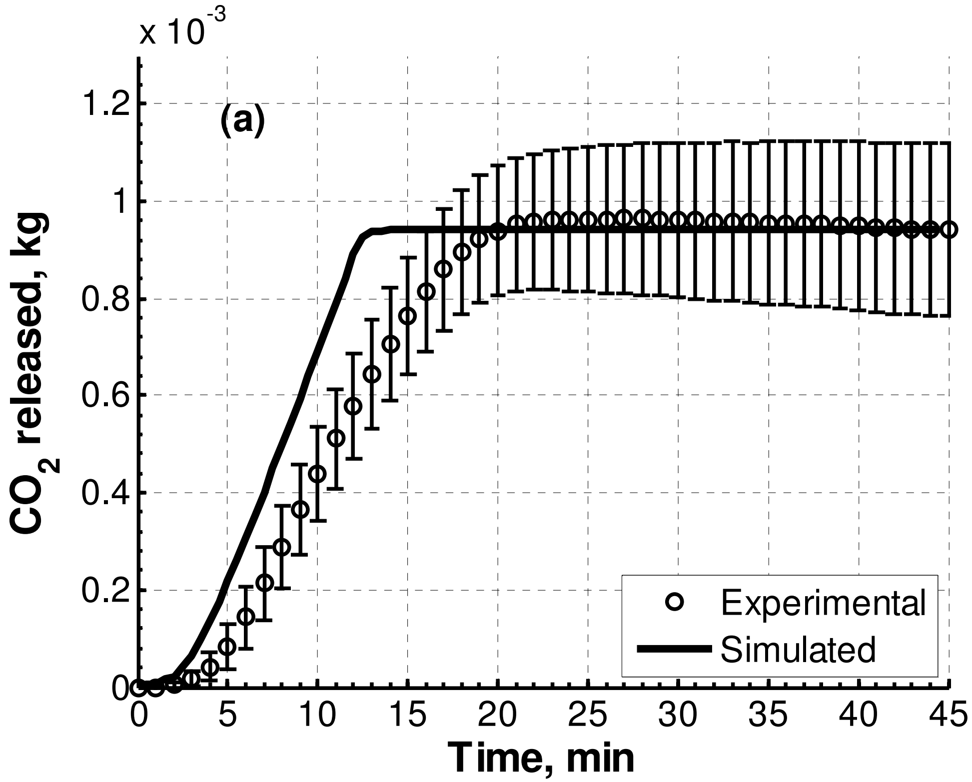


20 min



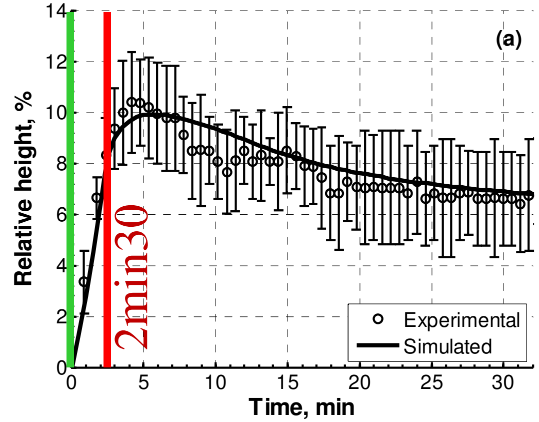
2/ Validation expérimentale

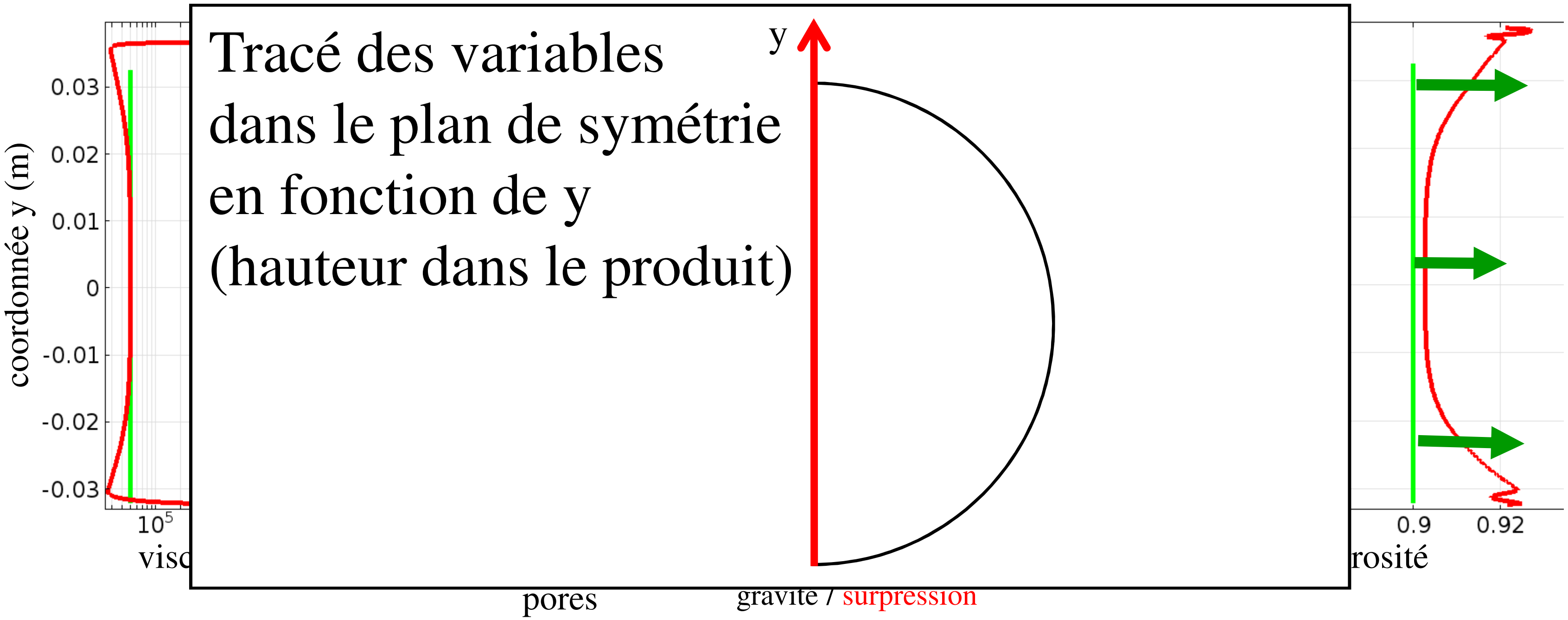
2.4/ Pertes en CO_2



3/ Interprétations

3.1/ Compréhension des phénomènes (1)

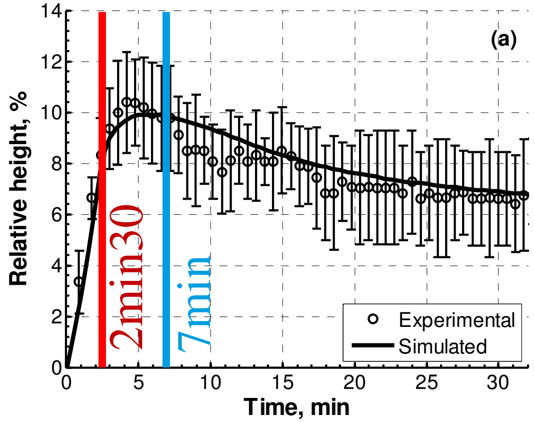


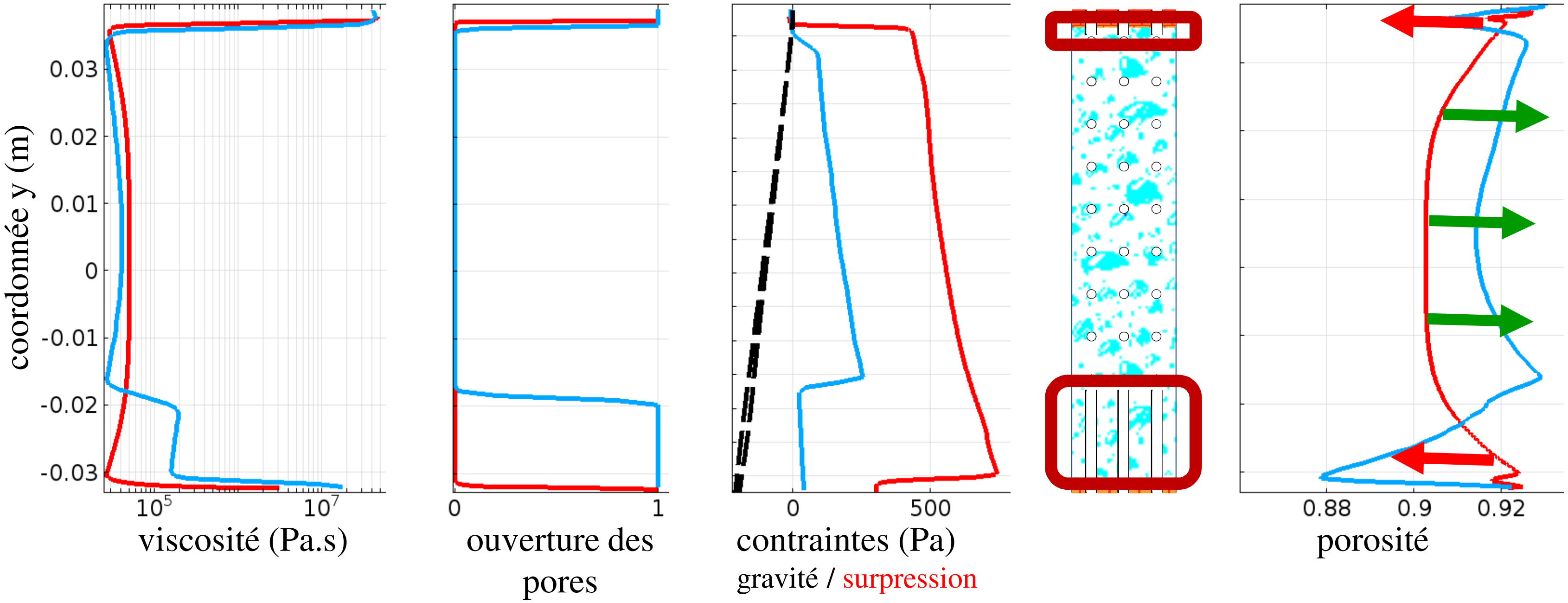


Phase d’expansion :
- porosité fermée
- haute pression
- faible viscosité

3/ Interprétations

3.1/ Compréhension des phénomènes (2)



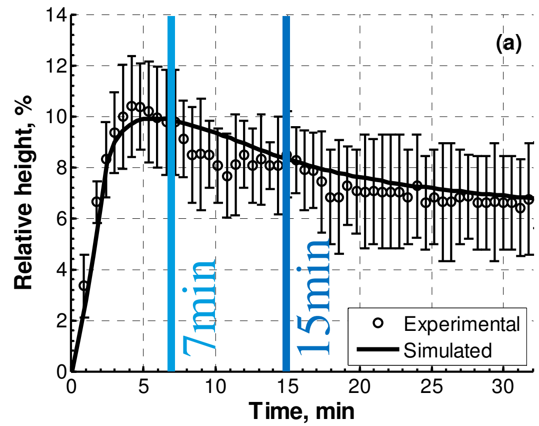


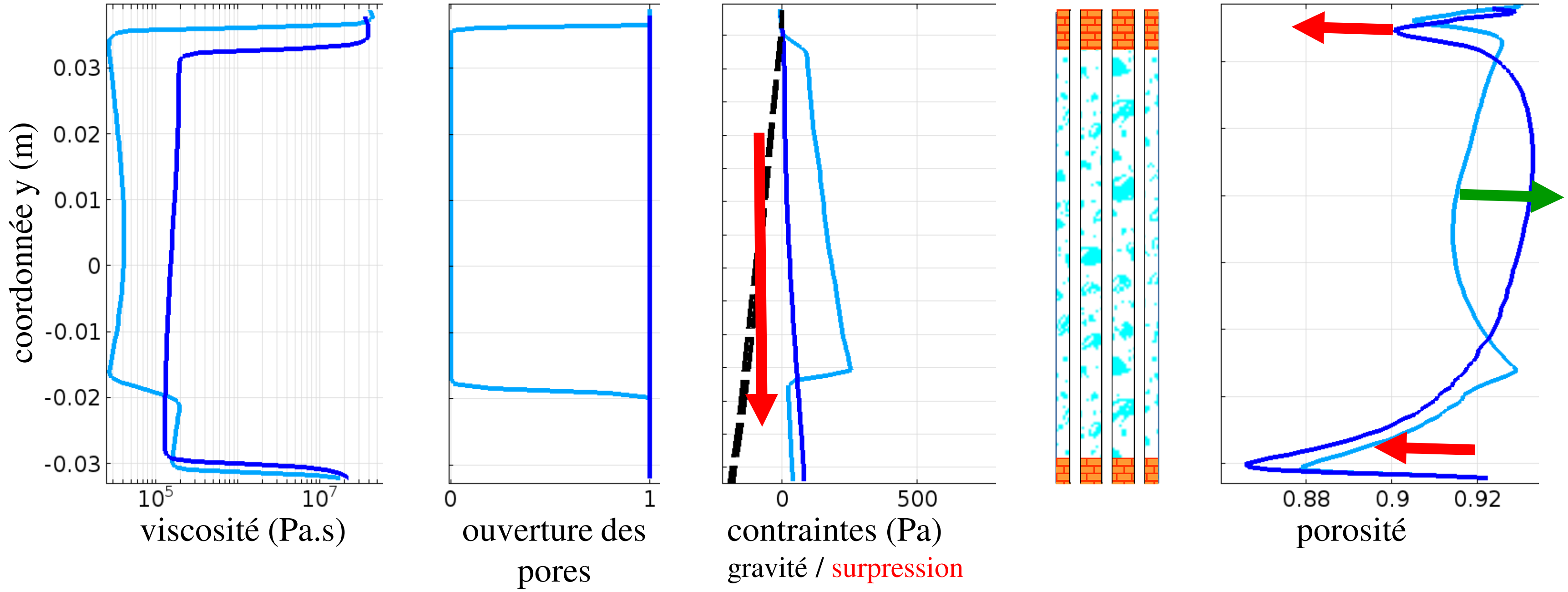
Phase de compression/expansion locale :

- phénomène de compression entre zones de surpression (cœur) et de haute viscosité (croûte)
- compression des zones de porosité ouverte mais encore liquide
- effet de la gravité

3/ Interprétations

3.1/ Compréhension des phénomènes (3)





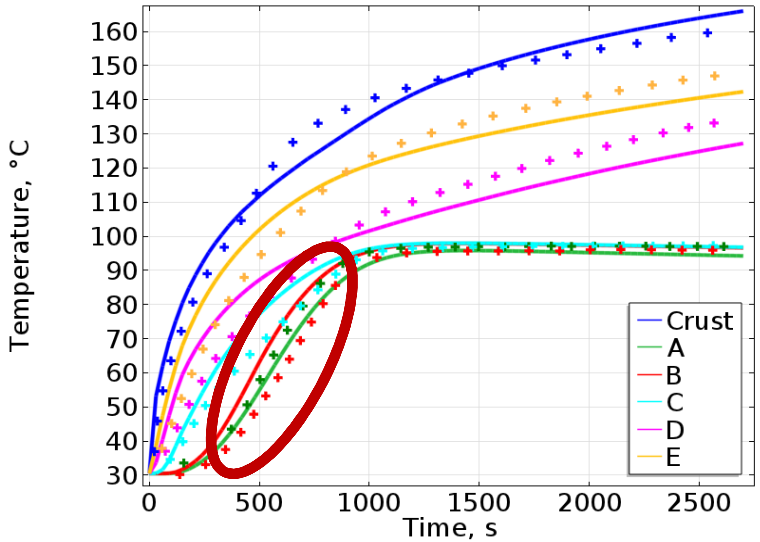
La hauteur du produit diminue :

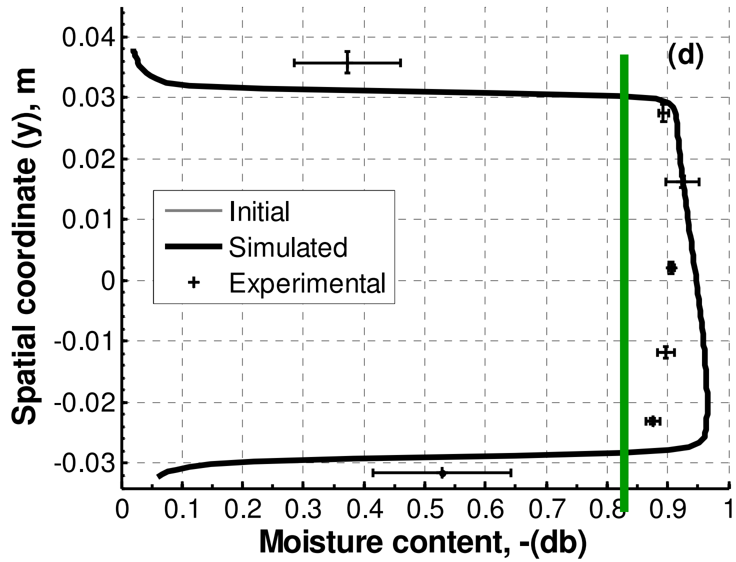
- porosité ouverte
- effet de la gravité et du retrait hydrique

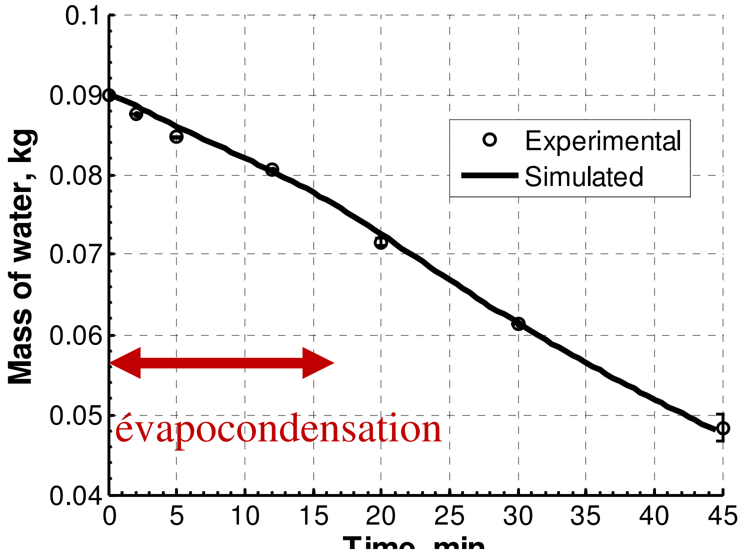
3/ Interprétations

3.2/ Rôle de l'évapo-condensation (1)

Il est nécessaire de prendre en compte :
- l'évapocondensation



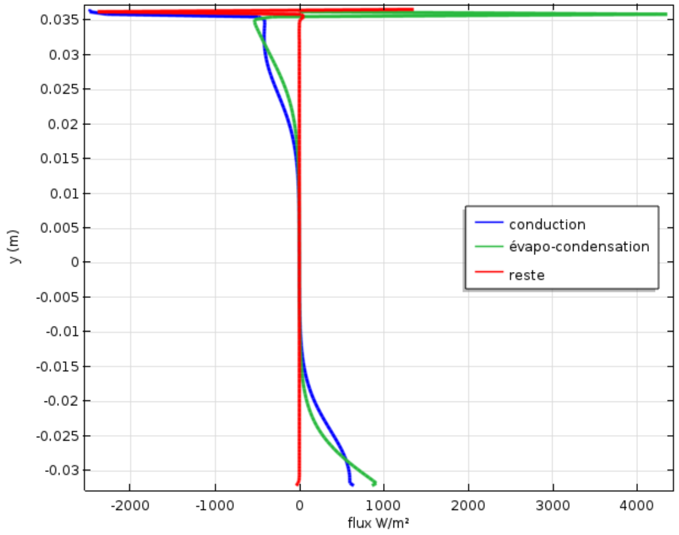




- enrichissement en eau à cœur
- montée en température plus rapide à cœur
- perte en masse plus rapide sans évapocondensation

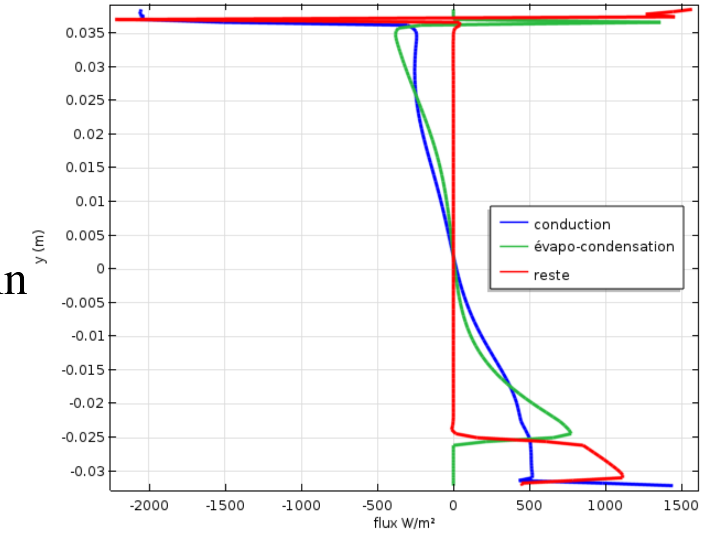
3/ Interprétations

3.2/ Rôle de l'évapo-condensation (2)

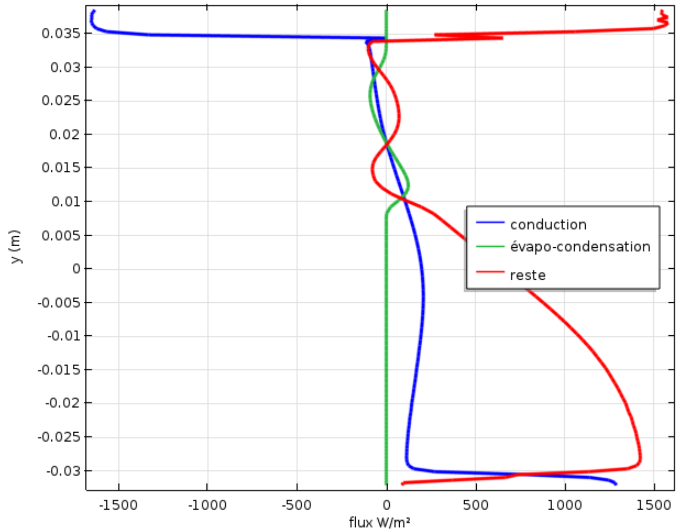


t = 2 min

t = 5 min

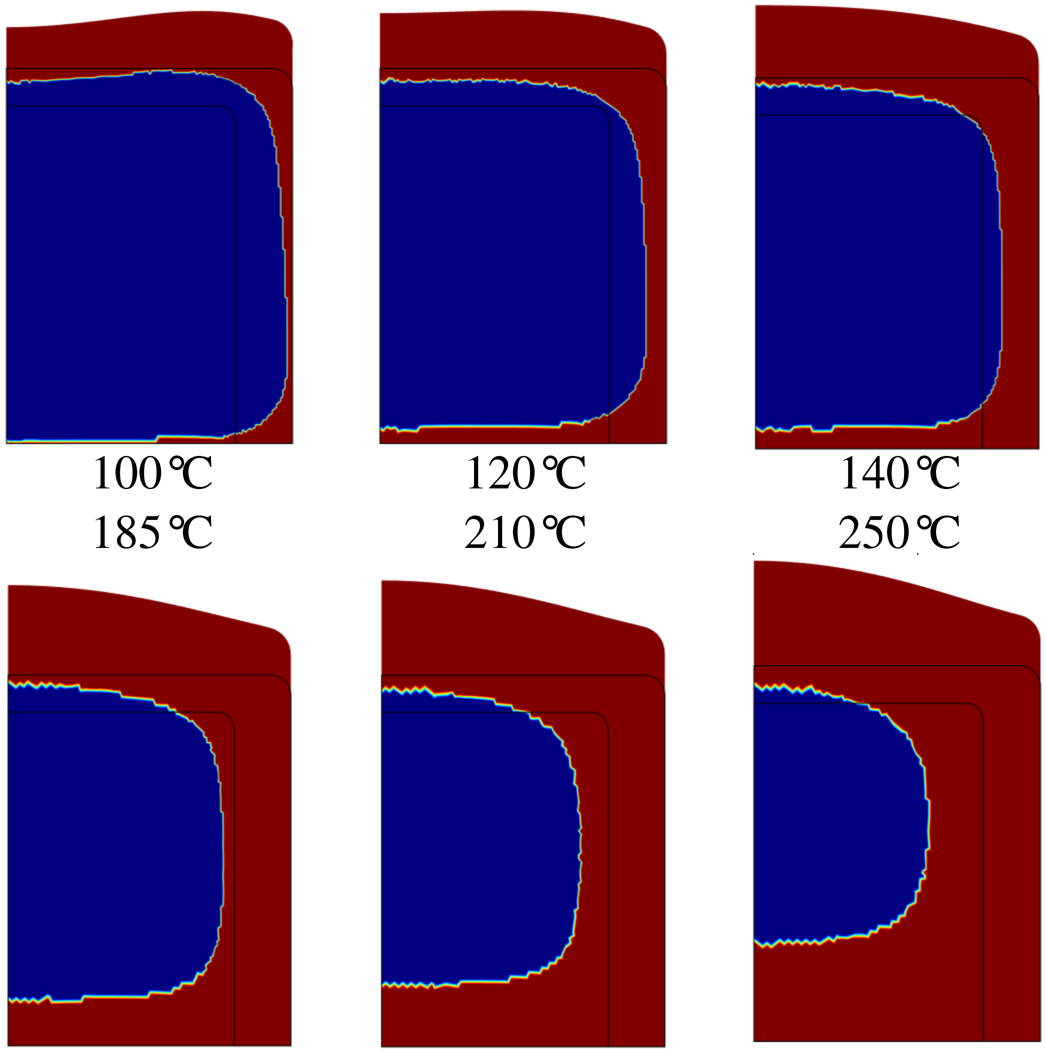


t = 12 min



3/ Interprétations

3.3/ Changement des conditions de cuisson



Teneur en eau en fin de cuisson (45 min)
$X_{wl} < 0.2$ kg.kg^{-1} en rouge

Merci