A goal-based angular adaptivity method for thermal radiation modelling in non gray media

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Motivations

Why angular adaptivity ?

‣ The angular dependence of the radiation field changes with space and optical thickness

thin media: very directional, non local transfer

thick media: close to isotropy, local transfer

Which critera to use for adapting ?

‣ In the framework of coupled flow/radiation problems, we want to compute accurately the radiative source terms affecting the energy balance

Adapt the angular resolution differently depending on the optical thickness and across space with the radiative source terms as a goal

Outline

- Motivations
- Numerical methods
- Goal-based angular adaptivity
- Results
- Conclusion

Numerical methods

Radiative transfer equation

- RTE without scattering at LTE, n=1, homogeneous medium

$$\boldsymbol{\Omega} \cdot \boldsymbol{\nabla}\, I(\nu, \boldsymbol{r}, \boldsymbol{\Omega}) = \kappa(\nu)\, (I_b(\nu, T(\boldsymbol{r})) - I(\nu, \boldsymbol{r}, \boldsymbol{\Omega})) \tag{1}$$

- Boundary condition for a diffuse opaque gray body

$$I(\nu, \boldsymbol{r}_w) = \varepsilon I_b(\nu, T(\boldsymbol{r}_w)) + \frac{1-\varepsilon}{\pi} \int_{\boldsymbol{\Omega}' \cdot \boldsymbol{n} < 0} I(\nu, \boldsymbol{r}_w, \boldsymbol{\Omega}') |\boldsymbol{\Omega}' \cdot \boldsymbol{n}\,| d\boldsymbol{\Omega}' \tag{2}$$

- Radiative source term affecting the energy balance

$$\boldsymbol{\nabla} \cdot \boldsymbol{q}^{rad}(\boldsymbol{r}) = \int_{\nu} \kappa(\nu) \left(4\pi I_b(\nu, T(\boldsymbol{r})) - \int_{\boldsymbol{\Omega}} I(\nu, \boldsymbol{r}, \boldsymbol{\Omega}) d\boldsymbol{\Omega}\right) d\nu \tag{3}$$

Numerical methods

FETCH

FETCH: Finite element solver for radiation transport

- Combines CG and DG methods for space discretisation
- Uses arbitrary angular discretisations (S_N, P_N, wavelets)
- Is implemented matrix free in **parallel**
- Is coupled with FEM flow solver Fluidity
- Has the ability to adapt its resolution in space and angle

Numerical methods

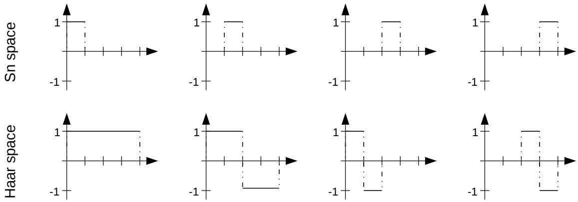
Angular discretisation

Angular adaptivity is based on hierarchical angular basis functions

- It provides an easy estimation of angular error
- No interpolations are required to couple angles across space

Haar wavelets

- $\mathbb{P}_0$ (piece-wise constant) hierarchical basis functions
- Mapping with discrete ordinate basis functions



Transformation matrix

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & -1 \end{pmatrix}$$

Figure: Haar wavelet and S_N basis functions over a 1D interval, order 3

Numerical methods

Validation

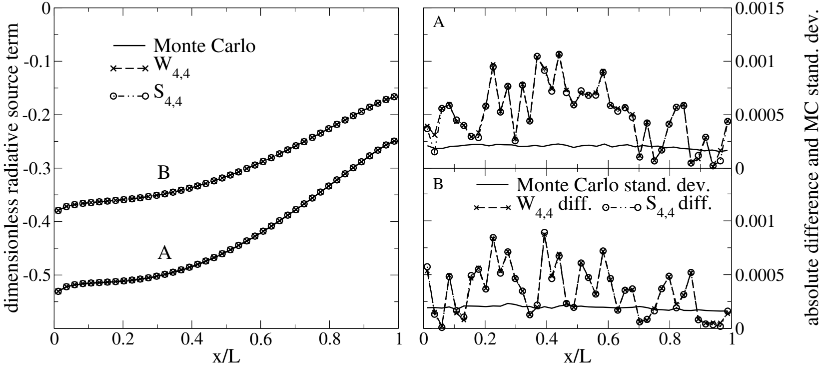


Figure: Benchmark test case, comparison between Monte Carlo, Haar wavelets and Discrete Ordinates

Numerical methods

Radiative property modelling

Global model

$$F(k) = \frac{\pi}{\sigma T_{\mathrm{ref}}^4} \int_{\nu/\kappa(\nu, T_{\mathrm{ref}}) \leqslant k} I_b(\nu, T_{\mathrm{ref}}) d\nu \tag{4}$$

$$\boldsymbol{\Omega} \cdot \boldsymbol{\nabla}\, I_i(\boldsymbol{r}, \boldsymbol{\Omega}) = k_i \left(a_i \frac{\sigma T^4(\boldsymbol{r})}{\pi} - I_i(\boldsymbol{r}, \boldsymbol{\Omega}) \right) \tag{5}$$

- computation of model parameters (k_i, a_i) from LBL absorption spectrum

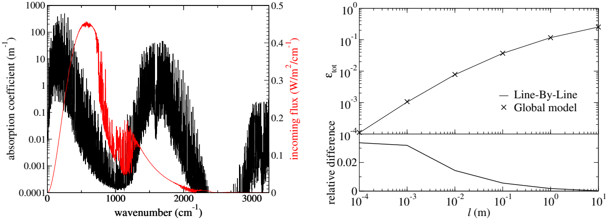


Figure: (Left) Absorption spectrum for 2% H_2O in air at 294.2 K and incoming flux BC. (Right) Emissivities for different column lengths.

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Goal-based angular adaptivity

Functional and adjoint problem $\langle \mathcal{L} I, I^* \rangle = \langle I, \mathcal{L}^* I^* \rangle$

- The adaptivity is based on a goal represented by a functional. Here the functional is the radiative source term affecting the energy balance

$$F_v(I(\nu, \boldsymbol{r}, \boldsymbol{\Omega})) = \int_{\boldsymbol{r}} \int_{\boldsymbol{\Omega}} \int_{\nu} \kappa(\nu)(I_b(\nu, T(\boldsymbol{r})) - I(\nu, \boldsymbol{r}, \boldsymbol{\Omega})) d\nu d\boldsymbol{\Omega} d\boldsymbol{r} \qquad (6)$$

- Forward I and adjoint I^* solutions are required in the goal-based adaptivity algorithm, where the adjoint problem is defined by

$$-\boldsymbol{\Omega} \cdot \boldsymbol{\nabla}\, I^*(\nu, \boldsymbol{r}, \boldsymbol{\Omega}) = \mathcal{S}_v^* - \kappa(\nu) I^*(\nu, \boldsymbol{r}, \boldsymbol{\Omega}) \qquad (7)$$

- The volume source is related to the integrand f of the goal functional F

$$\mathcal{S}_v^* = -\frac{\partial f_v}{\partial I} = \kappa(\nu) \qquad (8)$$

Goal-based angular adaptivity

Error estimation

- An error estimation in an arbitrary goal can be written as

$$F(I_{\mathrm{exact}}) - F(I) = -\int \mathcal{R}(I)(I^*_{\mathrm{exact}} - I^*)dP \tag{9}$$

- This error measure is computed for each DOF (space i, angle q, class k) and the resolution is increased locally if this error is above a given tolerance

$$e_{k,q,i} = \frac{R_{k,q,i}\ \epsilon^*_{k,q,i}}{\Delta F} \tag{10}$$

- We use the wavelet hierarchy to approximate the error in the adjoint solution

$$\epsilon^*_{k,q,i} = \begin{cases} I^*_{k,q,i}, & \text{if q belongs to the max order} \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

Goal-based angular adaptivity

Adaptivity algorithm

```
for $a = 1, N_a$ do
    Solve forward and ajoint problem and calculate error field
    for $k = 1, N_k$ do
        for $i = 1, N_i$ do
            for $q \in \mathcal{M}_{ik}$ do
                if $e_{k,q,i} < 0.01$ then
                    Remove basis function $q$ from node $i$
                else if $e_{k,q,i} > 1.0$ then
                    Add next level basis function to node $i$
                end if
            end for
        end for
    end for
end for
```

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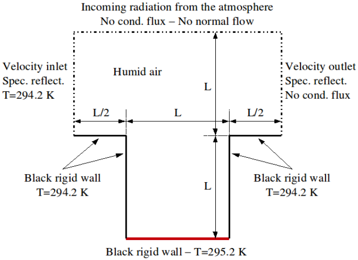
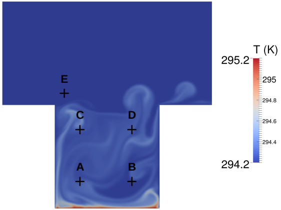
Results

Test case configuration

- Street canyon configuration and temperature snapshot obtained from uncoupled simulations at $Ra = 10^8$ and $Re = 5 \times 10^3$
- Discretised with 10^6 spatial elements, simulations run on 48 cores
- Results published in Soucasse *et al.*, JQSRT 200, 2017





- Accuracy and efficiency of the method
- Analysis of the distribution of the adapted angles
- Use of adapted resolution in coupled calculations

Results

Accuracy and efficiency

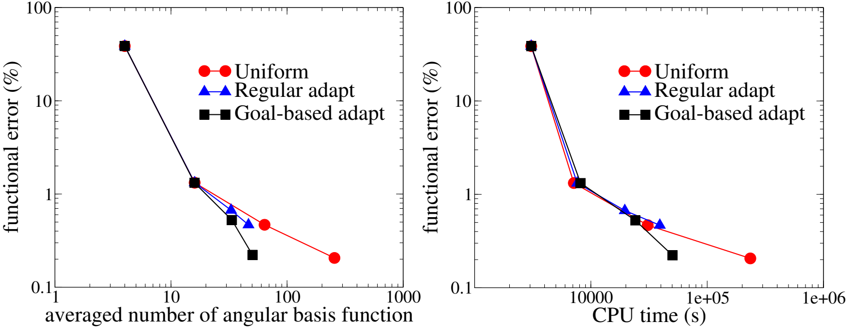
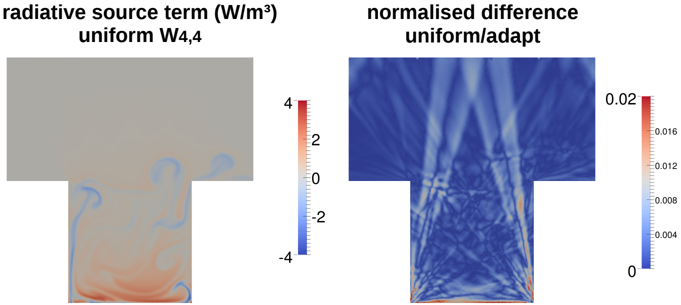


Figure: Functional error plot against averaged number of angles (left) ad CPU time (right).

- Gain of a factor 5 in CPU time and in angular resolution with goal-based adapted calculations without compromising the accuracy

Results

Accuracy and efficiency



radiative source term (W/m³)
uniform W4,4

normalised difference
uniform/adapt

Figure: Reference radiative source term $W_{4,4}$ (left) and difference with goal based adapted calculations $W_{4,4}^{\mathrm{GB}}$

- Patters of the radiative power field follow the thermal structures
- Local differences between reference and adaptivity solutions are around 2 % at most

Results

Distribution of the adapted angles

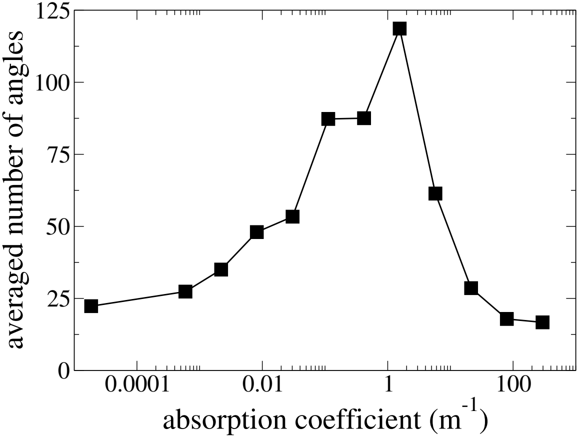


Figure: Averaged number of angular basis functions for each *k*-class

- The averaged angle number vary a lot with the optical thickness

Results

Distribution of the adapted angles

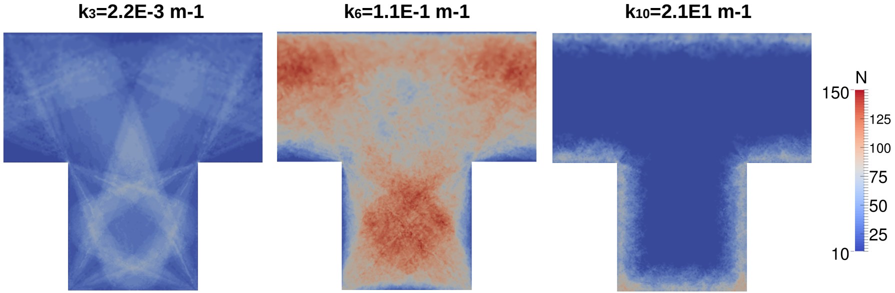


Figure: Spatial distribution of angular resolution for three *k*-classes $W_{4,4}^{\mathrm{GB}}$

- Thin: very directional but do not contribute much to the power
- Intermediate: very directional and contribute significantly to the power
- Thick: local transfer, focus on thermal gradients

Results

Distribution of the adapted angles

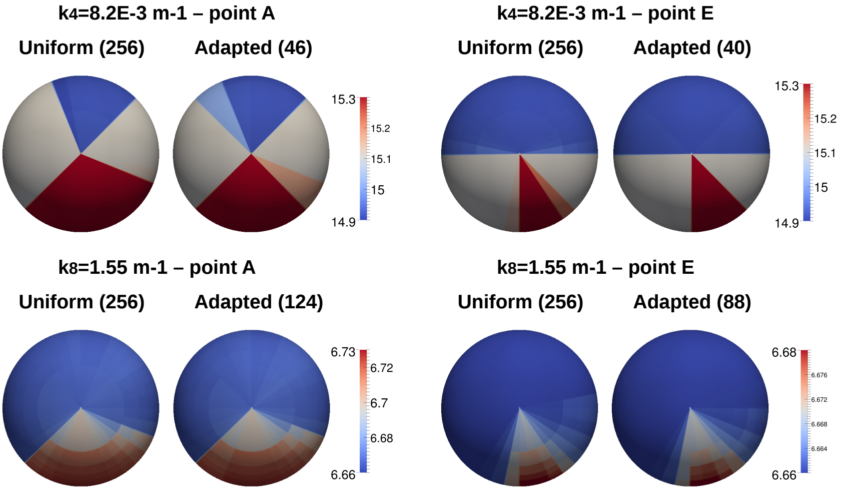


Figure: Angular distribution of the radiative intensity at different spatial points and *k*-class

Results

Coupled calculations

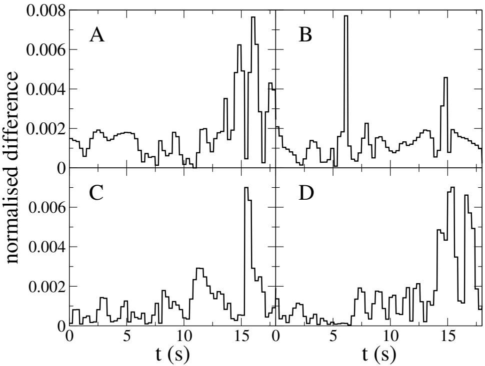


Figure: Difference in the radiative source term between uniform and adaped resolution during a coupled unsteady simulation

- Increase of differences with time
- Changing the adapted resolution with time does not help

Future works

- To enhance the performances of angular adaptivity with load balancing
- To apply the method to heterogeneous media encountered in combustion processes
- To test the adaptivity algorithm for highly directional thermal radiation problems (i.e. solar applications, fires)
- To combine angular adaptivity with spatial adaptivity

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