Subgrid-scale model for radiative transfer in turbulent participating media

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Motivations

DNS of turbulent flows coupled with thermal radiation

- Few coupled flow/radiation DNS in the literature
- Very fine spatial meshes, added to directional and spectral dependence for radiation

Radiation interactions

- Radiation emission is local (can be computed on the DNS mesh)
- Radiation absorption is non local (contribution of all turbulent length scales)
- Radiation absorption associated with small turbulent length scales is only significant for high optically thicknesses

Objective
Develop an accurate radiation model to be coupled with a flow DNS

- Motivations
- Radiative transfer model
    - Filtering of the blackbody intensity
    - Subgrid–scale model
    - Implementation
- Results
    - Homogeneous turbulent grey medium
    - Natural convection in a differentially heated cavity
    - Turbulent channel flow of hot combustion products
- Conclusion

Radiative transfer model

Filtering of the blackbody intensity

Radiative transfer equation and BC

- Linear dependence with the Planck function

$$\mathbf{u}\cdot\boldsymbol{\nabla} I_\nu(\mathbf{r},\mathbf{u}) = \kappa_\nu(\mathbf{r})\left[I_{\mathrm{b}\nu}(\mathbf{r}) - I_\nu(\mathbf{r},\mathbf{u})\right]$$

$$I_\nu(\mathbf{r}_w,\mathbf{u}) = \varepsilon_\nu(\mathbf{r}_w) I_{\mathrm{b}\nu}(T_w(\mathbf{r}_w)) + \frac{1-\varepsilon_\nu(\mathbf{r}_w)}{\pi}\int_{\mathbf{u}'\cdot\mathbf{n}_w<0} I_\nu(\mathbf{r}_w,\mathbf{u}') \mid \mathbf{u}'\cdot\mathbf{n}_w \mid \mathrm{d}\mathbf{u}'$$

Filtering

$$I_{\mathrm{b}\nu}(\mathbf{r}) = \overline{I_{\mathrm{b}\nu}}(\mathbf{r}) + I'_{\mathrm{b}\nu}(\mathbf{r})$$

$$I_\nu(\mathbf{r},\mathbf{u}) = \widetilde{I}_\nu(\mathbf{r},\mathbf{u}) + I''_\nu(\mathbf{r},\mathbf{u})$$

Filtered contributions
Deterministic ray-tracing method on a coarse mesh

Subgrid scales
Model in Fourier space

Radiative transfer model

Subgrid-scale model

The subgrid contribution is likely to be significant for high optical thicknesses. Therefore, two assumptions are made:
(1) the absorption coefficient is considered uniform
(2) the subgrid intensity leaving the boundaries is set to zero

Subgrid model in Fourier space

- Fourier transform

$$\hat{I}''_\nu(\mathbf{k},\mathbf{u}) = \int_{\mathbf{r}} \exp(-i\mathbf{k}\cdot\,\mathbf{r}) I''_\nu(\mathbf{r},\mathbf{u}) \mathrm{d}\mathbf{r}$$

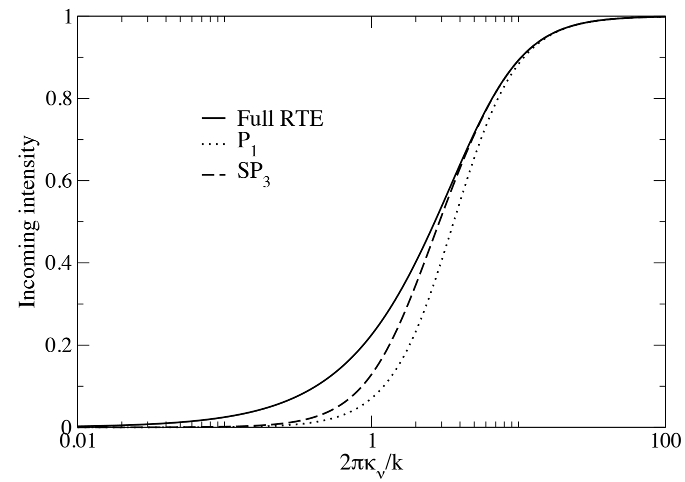
- RTE

$$(\kappa_\nu - i\mathbf{k}\cdot\mathbf{u})\hat{I}''_\nu(\mathbf{k},\mathbf{u}) = \kappa_\nu \hat{I}'_{\mathrm{b}\nu}(\mathbf{k})$$

- Analytic solution

$$\hat{G}''_\nu(\mathbf{k}) \equiv \int_{4\pi} \hat{I}''_\nu(\mathbf{k},\mathbf{u})\mathrm{d}\mathbf{u} = \kappa_\nu \hat{I}'_{\mathrm{b}\nu}(\mathbf{k}) \int_{4\pi} \frac{\mathrm{d}\mathbf{u}}{\kappa_\nu - i\mathbf{k}\cdot\mathbf{u}}$$

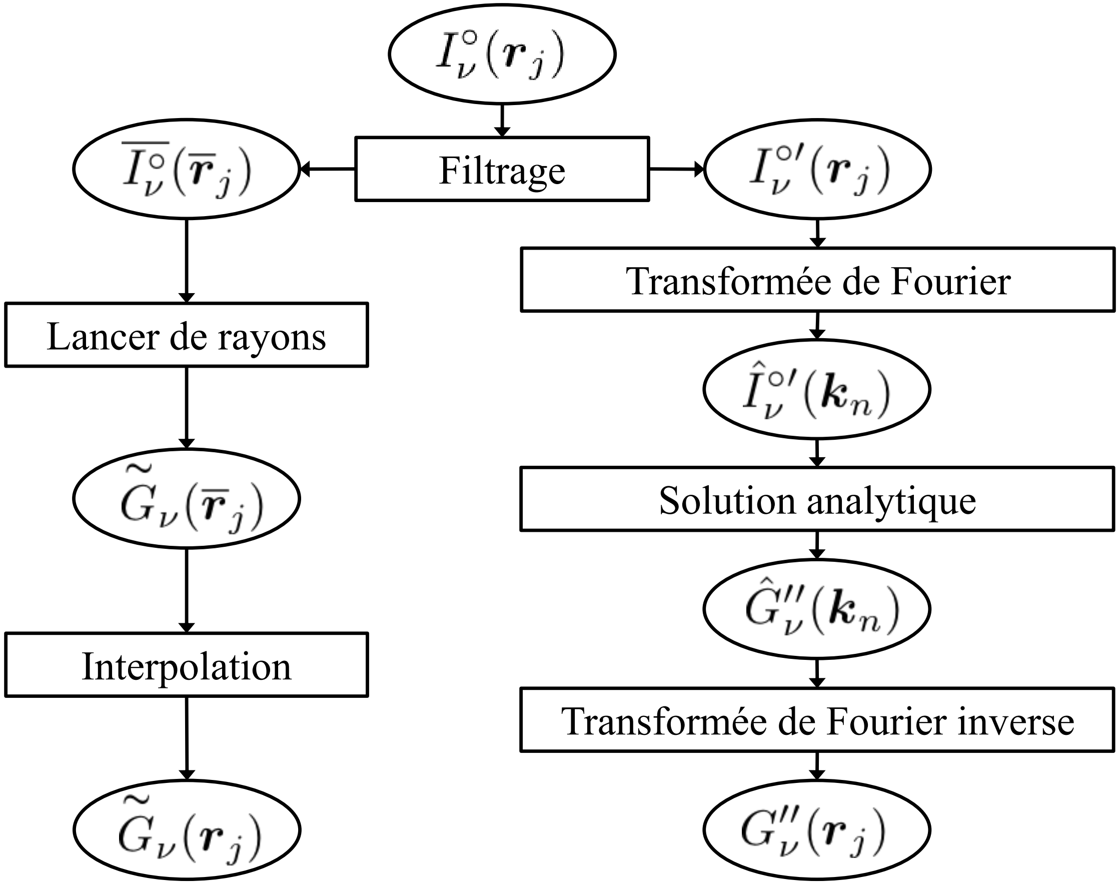
$$= 4\pi \hat{I}'_{\mathrm{b}\nu}(\mathbf{k}) \left(\frac{\kappa_\nu}{k}\right) \arctan\left(\frac{k}{\kappa_\nu}\right)$$



Subgrid intensity vs optical thickness of spatial fluctuations

Radiative transfer model

Implementation



$$P_\nu(\mathbf{r}) = \kappa_\nu(\mathbf{r})4\pi I_{\mathrm{b}\nu}(\mathbf{r}) - \kappa_\nu(\mathbf{r})(\widetilde{G}_\nu(\mathbf{r}) + G_\nu''(\mathbf{r}))$$

$\Rightarrow$ *Gain en temps de calcul de l'ordre de* $(N/\overline{N})^3$

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Results

- Homogeneous turbulent grey medium
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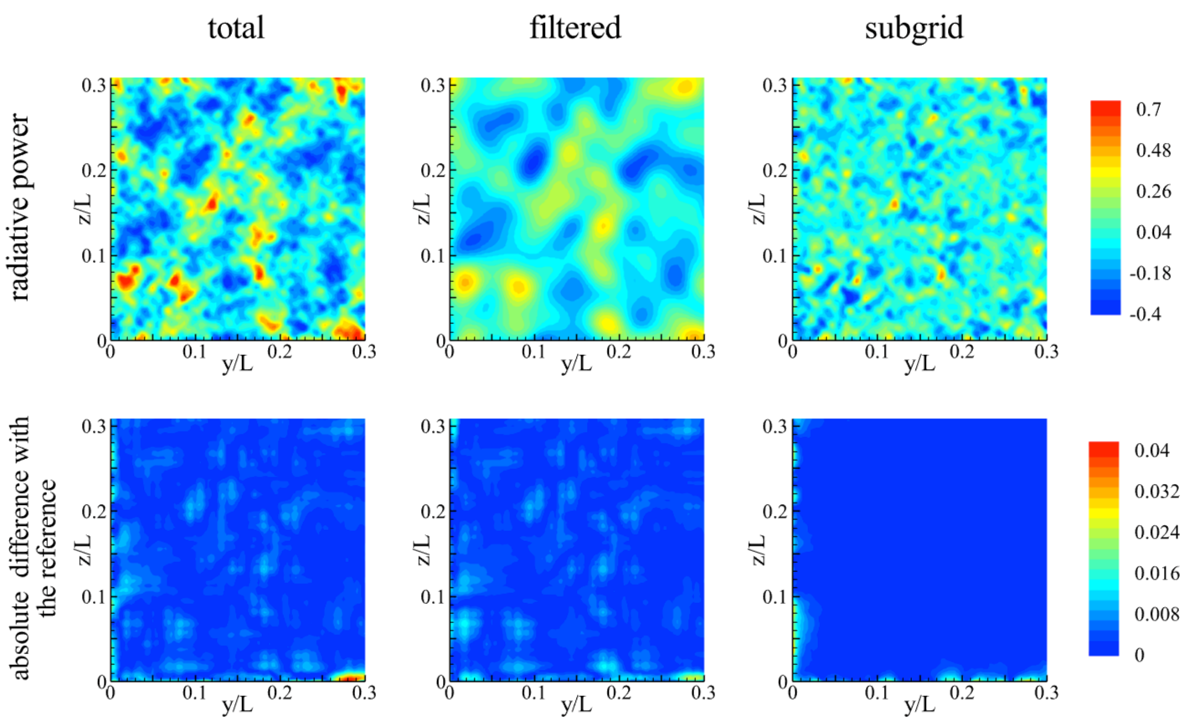
Conclusion

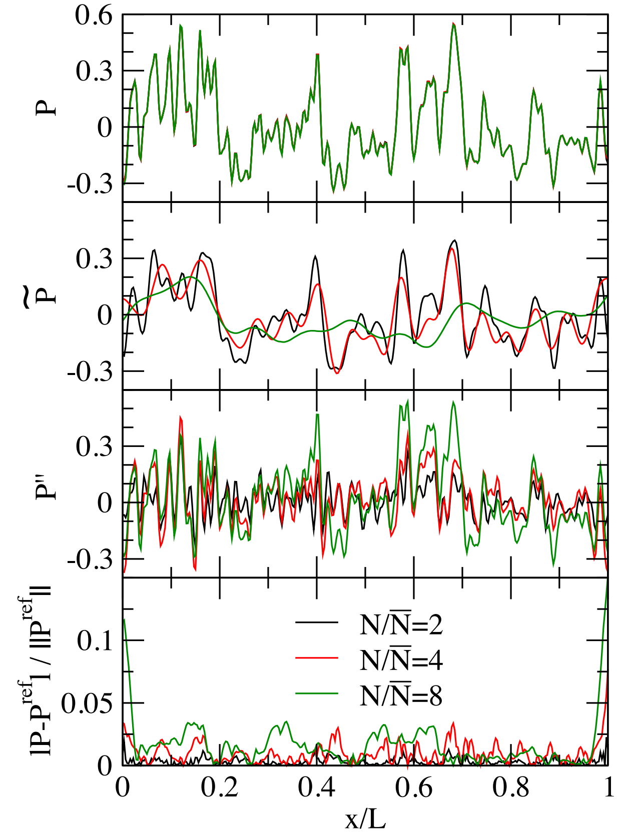
Results

Homogeneous turbulent gray medium

- Zero mean 3D temperature field satisfying:
  $$\langle \theta(\mathbf{r'})\theta(\mathbf{r}+\mathbf{r'})\rangle = \Theta^2 \exp(-\|\mathbf{r}\|/\Lambda)$$
- fine grid: 320^3, T_0 = 500 K, Θ = 50 K
- κL = 25, $\kappa\Lambda$ = 1.25



Dimensionless radiative power, x/L = 0.25, $N/\overline{N} = 4$

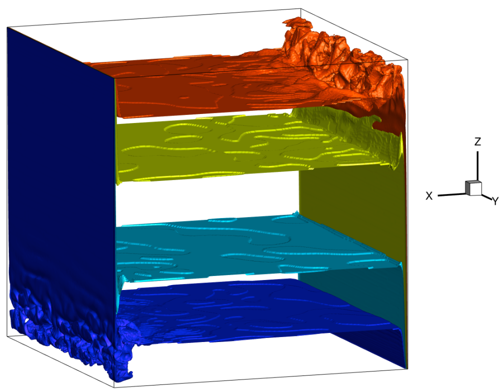


Effect of filtering level
y/L = z/L = 0.5

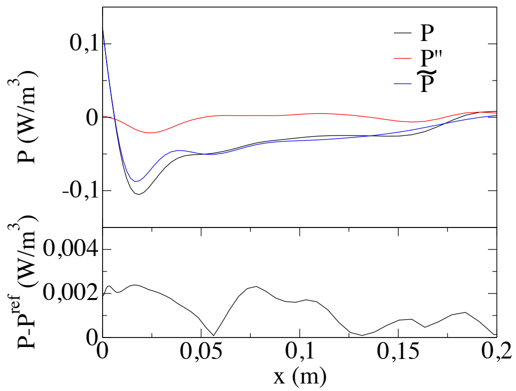
Results

Natural convection flow in a differentially heated cavity

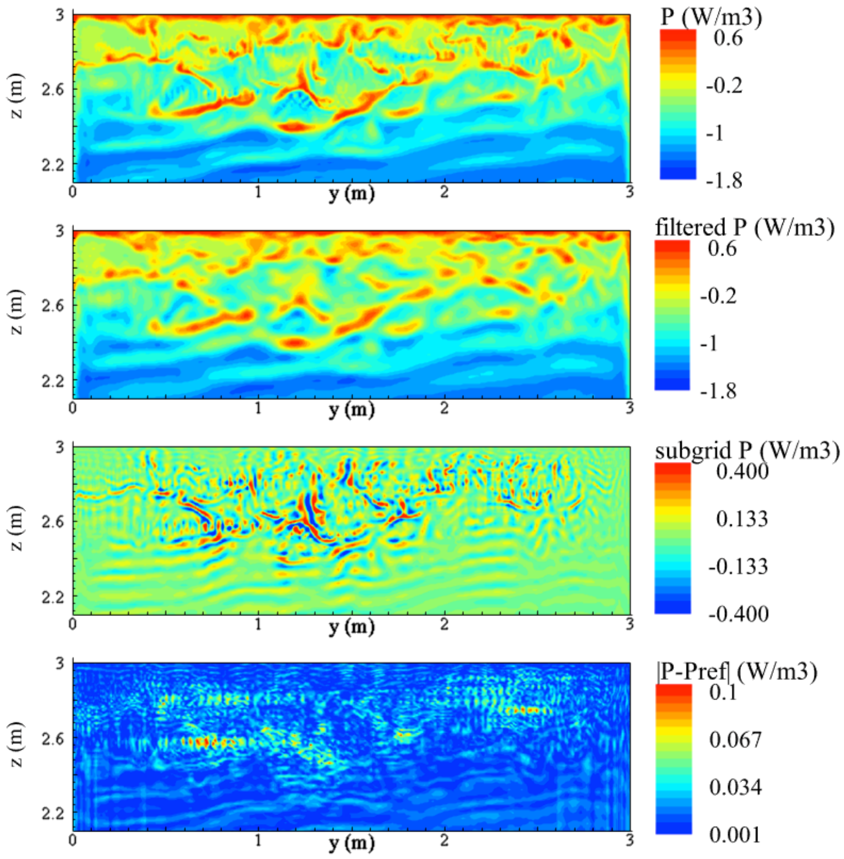
- fine grid: 320^3, coarse grid: 80^3, $Ra_L = 3x10^9$, $T_0 = 300$ K, L = 3 m
- Non grey uniform absorption coefficient (global model)



instantaneous iso-temperature surfaces (without radiation)



radiative power profile near the hot wall

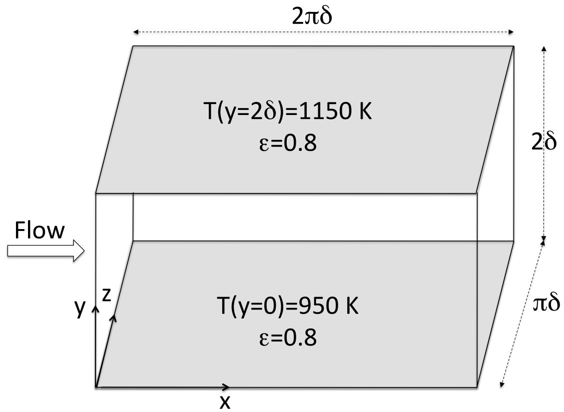


radiative power field in the B.L. near the hot wall

Results

Turbulent channel flow of hot combustion products

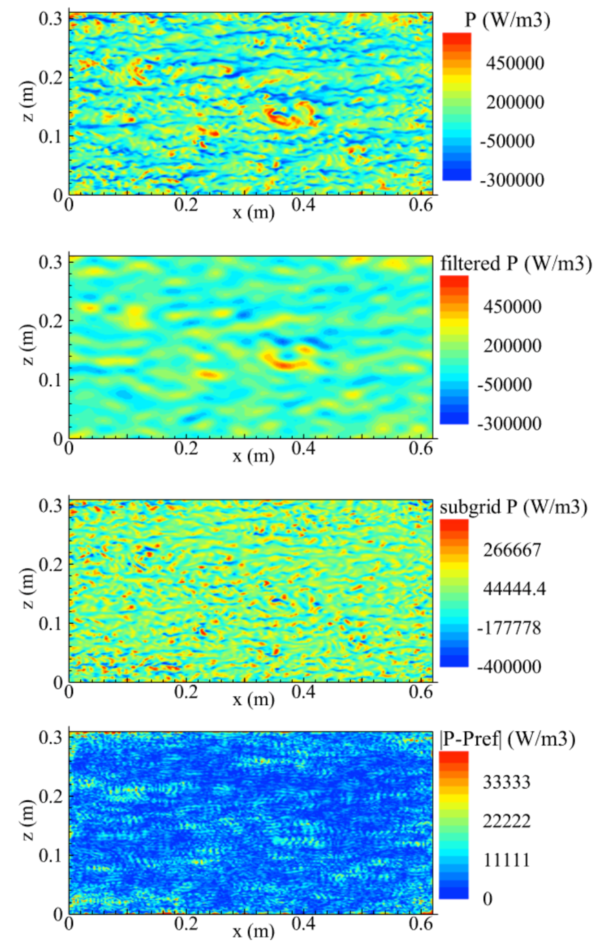
- Coupled calculations of Zhang et al. (2013)
- $Re_\delta = 1.175 \times 10^4$
- fine grid: 200x256x200
- coarse grid : 50x64x50



(p=40 atm, $x(H_2O)$ = 0.155, $x(CO_2)$ = 0.116, d = 10 cm)

Temperature dependent spectrum

- filtered contribution : sampling of the actual value of κ on the coarse mesh
- subgrid model : use of a uniform mean value



Radiative power field
in the B.L. near the cold wall

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Conclusion

Soucasse *et al.*, JCP 257, 2014

Strength

- Important CPU savings
- Accuracy (even near the walls and with temperature dependent spectra)
- Can be combined with other methods for the filtered part (Monte Carlo, DOM)

Drawback

- Use of structured grids (Fourier decomposition)

Perspectives

- How to combined this subgrid model with a LES of the flow ?