MICROSCALE SIMULATIONS OF CONDUCTIVE / RADIATIVE HEAT TRANSFERS IN POROUS MEDIA

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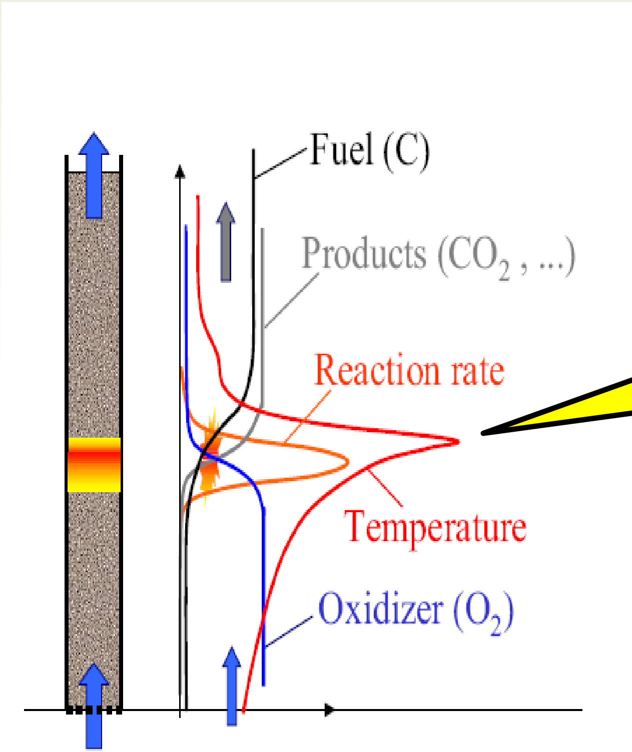
Context, motivation

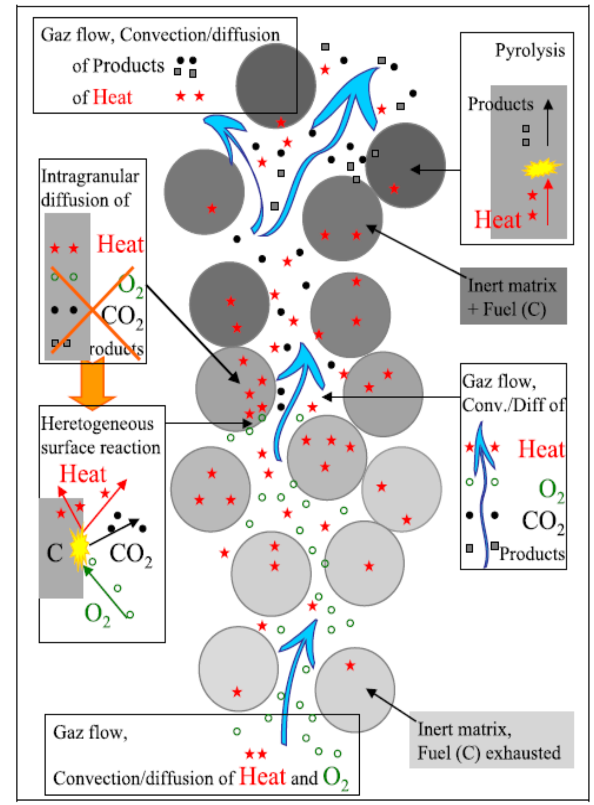


Smoldering in porous media

≈1400K (measured)

Significant radiative transfers

Microscale simulations



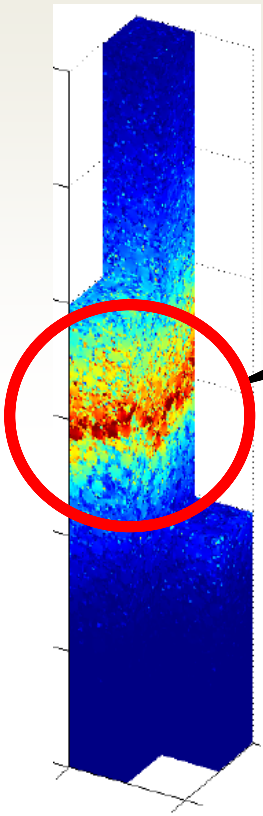
Heat conduction
convection/diffusion
no radiation

• no long-range tranfers, exchanges only between neighboring grains

⟹ we expect merely an additional equivalent conductivity $\lambda_{eff} = \lambda_{eff,c} + \lambda_{eff,r}\left(T^3\right)$

True ? **How much ?**

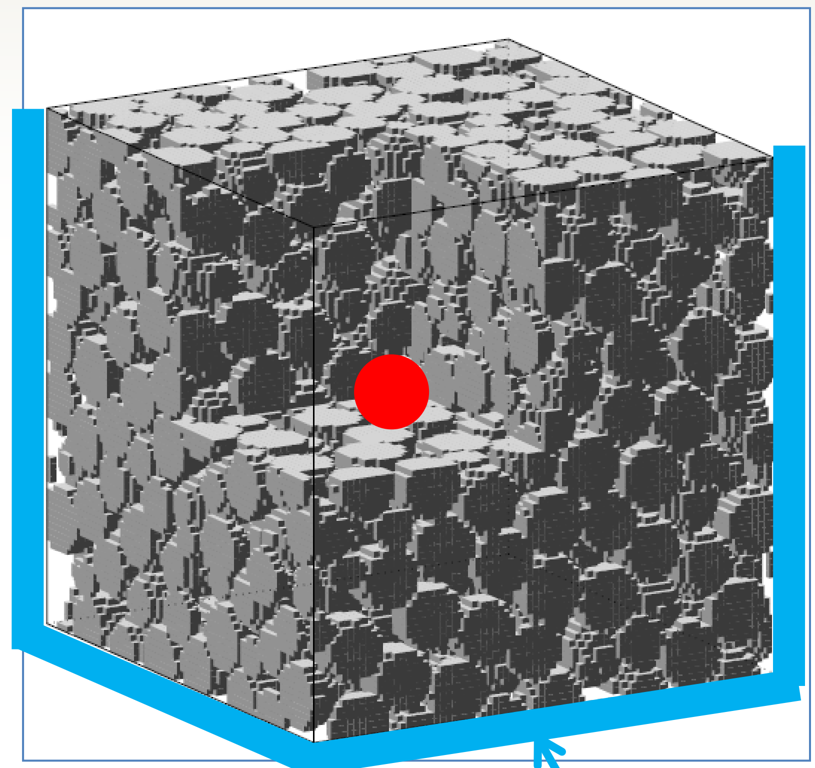
Reference situation



Reaction front

Heat source in grains currently burning

Continuous heat source in a single grain, within the packing



Oven at background temperature

Parameters:

- all set according to practical range
- Source power ***S*** [W]
- Background temperature ***T*.** [K]
- Grain size (*R*, = length unit)
- Solid thermal conductivity λ_s [W/mK]
- Bed porosity ε, effective conductivity $\lambda_{c,eff} = \tilde{\lambda}_c \, \lambda_s$

Numerical model

Conduction:

- conducting solid, Laplace equation
- no convection nor conduction in the gas
- time-explicit, finite-volume formulation
- discretization by a^3 cubic volume elements ($a = R/5$)

solid = conducting opaque black body
gas = vacuum

Radiative transfers:

- transparent gas phase
- opaque solid, black body
- the solid surfaces absorb all the incoming radiative flux
- the solid surfaces emit a flux with
  - an isotropic Lambert "cosθ" orientation distribution
  - a rate given by Stefan-Boltzmann law $E = \sigma T^4$ [W/m^2]
- Monte-Carlo simulation

Initial and boundary conditions:

- The bed is initially at background temperature $T_\bullet$
- External boundaries at constant background temperature $T_\bullet$
- Constant continuous heat supply S in the source

Numerical simulations

Simulation management: quasi-continous time scheme

- radiated **energy quantum: q_r** [J]
  set dynamically according to a cost/SNR compromise
  (corresponding to at most $\delta T = q_r/\rho c_p a^3 \approx 10^{-3}\ (T_S - T_\bullet)$ in a volume element)
- **time step δt** [s]
  set dynamically so each surface element emits $a^2 E\, \delta t \bullet q_r$
  (i.e. at most one quantum during δt)
- during a time step:
  each surface element emits 0 or 1 quantum (with a probability $a^2 E\, \delta t\ / q_r$)
  each quantum propagates till it hits a solid surface where it is absorbed
  the solid temperature is updated (-/+) in real time
- periodically, conduction is accounted for by an explicit finite-volume step

We monitor:

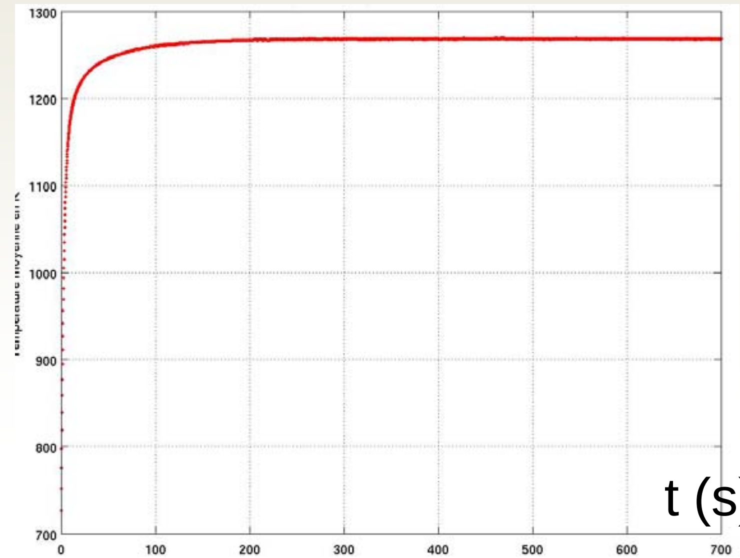
- the source temperature T_S
- the conductive and radiative and total outgoing fluxes

... until a steady regime is reached

Numerical simulations

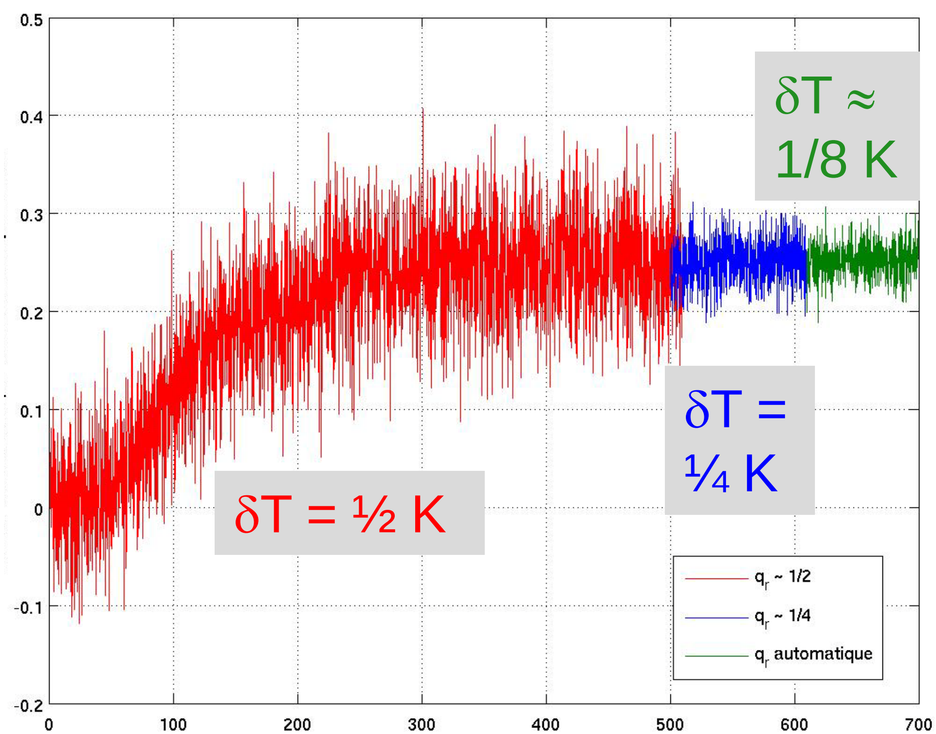
Example:

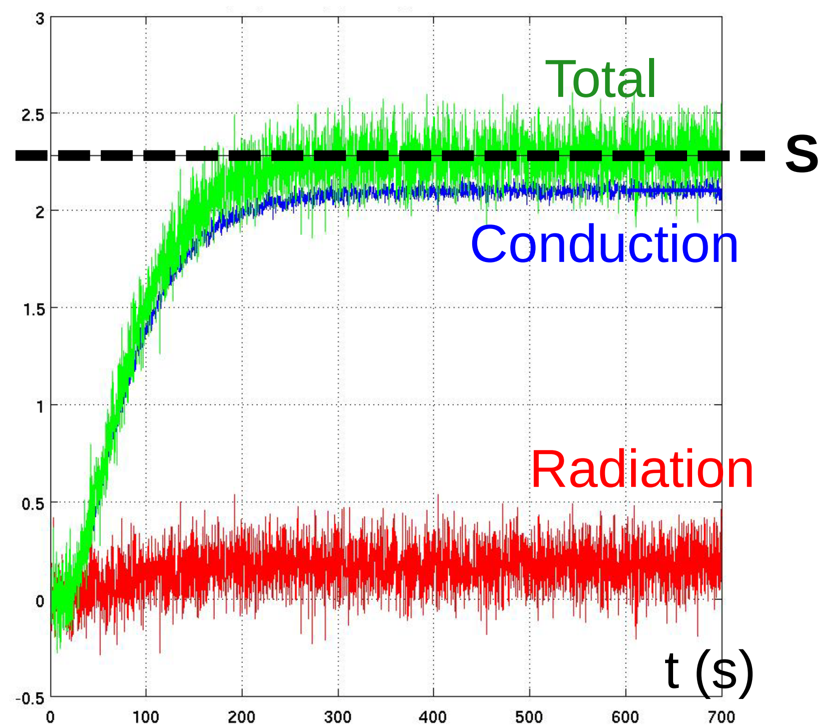
Mean source temperature T_S (K)



In practice:

- start with a coarse q_r (fast), and then
- refine q_r to improve SNR in steady state

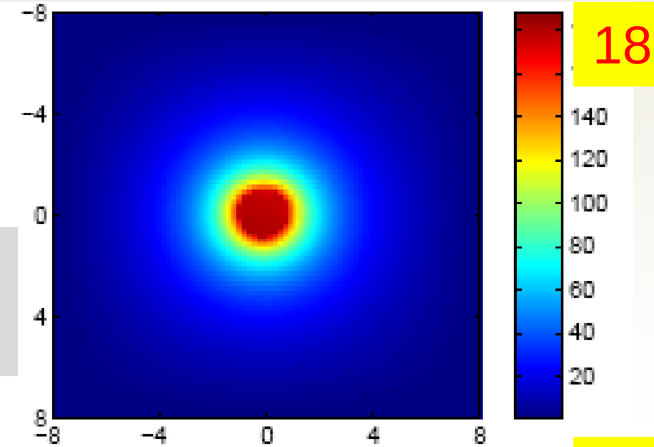




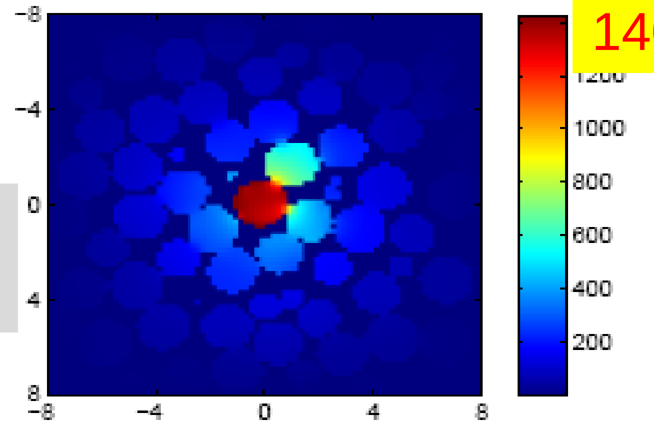
Phenomenology:

$T(r) - T_{\bullet}$ in 3 cases with the same parameters: S=2.28W, $T_{\bullet}$ =700K

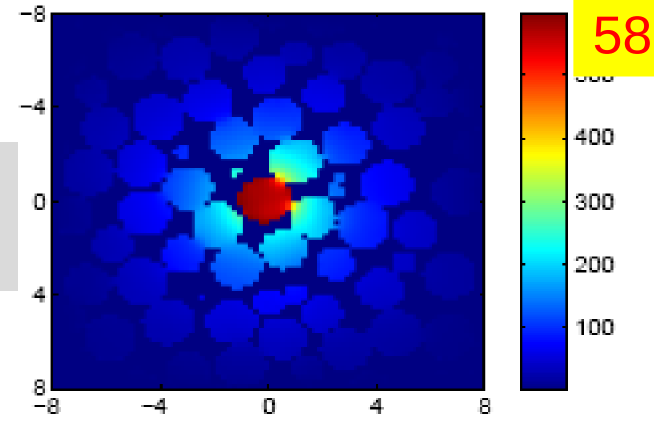
Reference case: conduction in plain solid



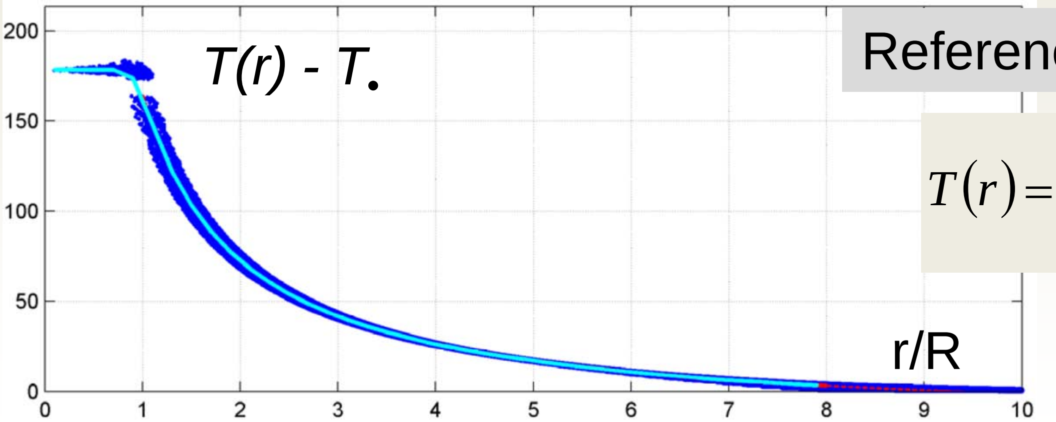
Source in a grain packing (conduction only)



Source in a grain packing (conduction + radiation)



Phenomenology: Radial temperature distribution



Reference case: conduction in plain solid

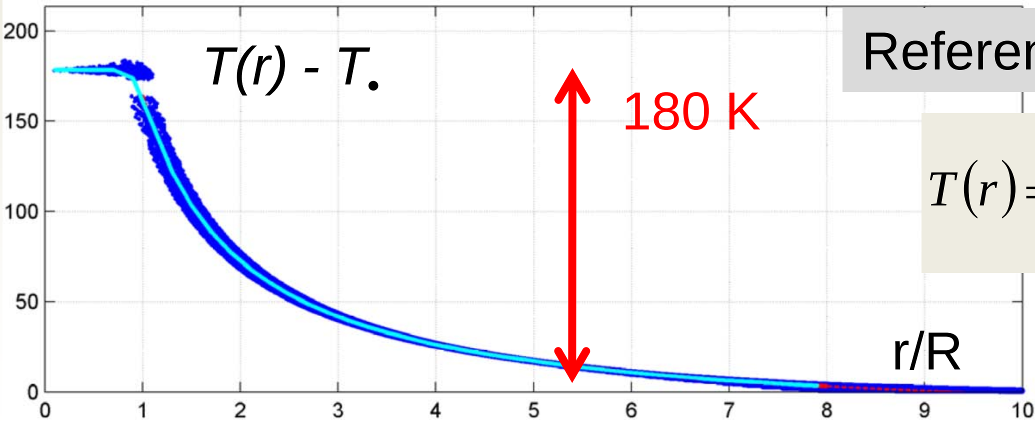
$$T(r) = T_\infty + \frac{S}{4\pi\lambda_s}\left(\frac{1}{r} - \frac{1}{r_\infty}\right), \quad \lambda_s = 0.979\mathrm{W/mK}$$

Analytical solution in a spherical domain.

Here, *r*. is an apparent distance for the application of the boundary condition,

➔ the solution applies only up to some distance to the oven walls (*r* • *8R*)

Phenomenology: Radial temperature distribution



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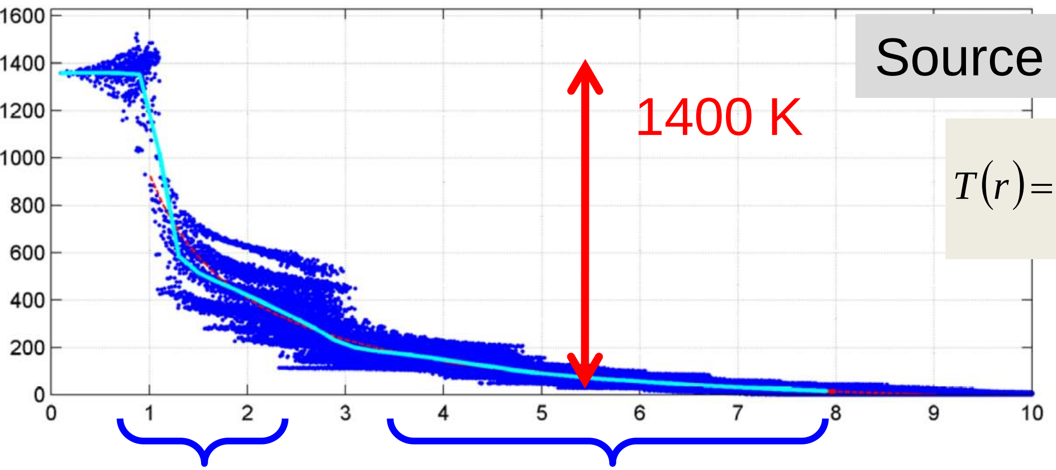


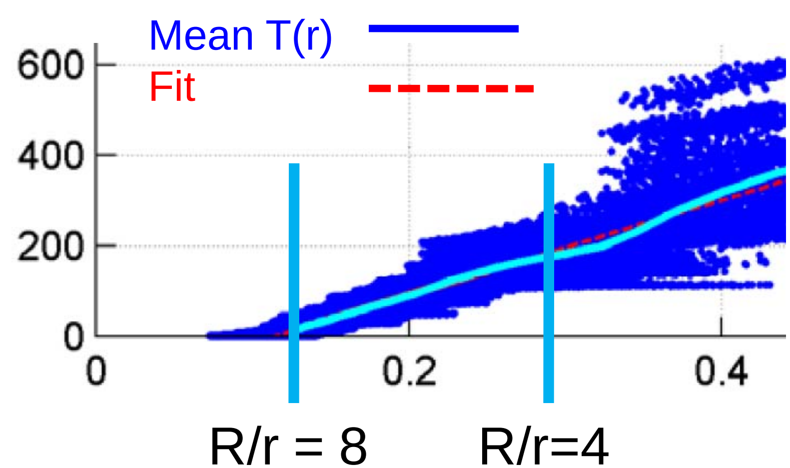
Source in a grain packing (conduction only)

$$T(r) = T_\infty + \frac{S}{4\pi\lambda_{c,eff}}\left(\frac{1}{r} - \frac{1}{r_\infty}\right), \quad \lambda_{c,eff} = 0.173\,\mathrm{W/mK}$$

Poor thermal contact between the source and the packing

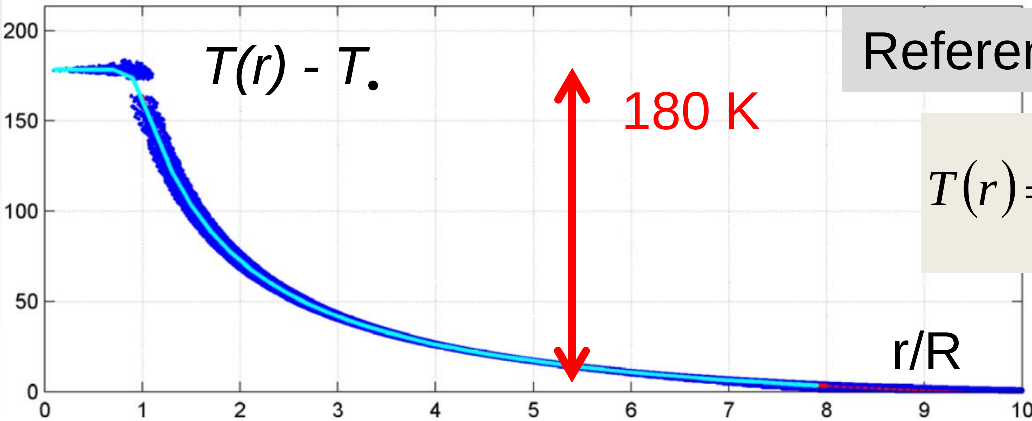
→ Sharp drop

Behavior of an equivalent continuous material



The fit applies far enough from the source and the oven walls (*4R • r • 8R*)

Phenomenology: Radial temperature distribution



Reference case: conduction in plain solid

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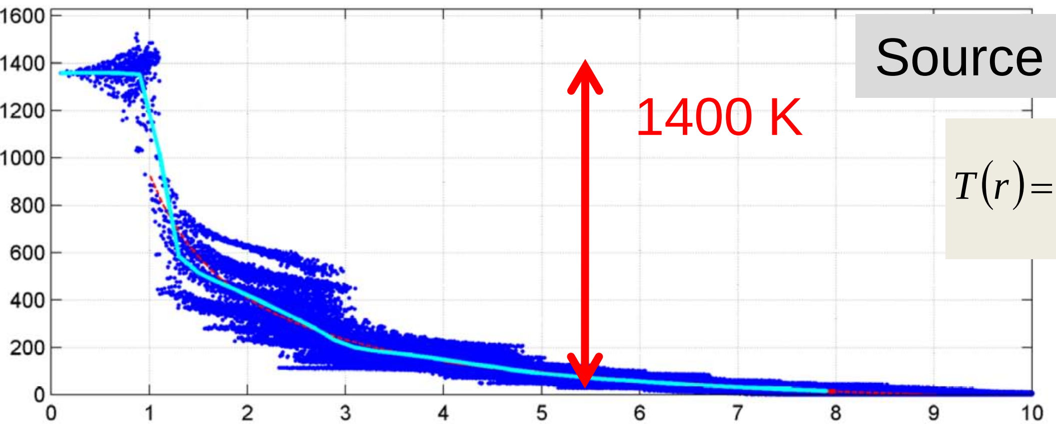
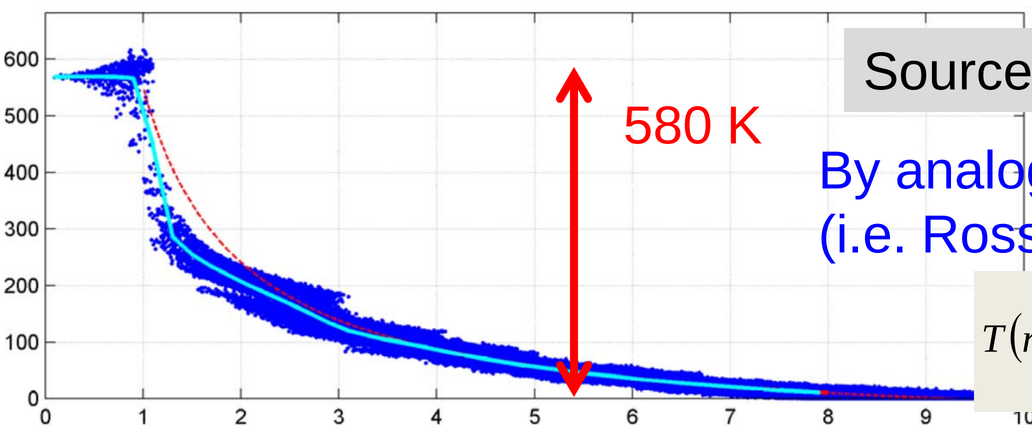


Source in a grain packing (conduction only)

$$T(r) = T_\infty + \frac{S}{4\pi\lambda_{c,eff}}\left(\frac{1}{r} - \frac{1}{r_\infty}\right), \quad \lambda_{c,eff} = 0.173\,\mathrm{W/mK}$$



Source in a grain packing (cond.+radiation)

By analogy, embedded (apparent) conductivity (i.e. Rosseland approximation)

Improved radiative transfer from the source into the packing

$$T(r) = T_\infty + \frac{S}{4\pi\lambda_{eff}}\left(\frac{1}{r} - \frac{1}{r_\infty}\right), \quad \lambda_{eff} = 0.295\,\mathrm{W/mK}$$

Semi-local analysis

<T> = mean temperature in concentric spherical shells of thickness R/5

$$S = -4\pi r^2 \lambda_{eff} \frac{d\langle T\rangle}{dr} \overset{?}{=} -4\pi r^2 \left(\lambda_{eff,c} + \lambda_{eff,r}\left(\langle T\rangle\right)\right)\frac{d\langle T\rangle}{dr}$$

R = 1 mm, S = 20.5 W

R = 1 mm, S = 9.86 W

R = 3 mm, S = 20.5 W

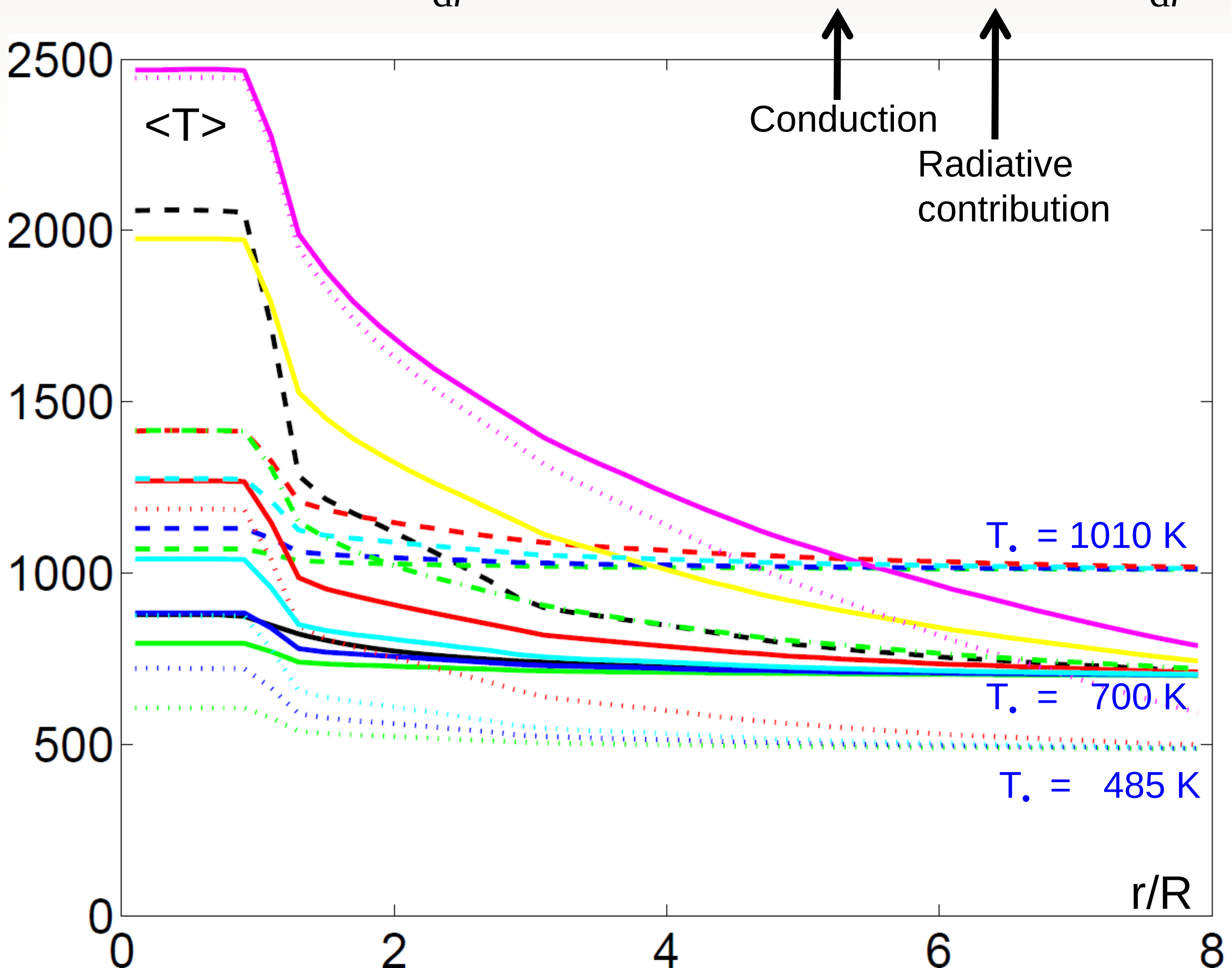
R = 1 mm, S = 2.28 W

R = 1/3 mm, S = 0.25 W

R = 1 mm, S = 0.53 W

R = 1 mm, S = 0.25 W

R = no radiation



Semi-local analysis

local effective conductivity in concentric spherical shells of thickness R/5

$$S = -4\pi r^2 \lambda_{eff} \frac{\mathrm{d}\langle T\rangle}{\mathrm{d}r}$$

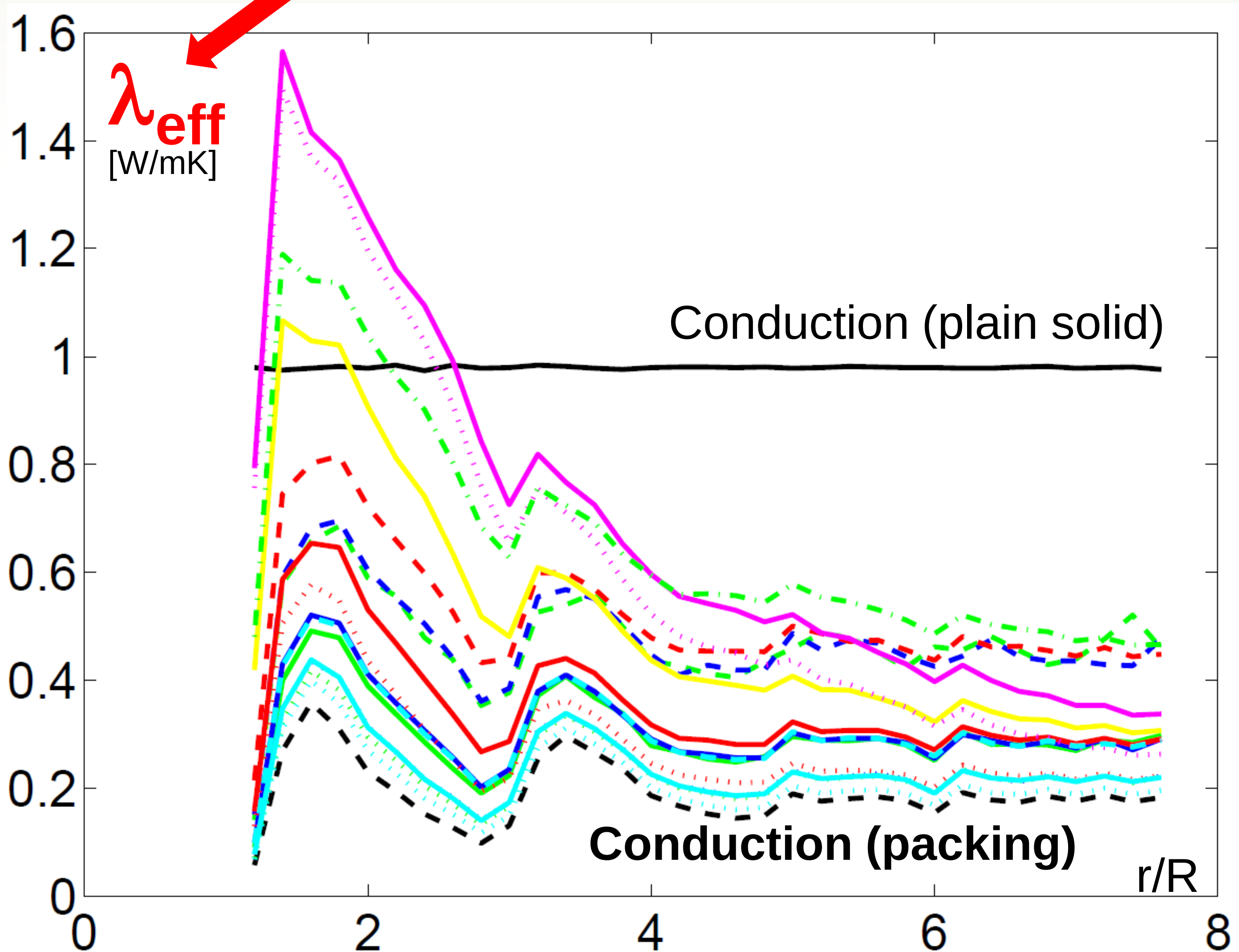
R = 1 mm, S = 20.5 W

R = 1 mm, S = 9.86 W

R = 3 mm, S = 20.5 W
R = 1 mm, S = 2.28 W

R = 1/3 mm, S = 0.25 W
R = 1 mm, S = 0.53 W
R = 1 mm, S = 0.25 W

T. = 480, 700 or 1010 K



Semi-local analysis

radiative contribution to the effective conductivity

$$S = -4\pi r^2 \lambda_{eff} \frac{d\langle T \rangle}{dr} = -4\pi r^2 \left(\lambda_{eff,c} + \lambda_{eff,r}(\langle T \rangle)\right) \frac{d\langle T \rangle}{dr}$$

R = 1 mm, S = 20.5 W

R = 1 mm, S = 9.86 W

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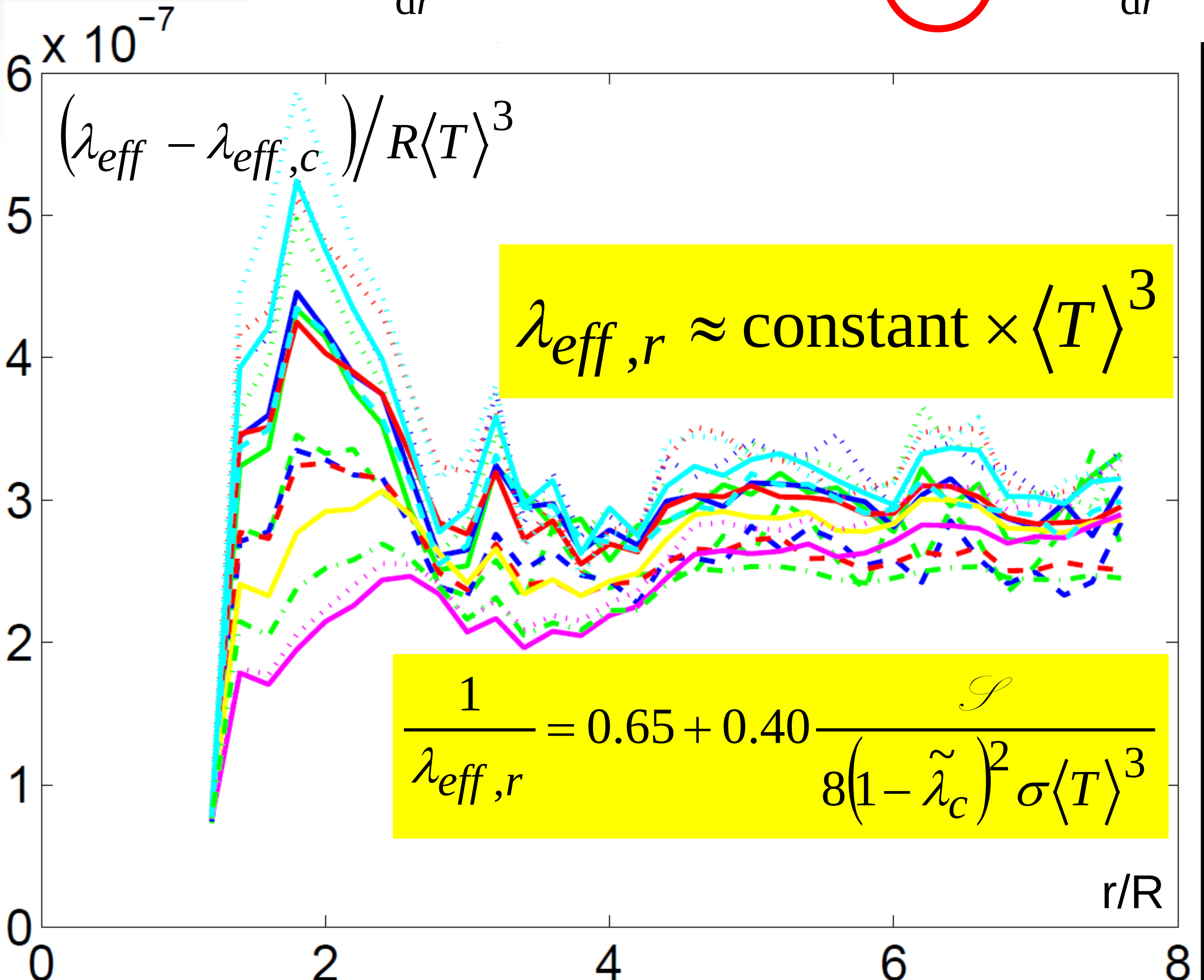
$T_{\bullet}$ = 480, 700 or 1010 K



$$\left(\lambda_{eff} - \lambda_{eff,c}\right) / R\langle T \rangle^3$$

$$\lambda_{eff,r} \approx \text{constant} \times \langle T \rangle^3$$

$$\frac{1}{\lambda_{eff,r}} = 0.65 + 0.40 \frac{\mathscr{S}}{8\left(1-\tilde{\lambda}_c\right)^2 \sigma \langle T \rangle^3}$$

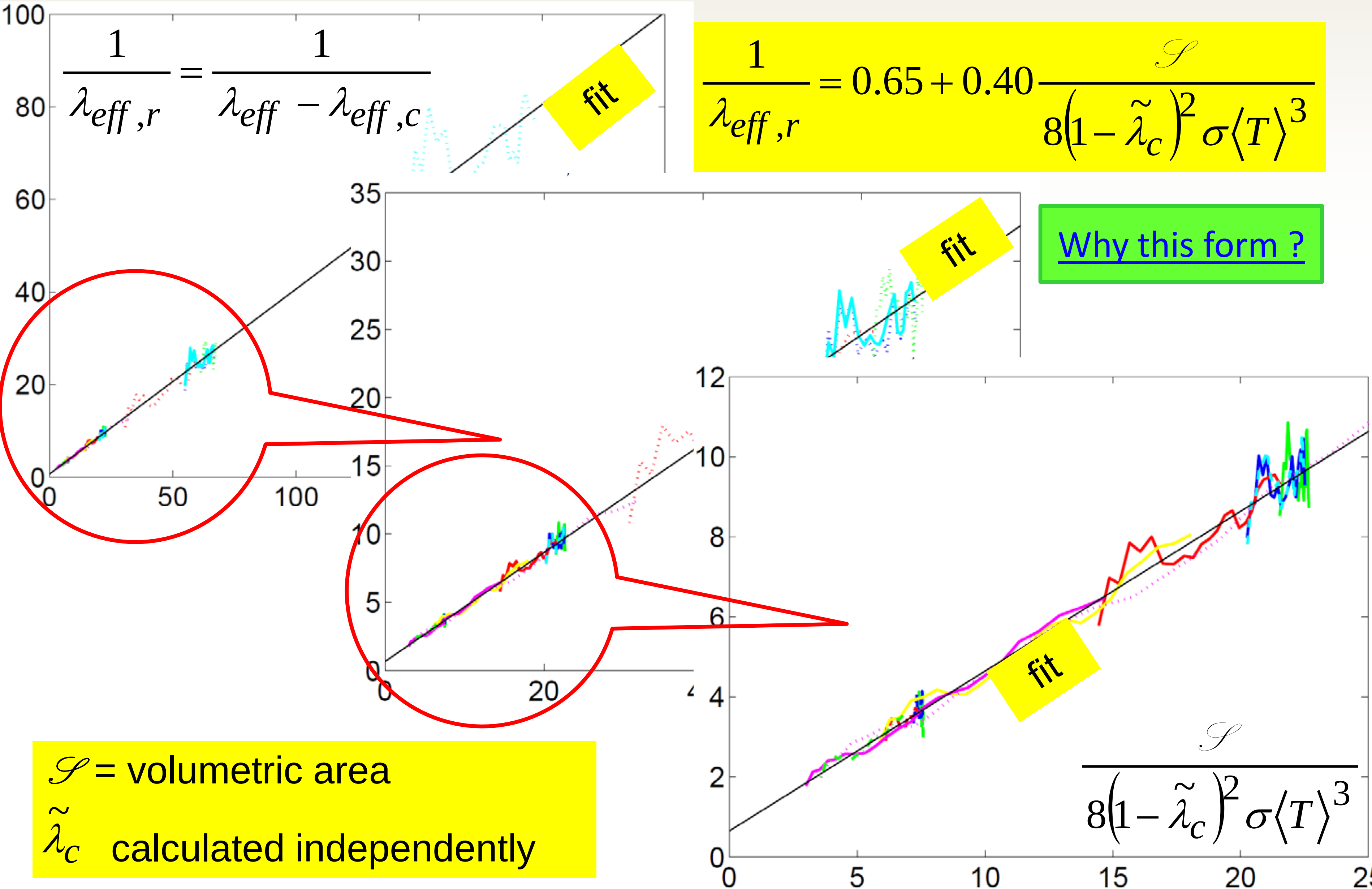
Semi-local analysis

radiative contribution to the effective conductivity

$$\frac{1}{\lambda_{eff,r}} = \frac{1}{\lambda_{eff} - \lambda_{eff,c}}$$

$$\frac{1}{\lambda_{eff,r}} = 0.65 + 0.40\,\frac{\mathscr{S}}{8\left(1-\tilde{\lambda}_c\right)^2 \sigma \langle T \rangle^3}$$

Why this form ?



$$\frac{\mathscr{S}}{8\left(1-\tilde{\lambda}_c\right)^2 \sigma \langle T \rangle^3}$$

$\mathscr{S}$ = volumetric area

$\tilde{\lambda}_c$ calculated independently

Generalization: reconstructed media

Thresholded Gaussian fields:

porosity
correlation length

(and more ...)

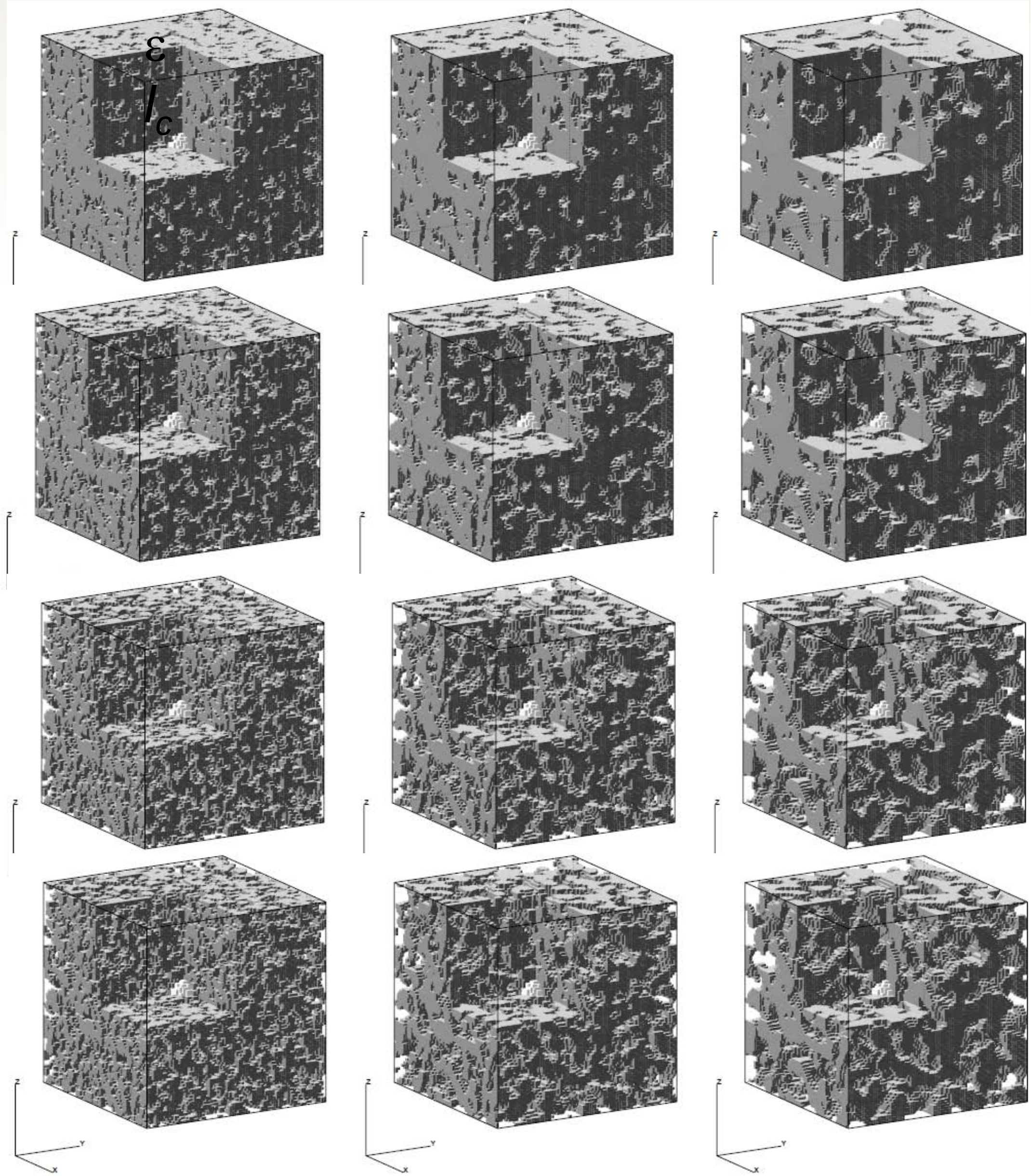
$l_c = 2$ $l_c = 3$ $l_c = 4$

$\varepsilon = 0.25$

$\varepsilon = 0.40$

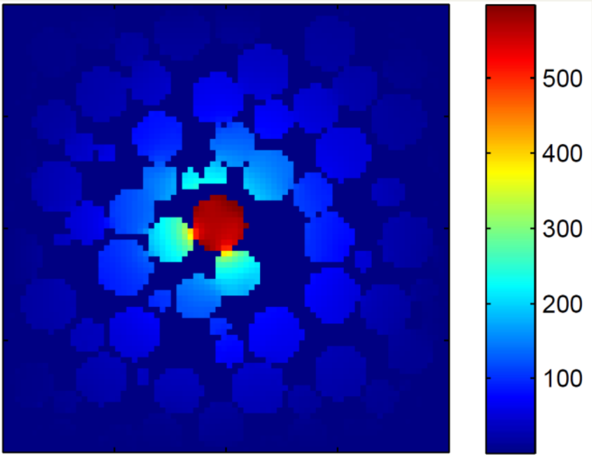
$\varepsilon = 0.60$

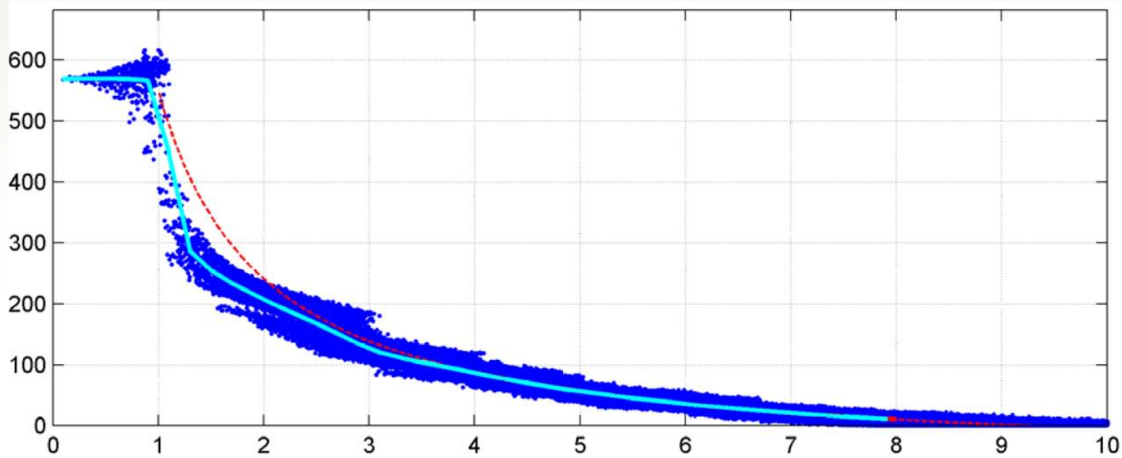
$\varepsilon = 0.80$



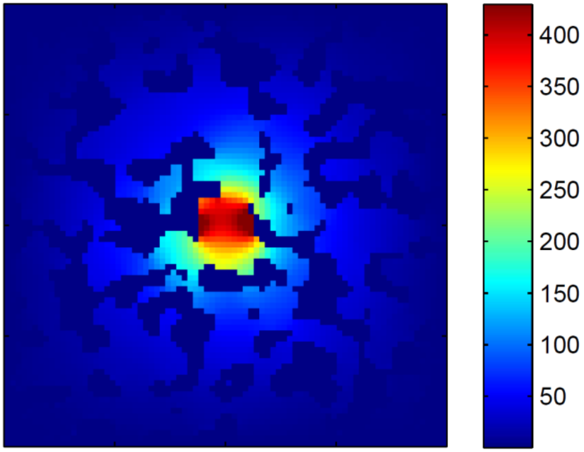
Comparison Packing / Reconstructed

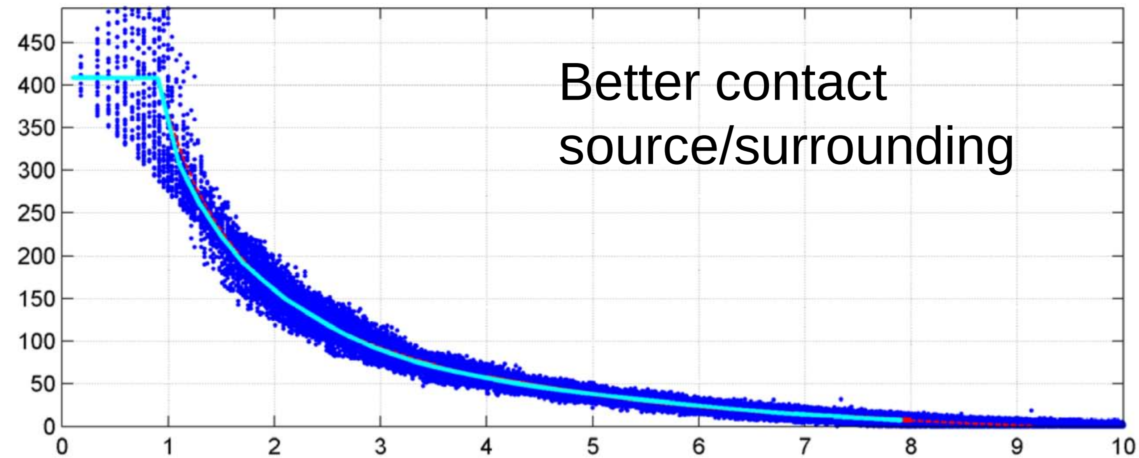
Packing: ε = 0.41 $\tilde{\lambda}_c = 0.181$ S=2.28 W T. = 700K





Reconstructed: ε = 0.40 $\tilde{\lambda}_c = 0.349$ S=2.28 W T. = 700K

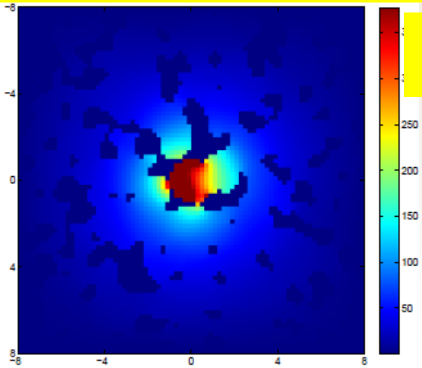




Comparison Without / With radiation

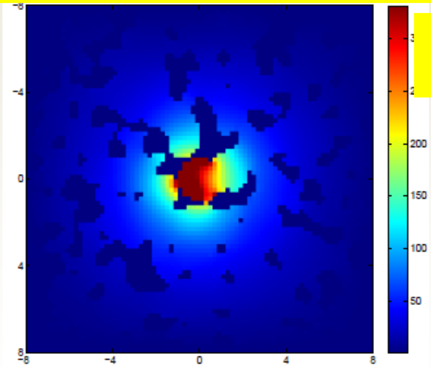
S = 2.28 W

T• = 700 K



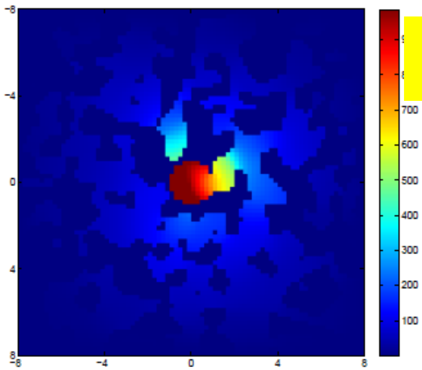
360 K

ε = 0.25



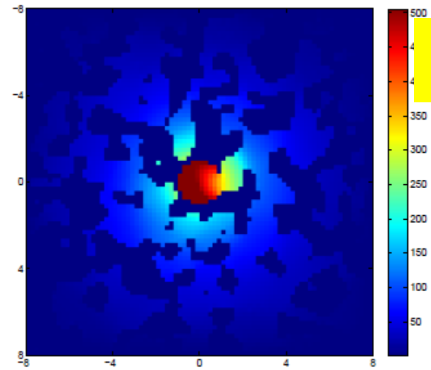
320 K

ε = 0.25



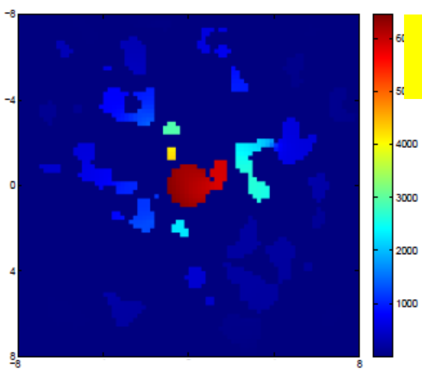
950 K

ε = 0.50



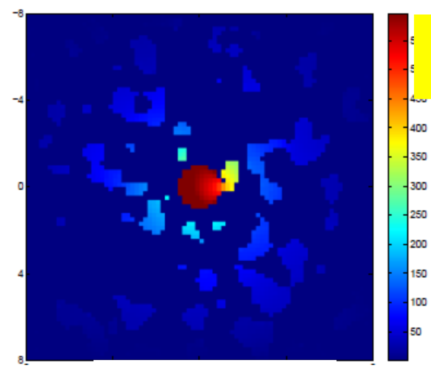
500 K

ε = 0.50



6200 K

ε = 0.80



570 K

ε = 0.80

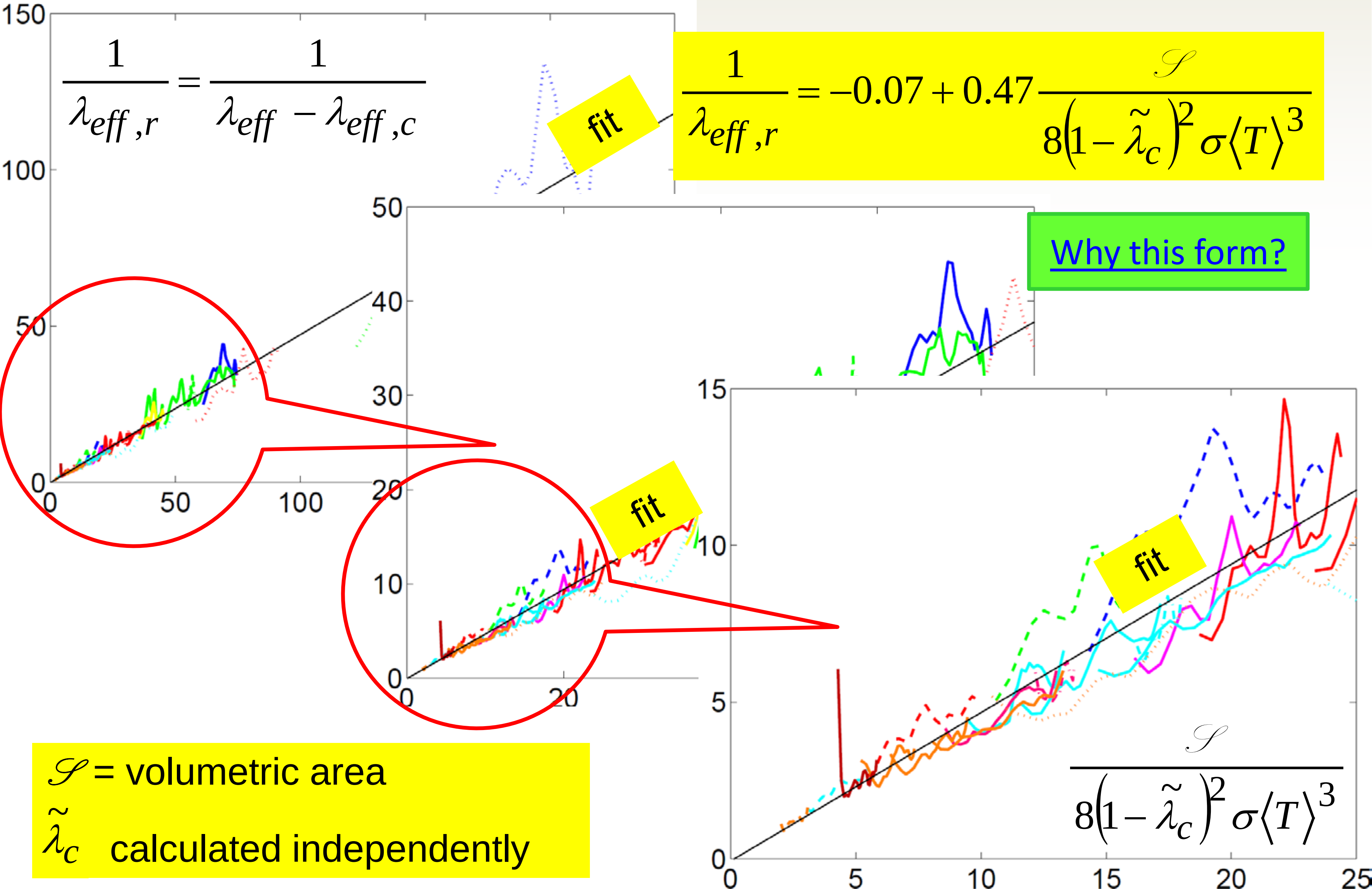
Semi-local analysis

radiative contribution to the effective conductivity

$$\frac{1}{\lambda_{eff,r}} = \frac{1}{\lambda_{eff} - \lambda_{eff,c}}$$

$$\frac{1}{\lambda_{eff,r}} = -0.07 + 0.47\frac{\mathscr{S}}{8\left(1-\tilde{\lambda}_c\right)^2 \sigma \langle T \rangle^3}$$

Why this form?



$$\frac{\mathscr{S}}{8\left(1-\tilde{\lambda}_c\right)^2 \sigma \langle T \rangle^3}$$

$\mathscr{S}$ = volumetric area

$\tilde{\lambda}_c$ calculated independently

Conclusion

- Radiation may contribute significantly to heat transfers in the target applications.
- The homogenizable part of their contribution is well described in a wide range of structures and temperatures by Rosseland approximation, with

$$\lambda_{eff} \approx \lambda_{eff,c} + \frac{8\left(1-\tilde{\lambda}_c\right)^2 \sigma \langle T \rangle^3}{\omega \mathscr{S}}$$

- This model involves only intrinsic dimensionless geometrical parameters:

  conductivity coefficient $\tilde{\lambda}_c$, volumetric area $\mathscr{S}$,

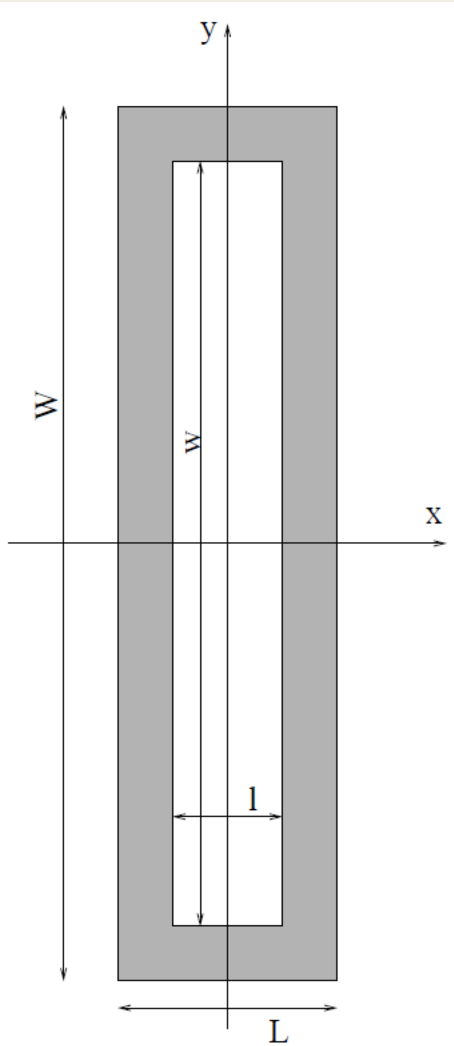
  and a fairly constant shape factor ω

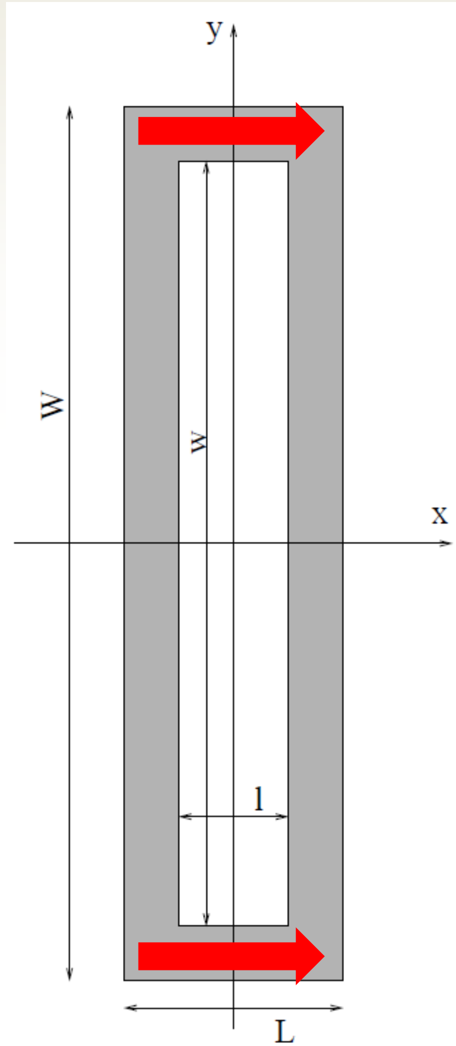
  $\omega = 0.40$ for unconsolidated grain packings

  $\omega = 0.47$ for consolidated reconstructed media ($\varepsilon = 0.20 \sim 0.80$)

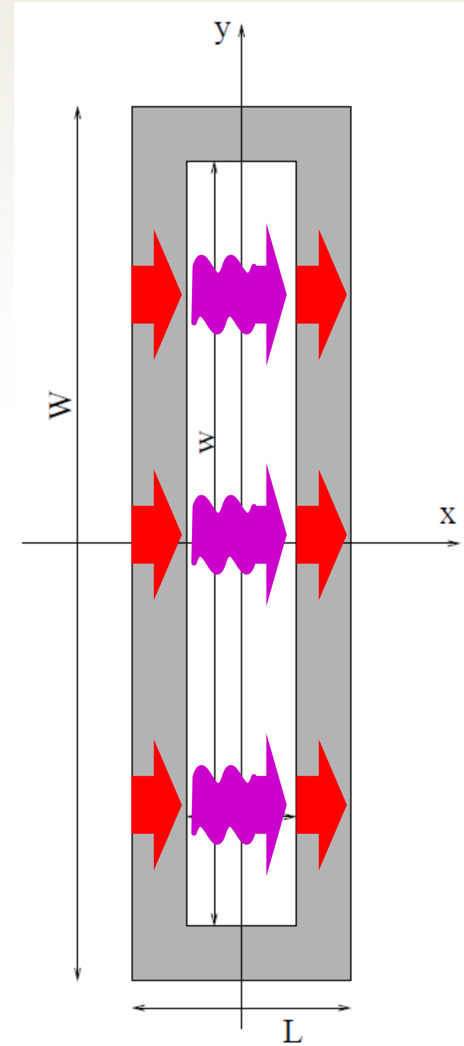
- Further work is desirable
  - to catalogue ω for other structures (e.g., foams)
  - to theoretically justify/improve the form of the heuristic formula
  - to address semi-transparent solid materials

Interpretation model: periodic vacuolar medium





$$\lambda_{eff,c} = \frac{W - w}{W} \lambda_s$$



$$\frac{1}{\lambda_{eff,r}} = \frac{W}{w} \left[\frac{L - l}{L \lambda_s} + \frac{1}{4\sigma \langle T \rangle^3 L} \right]^{-1}$$

Interpretation model: intrinsic formulation

$$\frac{L}{\lambda_{eff,r}} = \frac{W}{w}\left[\frac{L-l}{\lambda_s} + \frac{1}{4\sigma\langle T\rangle^3}\right]$$

Conduction in the solid slab

Radiation in the void layer

In terms of the intrinsic dimensionless geometrical parameters

$$\begin{cases} \varepsilon \\ \mathscr{S}_x = \dfrac{2w}{WL} \\ \tilde{\lambda}_c = \lambda_{eff,c}/\lambda_s \end{cases}$$

$$\frac{1}{\lambda_{eff,r}} = \frac{1-\tilde{\lambda}_c-\varepsilon}{\left(1-\tilde{\lambda}_c\right)^2\lambda_s} + \frac{\mathscr{S}_x}{8\left(1-\tilde{\lambda}_c\right)^2\sigma\langle T\rangle^3}$$

Not yet fully general: $\mathscr{S}_x$ is an ad-hoc volumetric area, for a transfer along x.

➔Introduction of a shape factor ω , multiplied by the whole volumetric area,

$$\frac{1}{\lambda_{eff,r}} = \frac{1-\tilde{\lambda}_c-\varepsilon}{\left(1-\tilde{\lambda}_c\right)^2\lambda_s} + \frac{\omega\,\mathscr{S}}{8\left(1-\tilde{\lambda}_c\right)^2\sigma\langle T\rangle^3}$$

Packing

Reconstructed