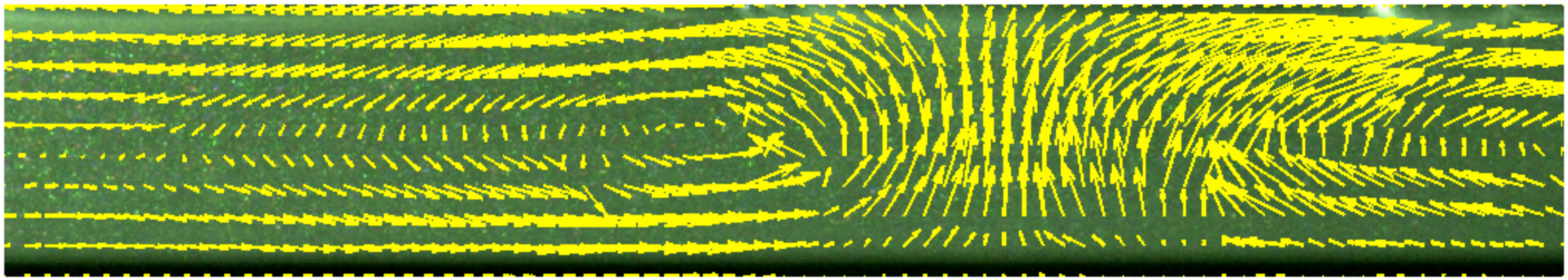
Rayleigh-Bénard convection in viscoplastic and/or shear thinning fluids :scaling properties, cross over from supercritical to subcritical behaviour

Cathy Castelain, Zineddine Kebiche, Teo Burghelea



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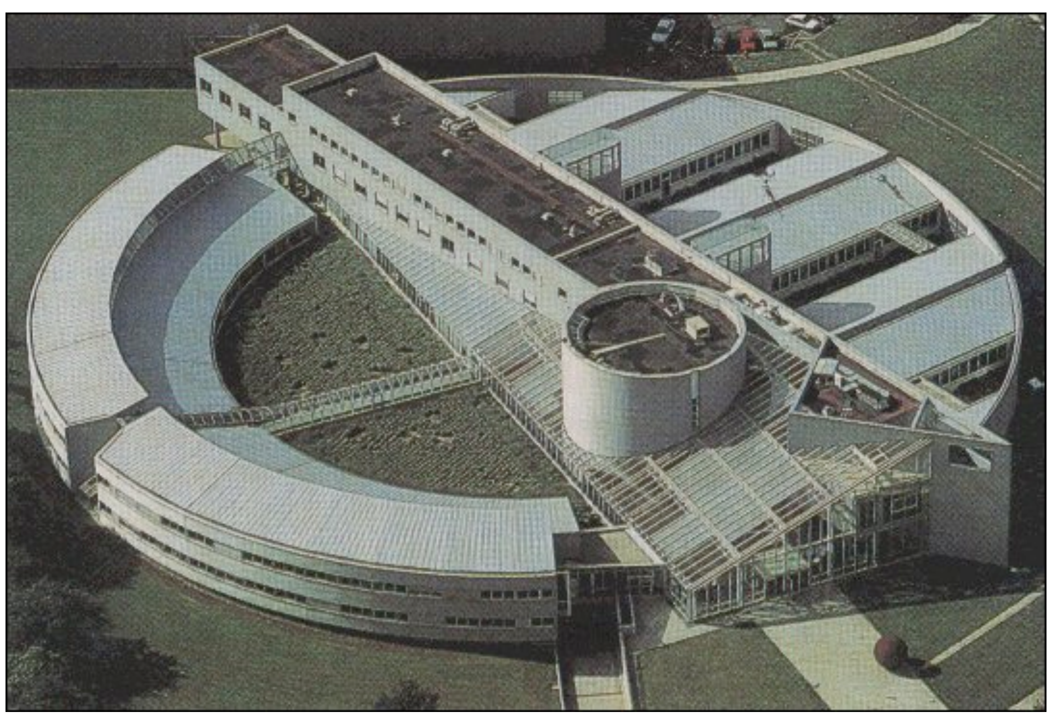

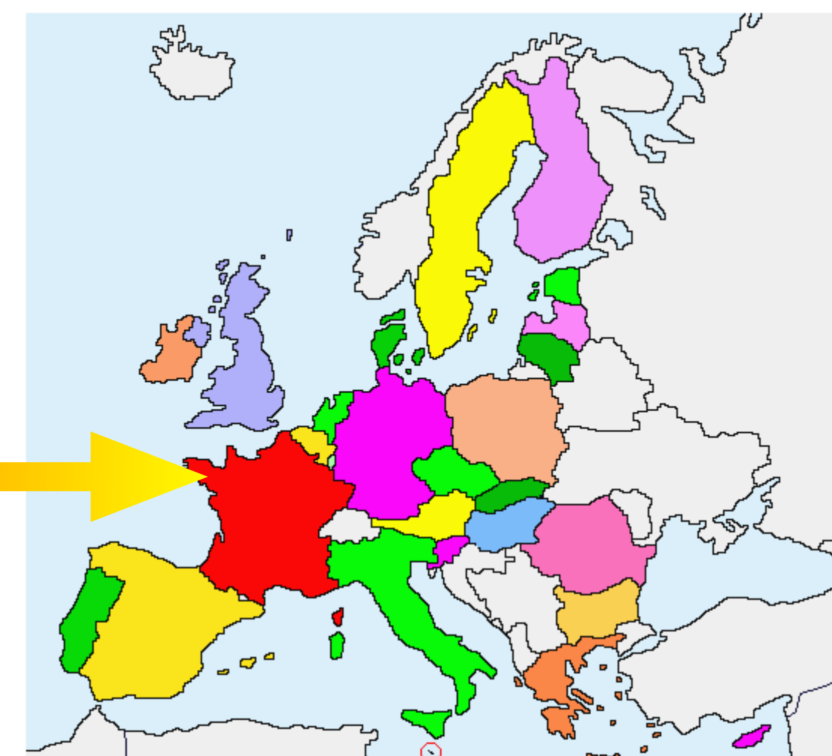

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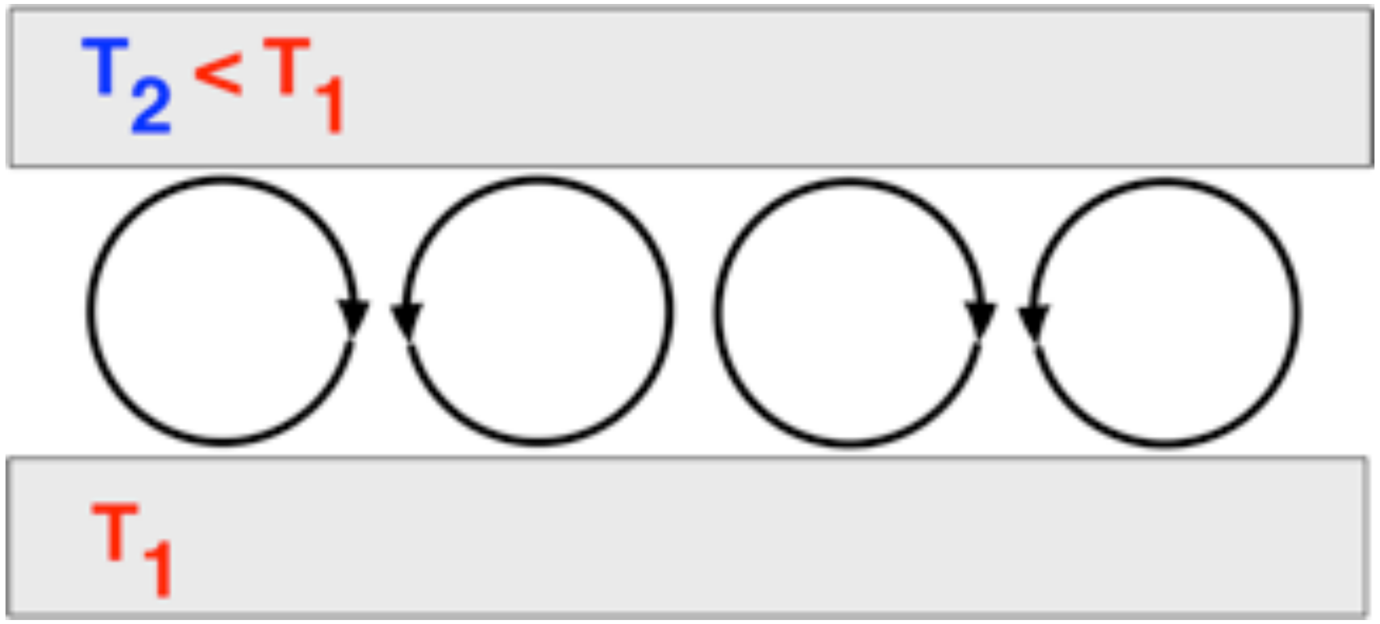
1. Introduction

The Rayleigh-Bénard convection is a paradigm of pattern forming systems



It can be triggered in a fluid heated from below when the **buoyancy stresses** overcome the **viscous stresses**

$$Ra = \frac{\alpha \Delta T g d^3}{k \cdot \nu} > Ra_c$$

Whereas there exists a large number of studies of the R-B convection in a Newtonian fluid, much less is known in the case of Non-Newtonian fluids

The challenge

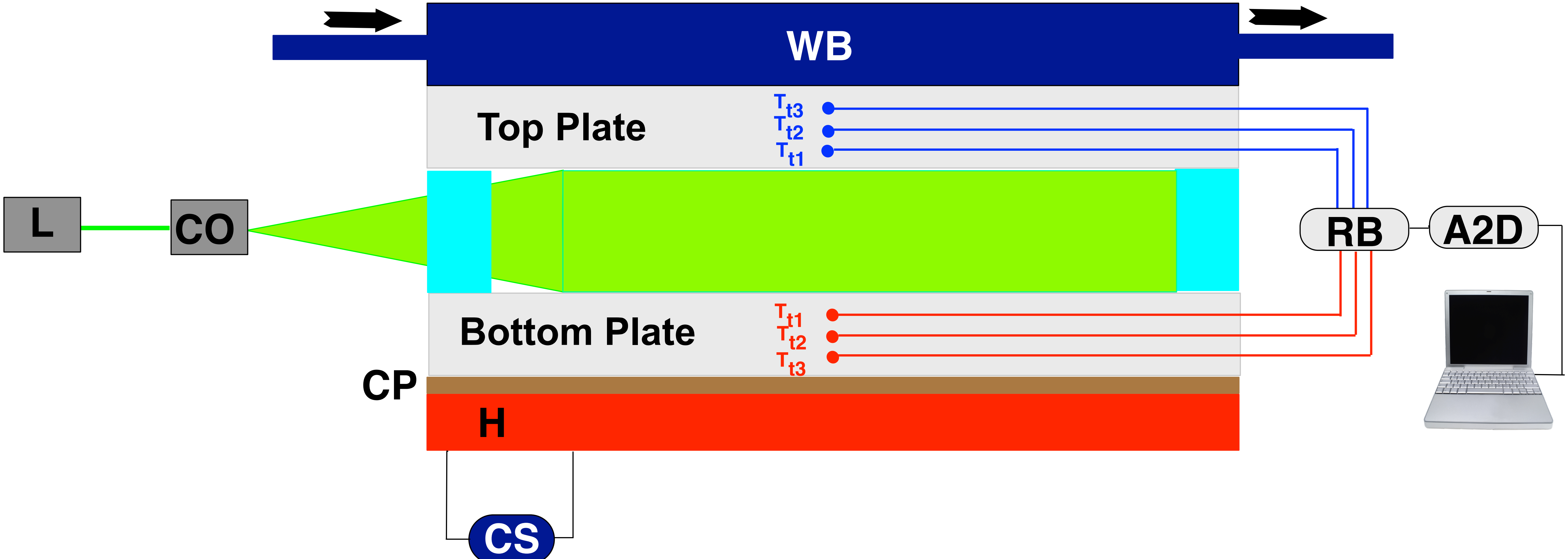
Account for the non linear stress- rate of strain relationship and time dependence (thixotropy) and their coupling to the heat transfer problem.

Two particular classes: - viscoplastic (yield) fluids

Fluids that do not flow unless a minimum stress is applied onto them

- shear thinnng fluids

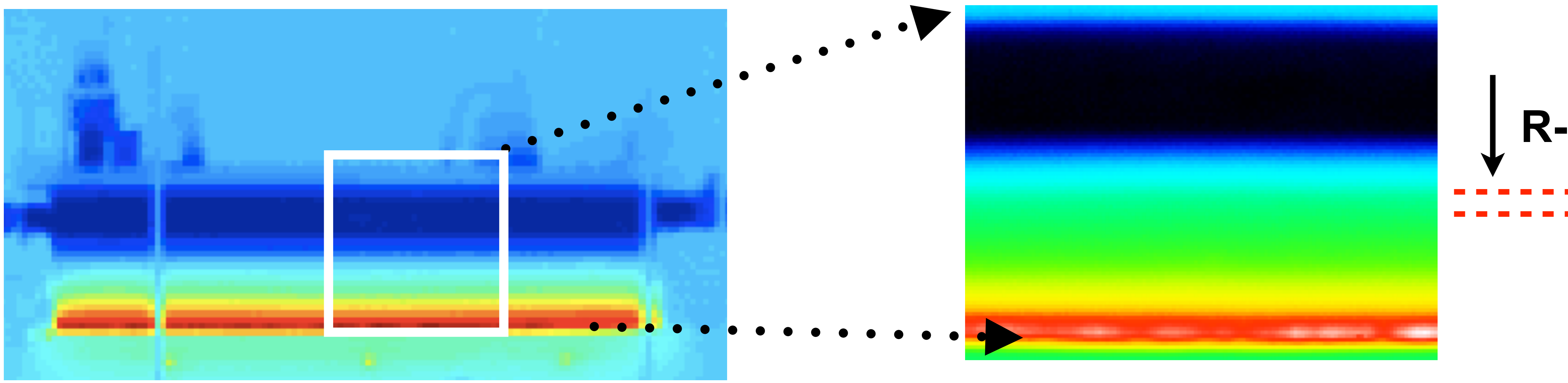
2. Experimental setup and techniques

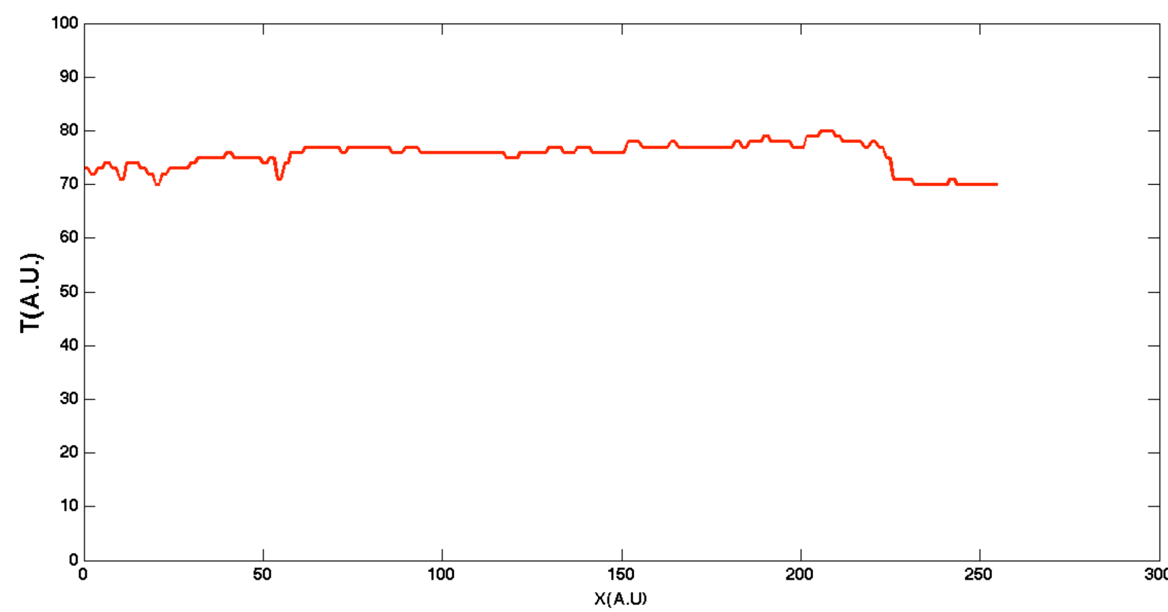


Methods:

(1) Digital Particle Image Velocimetry (DPIV) - Local scale assessment of the stability.
(2) Measurements of the temperature gradient - Integral scale assessment of the stability.
(3) Infrared imaging - For calibration purposes only.

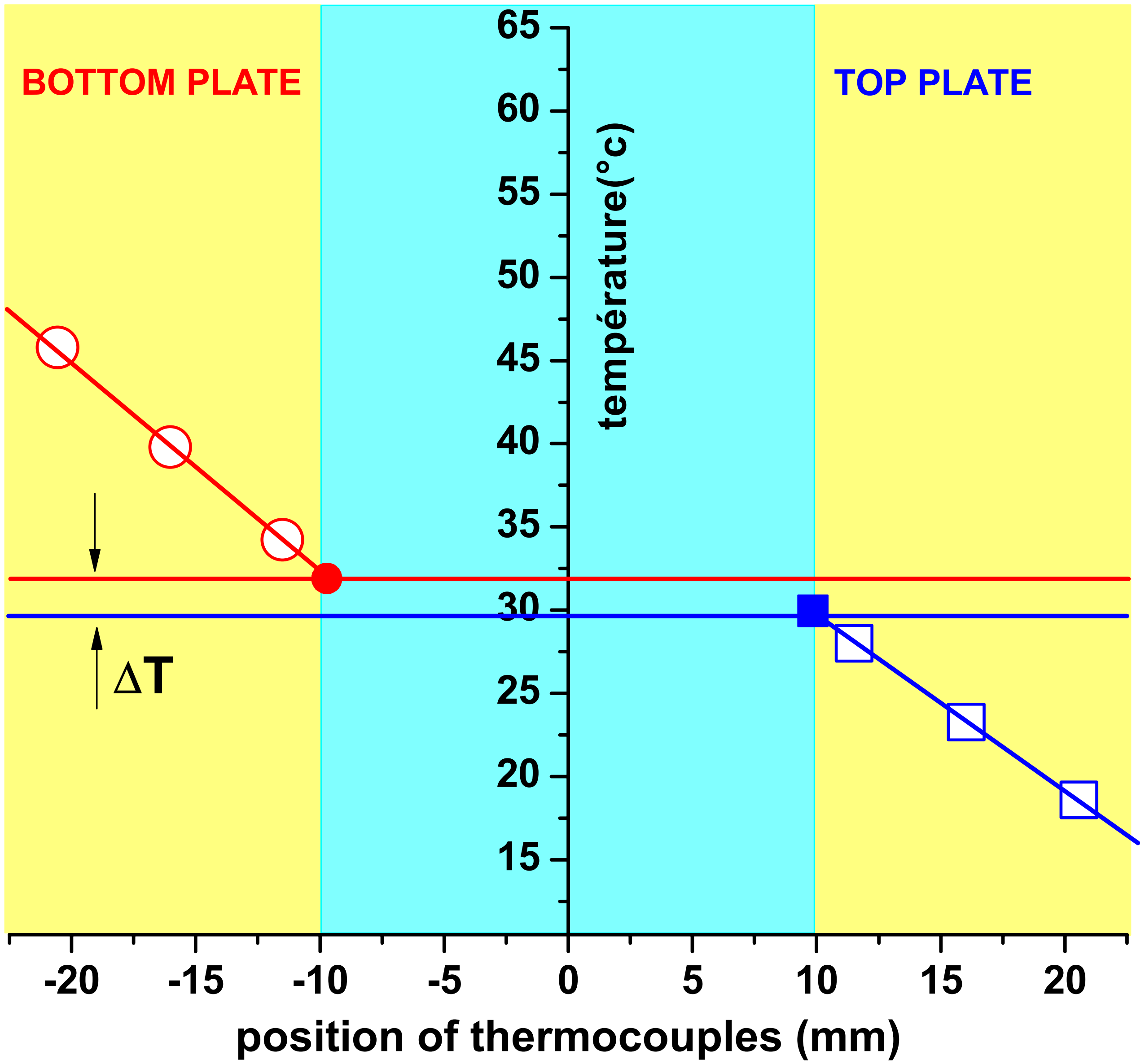
2.1 Uniformity of the temperature distribution along the R-B cell- infrared imaging





The temperature distribution along the cell is quite homogeneous (the mean gradient is smaller than 5 %)

2.2 Integral measurements of the temperature gradient



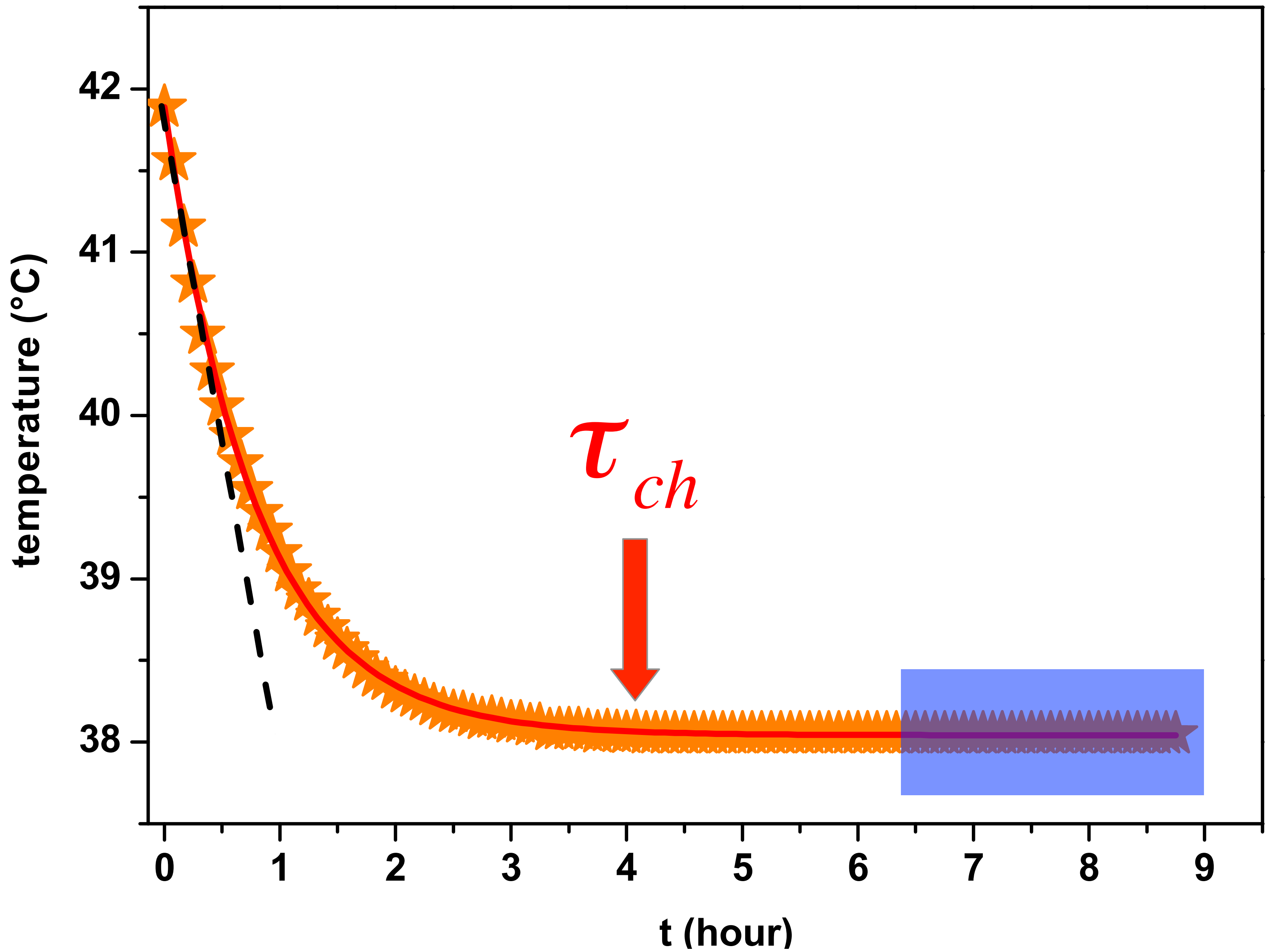
The procedure:

(1) Three thermocouples are embedded in each plate at precise vertical positions.

(2) Their readings are linearly extrapolated at the contact points between the plates and the fluid.

The temperature measurements are non-invasive

2.3 Carefully avoid the transients

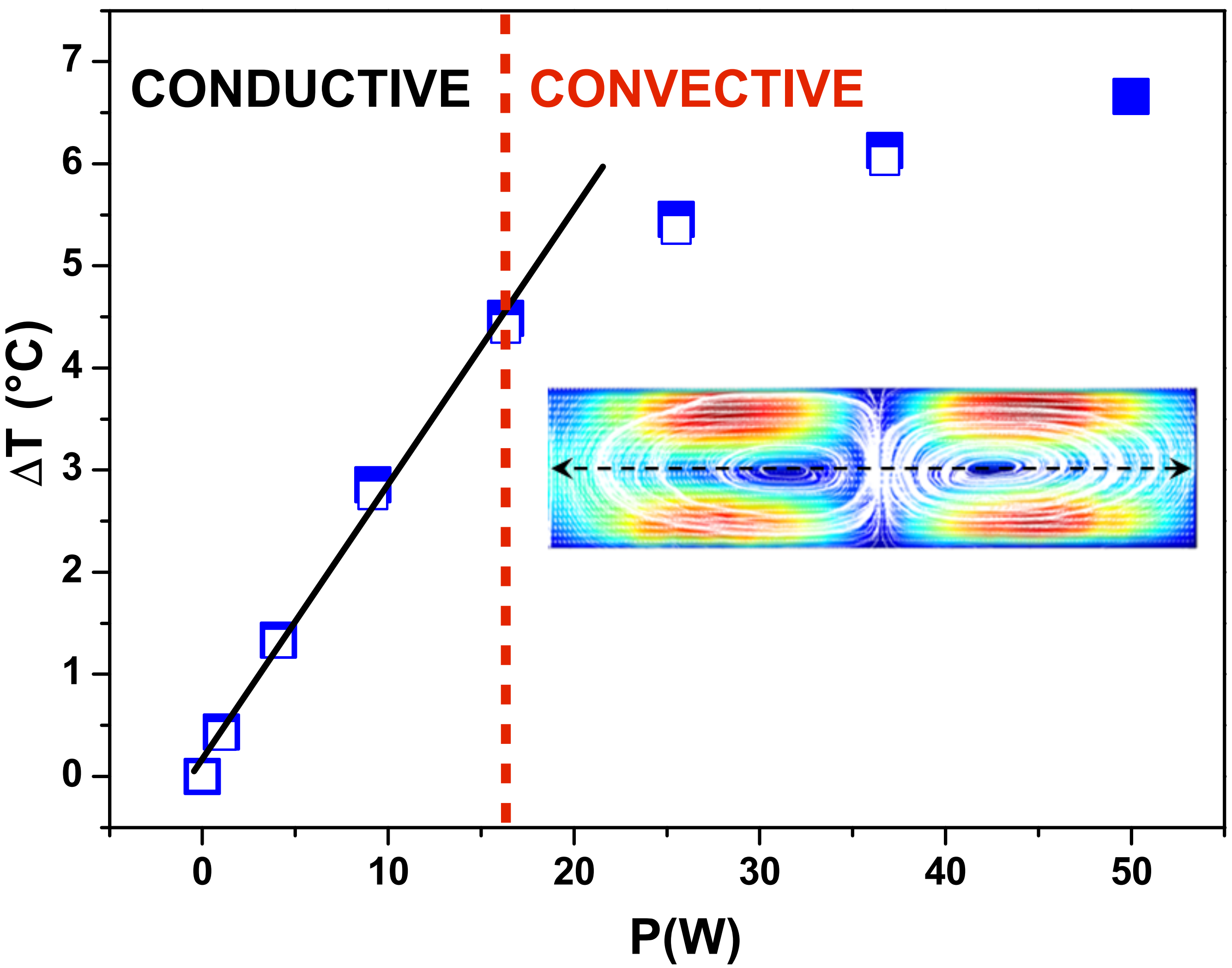


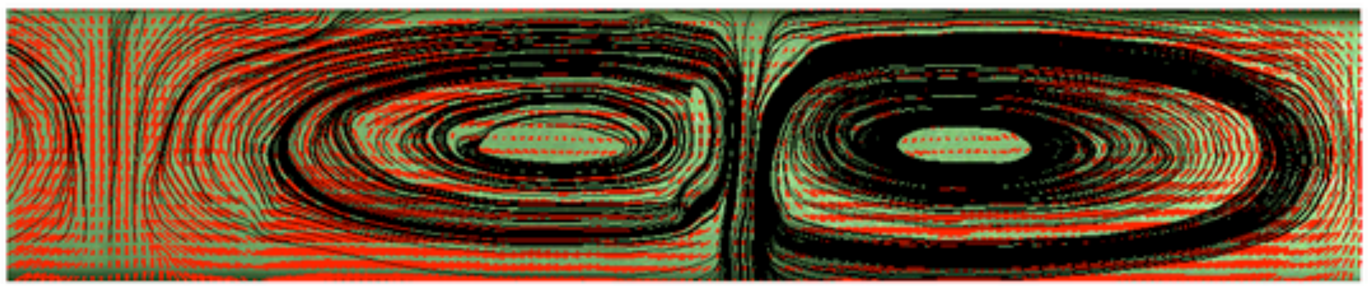
A thermal equilibrium is allowed to set in prior to measuring the temperature difference the between plates.

The temperature was measured ONLY in a steady state

2.4 Validation of the experimental setup/techniques with a Newtonian fluid

Glycerin: $\alpha = 5 \cdot 10^{-4} K^{-1},\ g = 9.8 m^2/s,\ k = 1.37 \cdot 10^{-7} m^2/s\ \ \nu = 872 \cdot 10^{-6} m^2 s^{-1}$



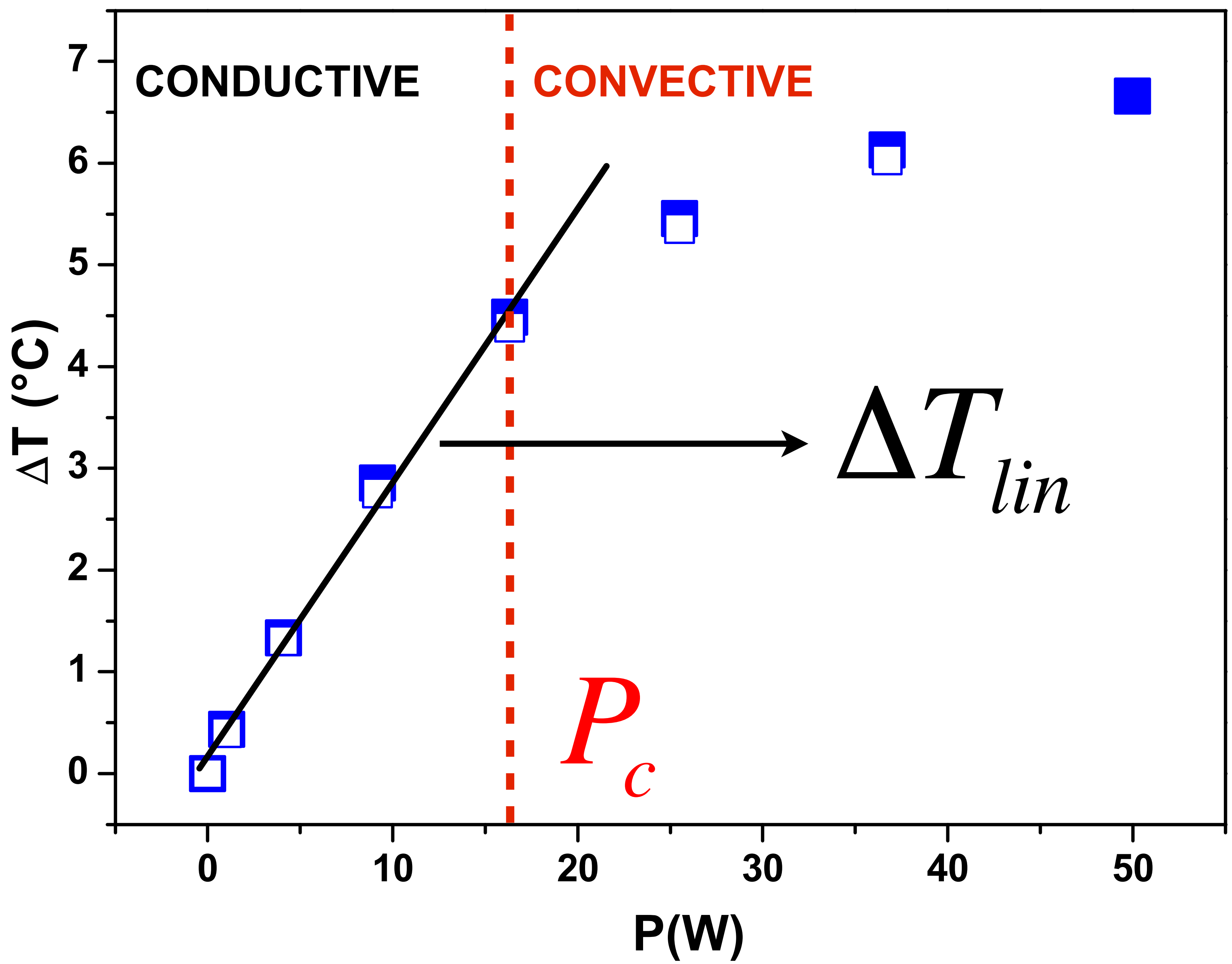


$$\Delta T_c = 4.8^o C, \quad Ra_c = 1774$$

A deeper insight into the validation results with the Newtonian fluid:

THE NATURE OF THE BIFURCATION TOWARDS CONVECTIVE STATES

(FOCUS FIRST on INTEGRAL MEASUREMENTS)

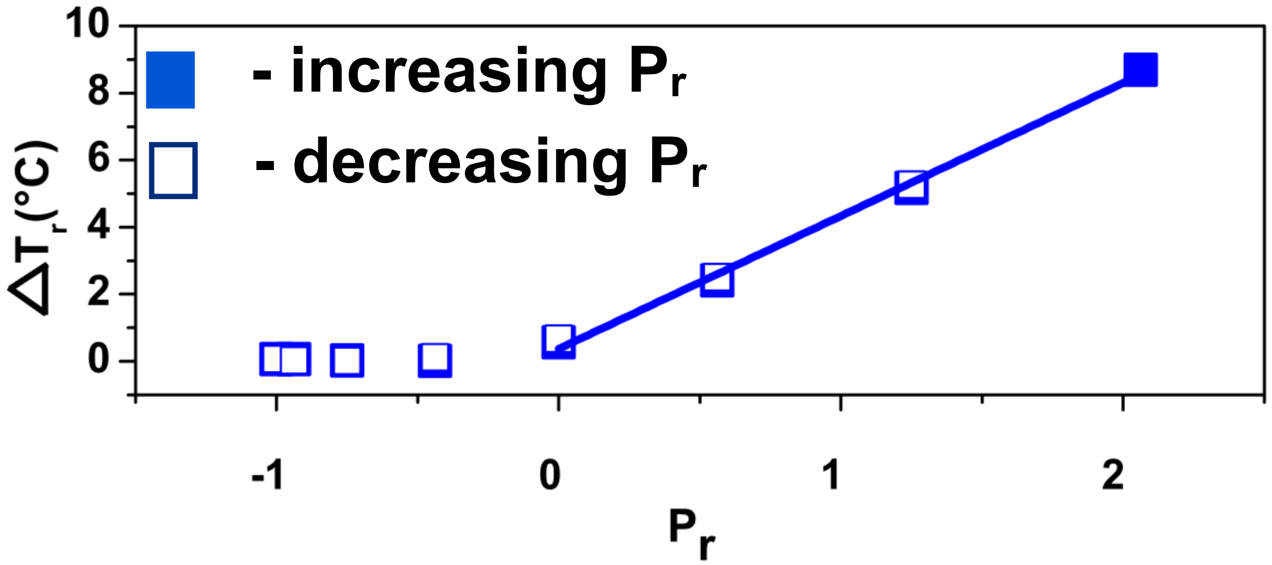


Reduced order parameter

$$\Delta T_r = \frac{\Delta T}{T_{lin}} - 1$$

Reduced control parameter

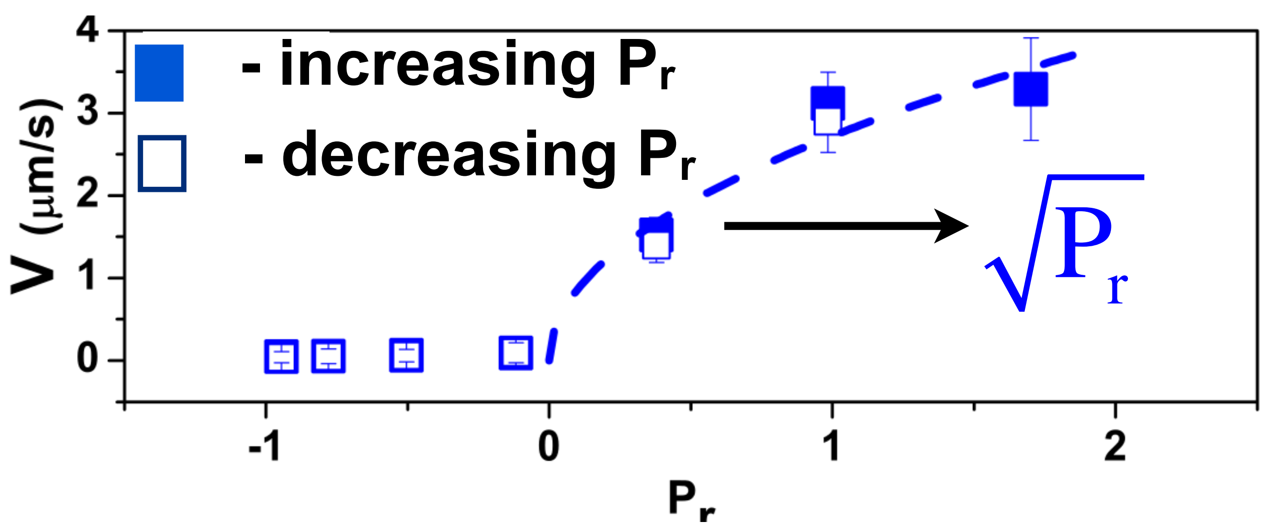
$$P_r = \frac{P}{P_c} - 1$$



$$\Delta T_r = \frac{\Delta T}{T_{lin}} - 1$$

(1) As expected, the reduced temperature scales linearly with the control parameter above the onset of the R-B bifurcation.
(2) The transition is reversible upon increasing/ decreasing control parameter (imperfect bifurcation).

FOCUS on LOCAL MEASUREMENTS of the CONVECTION AMPLITUDE



Landau Equation

$$\varepsilon = P_r = P / P_c - 1, \quad \xi = V$$

$$\varepsilon\xi - a\xi^3 + h = 0$$

As an additional validation of our setup and techniques, we have confirmed that the transition to R-B convection in a Newtonian fluid emerges as an imperfect (continuous) bifurcation described by the Landau theory

Theoretical predictions for the R-B convection in yield stress fluids

(1)

J. Fluid Mech. (2006), *vol.* 566, *pp.* 389–419. © 2006 Cambridge University Press 389
doi:10.1017/S002211200600200X Printed in the United Kingdom

Yield stress effects on Rayleigh–Bénard convection

By J. ZHANG[1], D. VOLA[2] AND I. A. FRIGAARD[1,3]†

Within the Bingham framework, the system is linearly stable

(2)

J. Non-Newtonian Fluid Mech. 158 (2009) 36–45
Contents lists available at ScienceDirect
Journal of Non-Newtonian Fluid Mechanics
journal homepage: www.elsevier.com/locate/jnnfm
ELSEVIER

Weakly nonlinear viscoplastic convection

Neil J. Balmforth[a,b], Alison C. Rust[c,*]

A finite amplitude perturbation may trigger the R-B instability in spite of a finite yield stress.

Experimental study for the R-B convection in yield stress fluids

PHYSICS OF FLUIDS **25**, 023101 (2013)

Rayleigh-Bénard convection for viscoplastic fluids

Mohamed Darbouli, Christel Métivier, Jean-Michel Piau, Albert Magnin, and Ahmed Abdelali
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(Received 7 May 2012; accepted 20 December 2012; published online 8 February 2013)

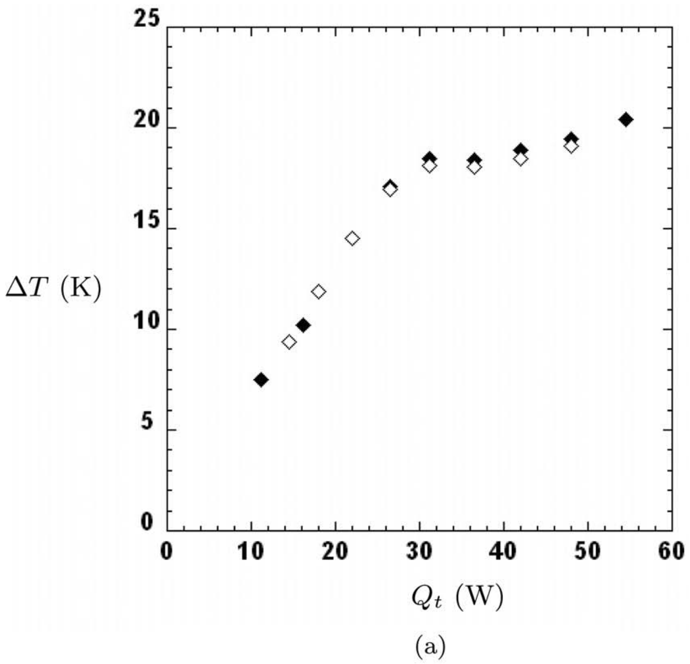
-Détermination expérimentale de l'apparition de l'instabilité
-Construction d'un nombre de Rayleigh
-Effet du glissement à la paroi

TABLE I. Identification of the gels coefficients and the values of yield stress obtained by both methods: Flow and oscillatory measurements at $T = 293$ K.

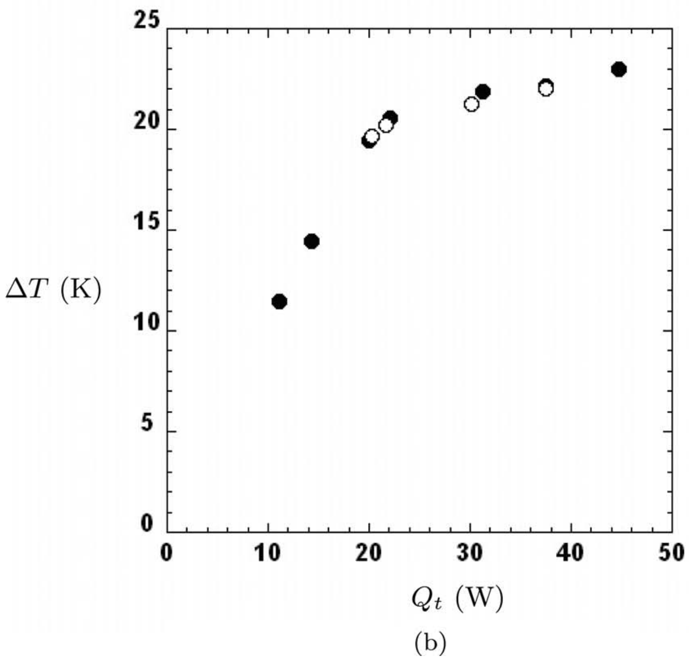
τ_y (Pa)	$K(Pa.s^n)$	n	G' (Pa)	G'' (Pa)	τ_c (Pa)
0.104	0.47	0.41	3.25	0.63	0.117
0.045	0.4	0.43	2.1	0.4	0.043
0.031	0.26	0.46	0.77	0.23	0.029
0.01	0.11	0.6			
0.009	0.093	0.62	0.5	0.15	0.0089
0.006	0.073	0.68	0.45	0.16	0.0067
0.0047	0.039	0.75			

$$Ra_g = \frac{\rho g \beta \, \Delta T \, d}{\tau_y} = Y^{-1}.$$

$Nu \geq 1$. The critical $1/Y$ values are determined: $1/Y_c^S \approx 40$ with slip conditions and $1/Y_c^{NS} \approx 80$ with adherence conditions highlighting the destabilizing effect of wall slip as discussed previously.



(a)



(b)

FIG. 5. Temperature difference ΔT as a function of the total heat input Q_t, for different values of d, τ_y, and slip condition (untreated copper alloy, glass, and PMMA surfaces). The black (resp. white) symbols represent the results obtained by increasing (resp. decreasing) Q_t. (a) $d = 0.01$ m, $\tau_y = 0.006$ Pa; (b): $d = 0.017$ m, $\tau_y = 0.01$ Pa.

The quests for today:

3. Experimental Results for a yield stress fluid (Carbopol 980)

- Observe and characterize experimentally the Rayleigh-Bénard convection in a yield stress fluid **(Carbopol 980)**.

- Relate the observations to the rheological properties of the gel (the yielding picture).

- Compare the results with the existing theoretical predictions.

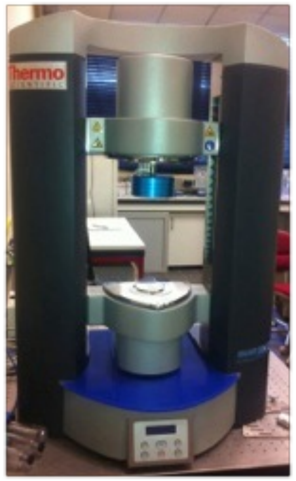
The yielding process

Thermorheology

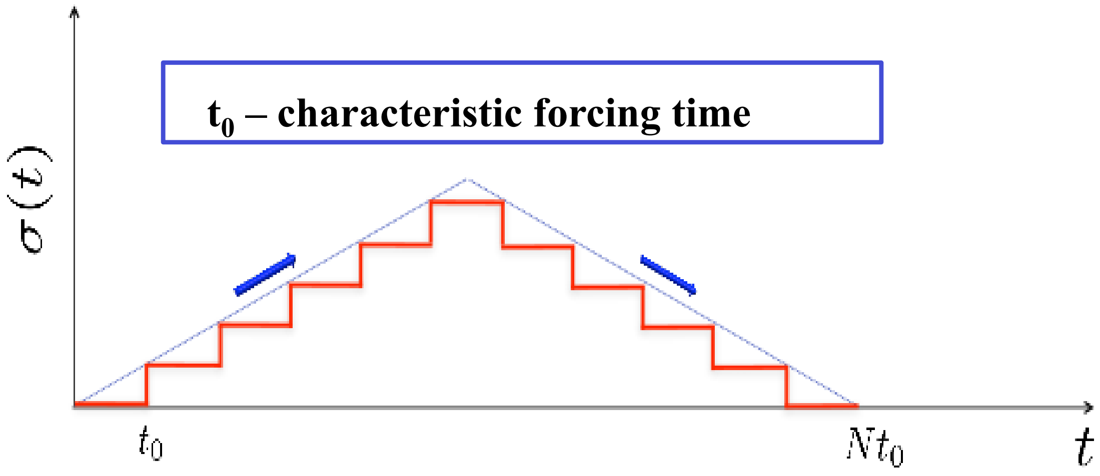
Concentration Dependence, Physico-Chemical Properties

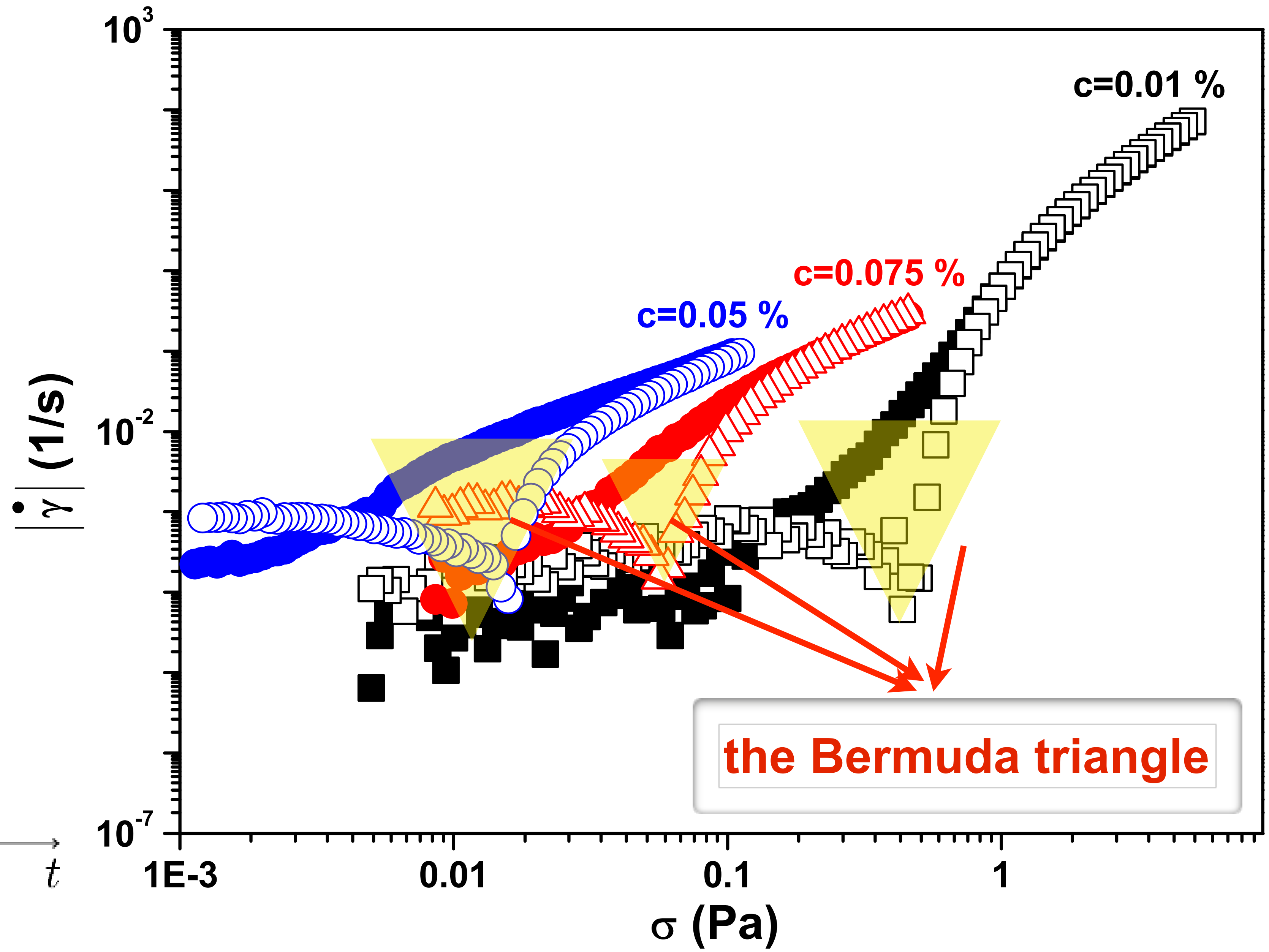
3.1. Measurements of the yield stress

Mars III rheometer
+
nano torque module



Controlled Stress Flow Ramps

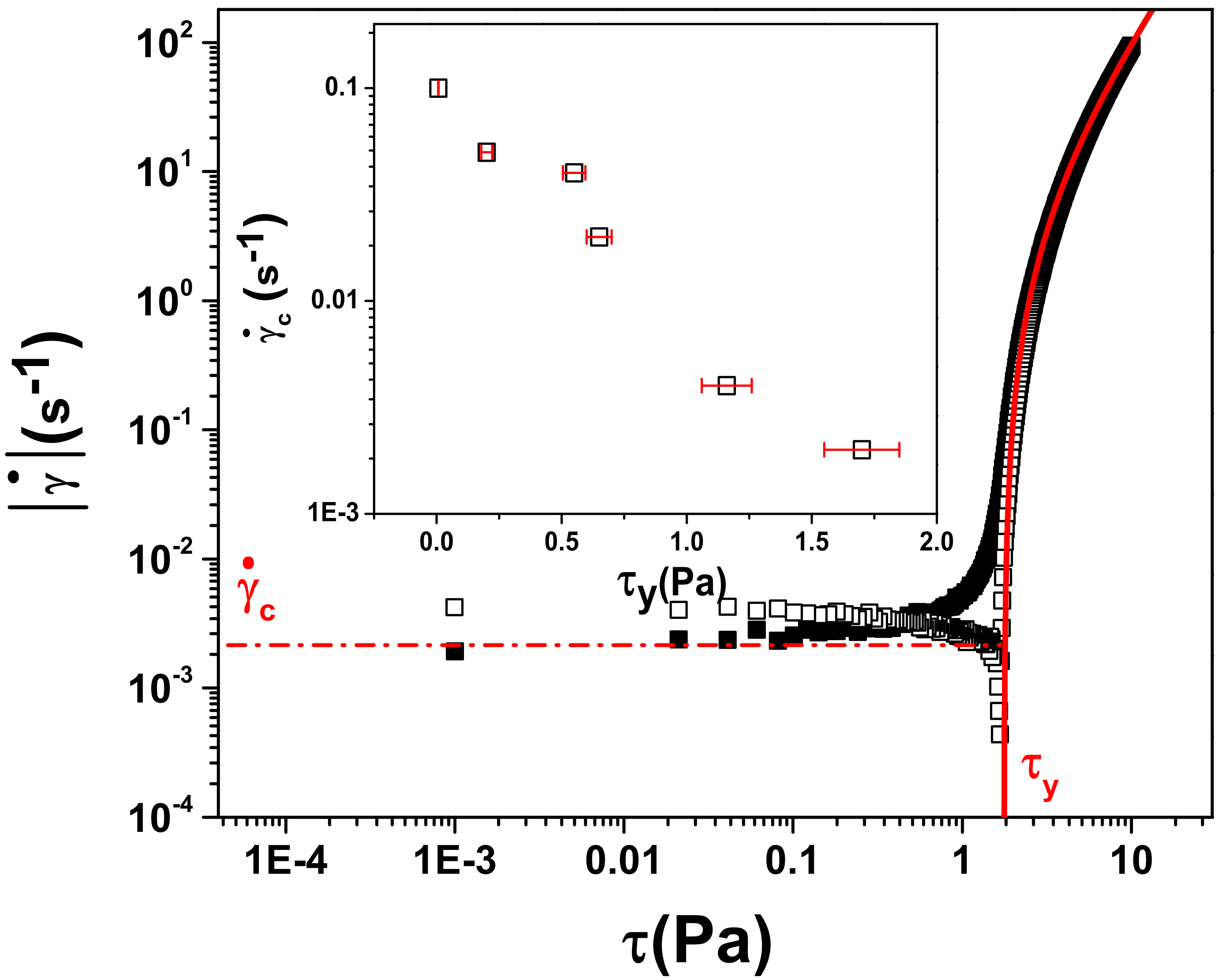




(1) The flow curves are irreversible upon increasing/decreasing the applied stresses.
(2) The R-B convection lives in the irreversible range of stresses (the Bermuda triangle)

PLEASE NOTE:

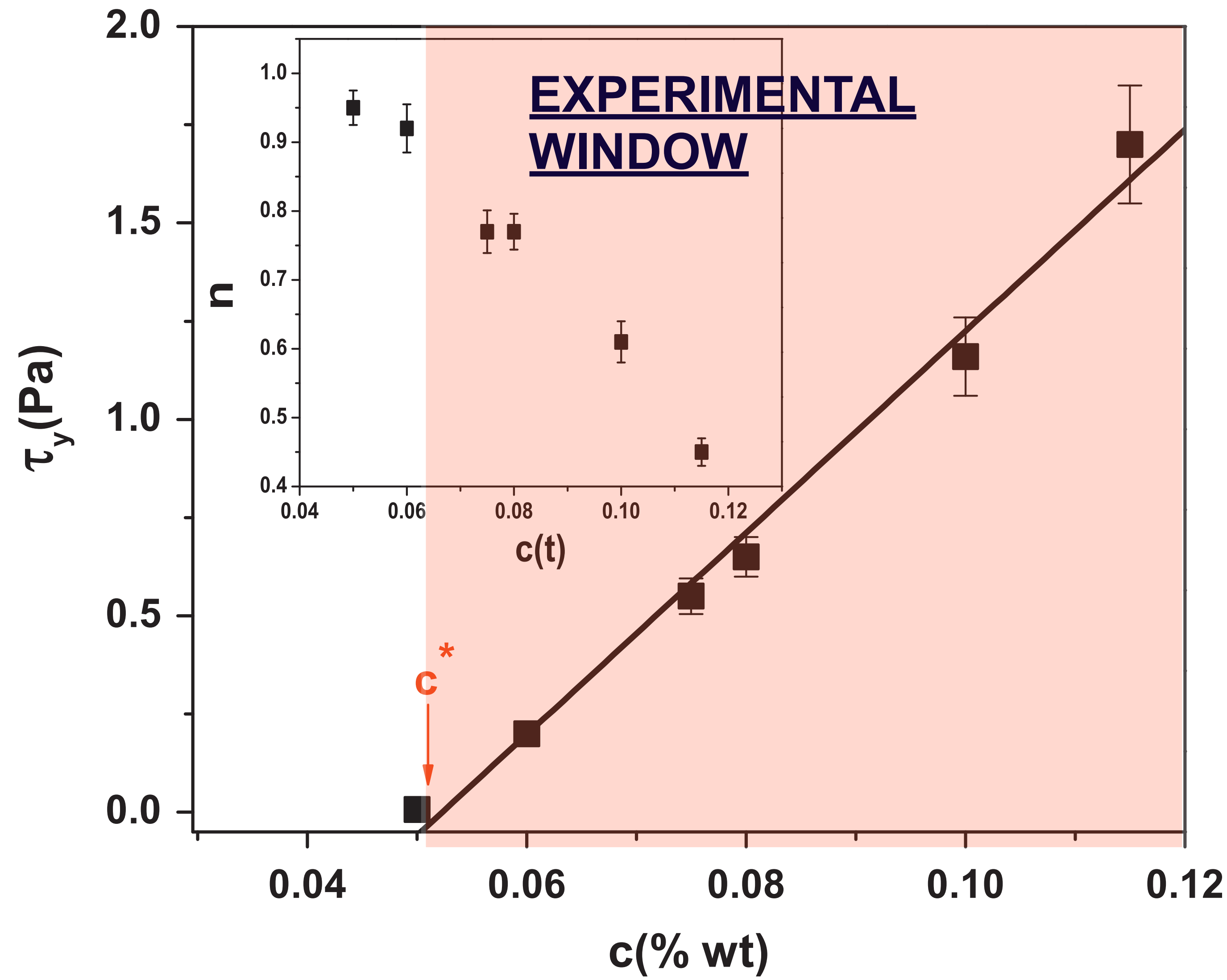
c (wt%)	β (10^{-4} K^{-1})	c_p (J kg^{-1} K^{-1})	κ (10^{-7} m^2 s^{-1})	α (W/mK)	ρ (kg m^{-3})	τ_y (Pa)	K (Pa s^n)	n	$\dot{\gamma}_c$ (s^{-1})
0.05	2	4231.63	1.5	0.61	961	$0.007 \pm 7 \times 10^{-4}$	0.046	0.95	0.1
0.06	2	4245.77	1.48	0.61	970	0.2 ± 0.022	0.054	0.92	0.05
0.075	2	4202.79	1.44	0.6	990	0.55 ± 0.045	0.079	0.77	0.04
0.08	2	4176.13	1.45	0.6	990	0.65 ± 0.05	0.118	0.77	0.02
0.1	2	4119.47	1.44	0.6	1010	1.16 ± 0.1	0.305	0.61	0.004
0.115	2	3998.44	1.48	0.61	1030	1.7 ± 0.15	0.727	0.45	0.002



PLEASE NOTE:

We have first made sure that we are in the right concentration regime

A clear yield stress behavior is involved in ALL our experiments



Increasing Shear Thinning Behaviour

3.1 Observation of the R-B convection in a Carbopol gel

Note:

- The same experimental procedure as in the Newtonian case is employed.

$$\Delta T < \Delta T_c$$

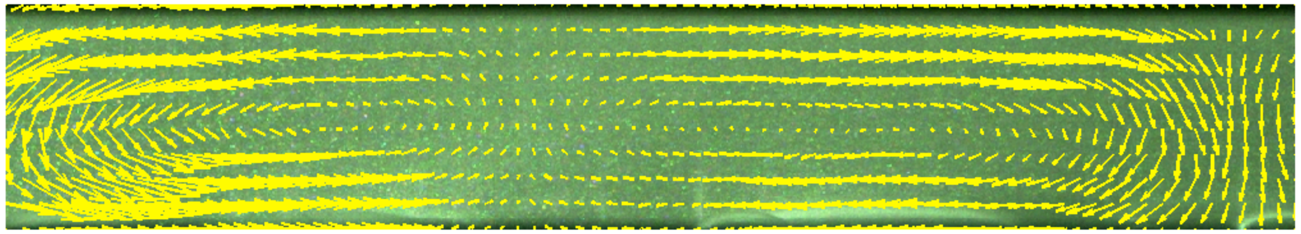
I=0.5A , $\Delta.T = 2.59^{\circ}c$

I=0.8A , $\Delta.T = 4.08^{\circ}c$

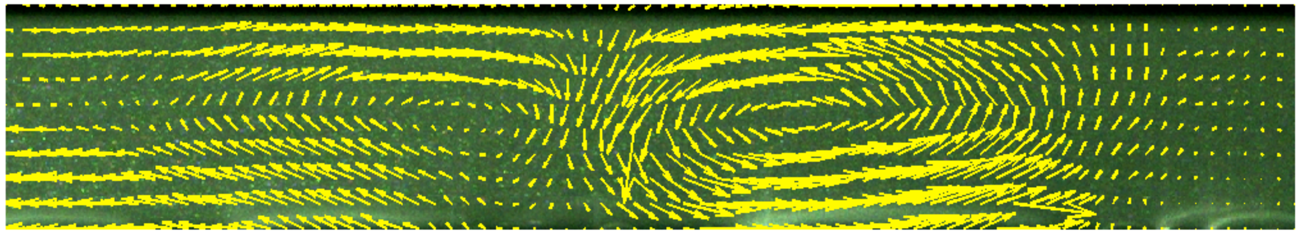
No convection detected by local PIV measurements within a 1 microns/s resolution

Flow structure and dynamic slightly above the onset

$$\Delta T = 6.5^{o} C$$



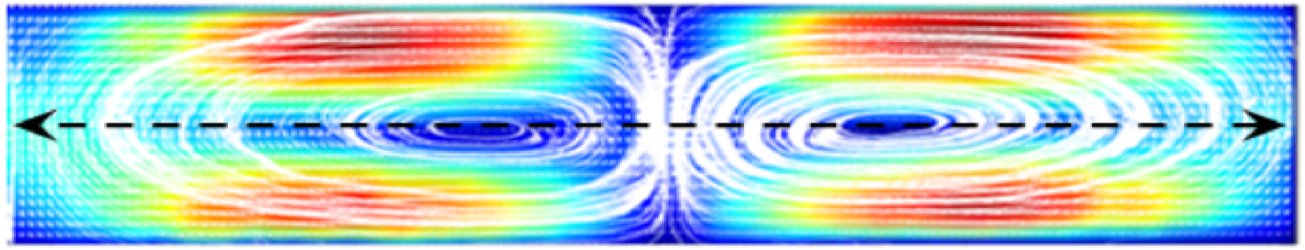
$$\Delta T = 6.72^{o} C$$



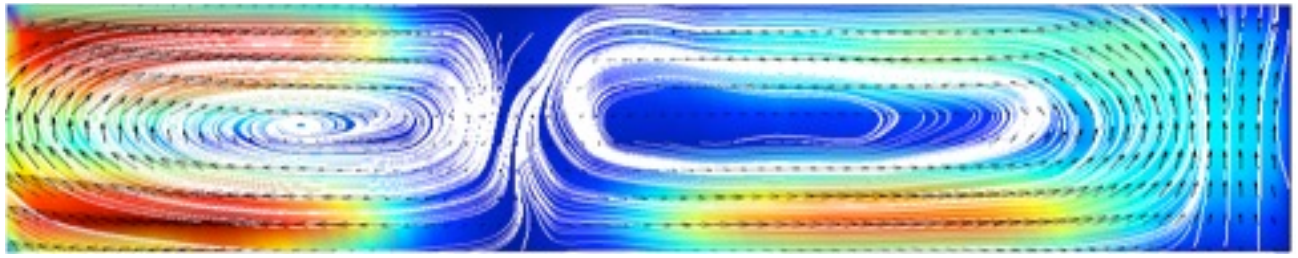
$$\Delta T > \Delta T_c$$

3.2 Newtonian versus Viscoplastic Convective Flow Patterns: a qualitative look

Newtonian Convective Pattern



Viscoplastic Convective Pattern



The flow patterns are qualitatively similar, but at a closer look...

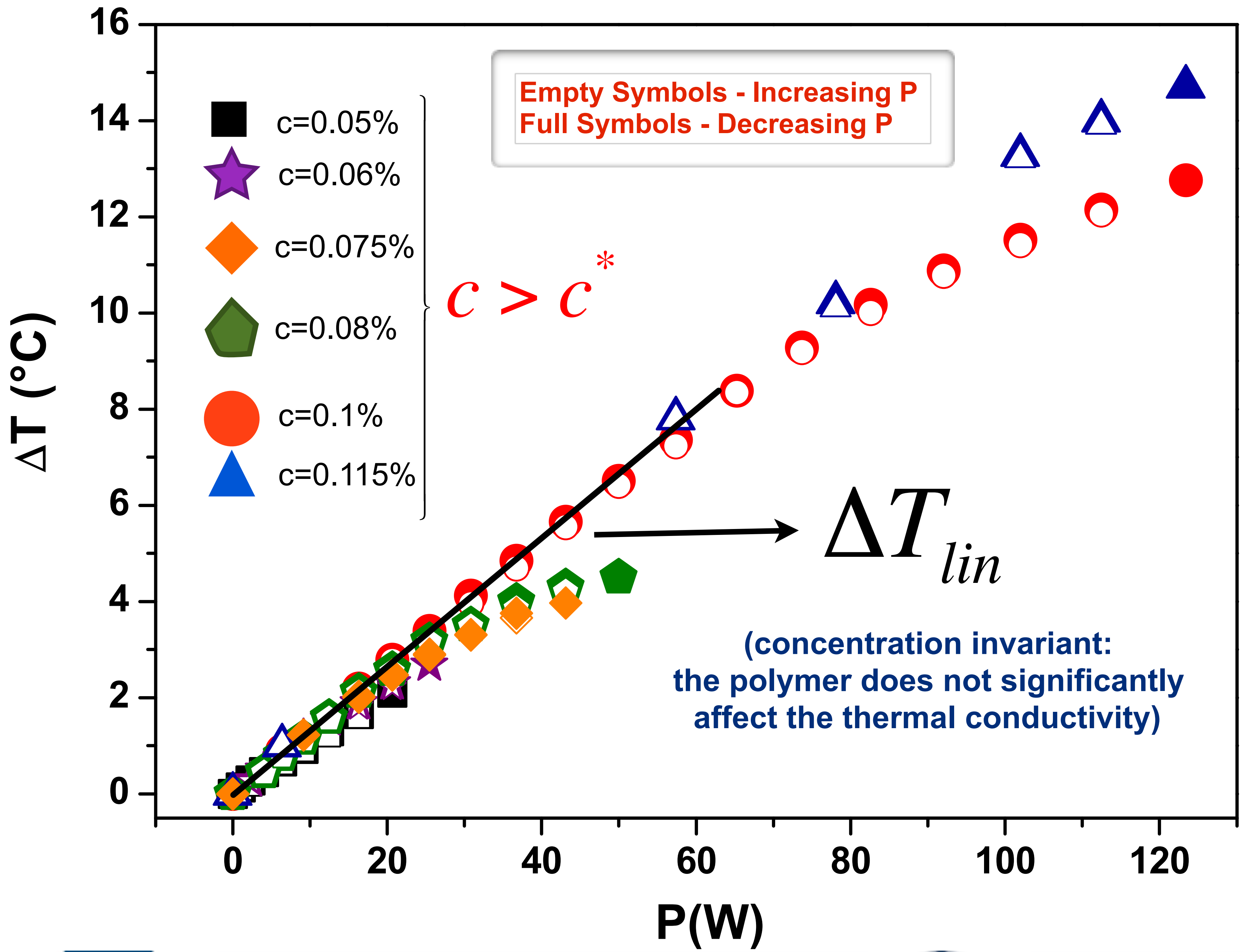
In the vicinity of the boundaries, the convection rolls are "flat" in the viscoplastic case

A plausible reason

Topological differences between the Newtonian and the Viscoplastic boundary layer

3.3 The physical nature of the R-B bifurcation in viscoplastic fluids

(FOCUS FIRST on INTEGRAL MEASUREMENTS)

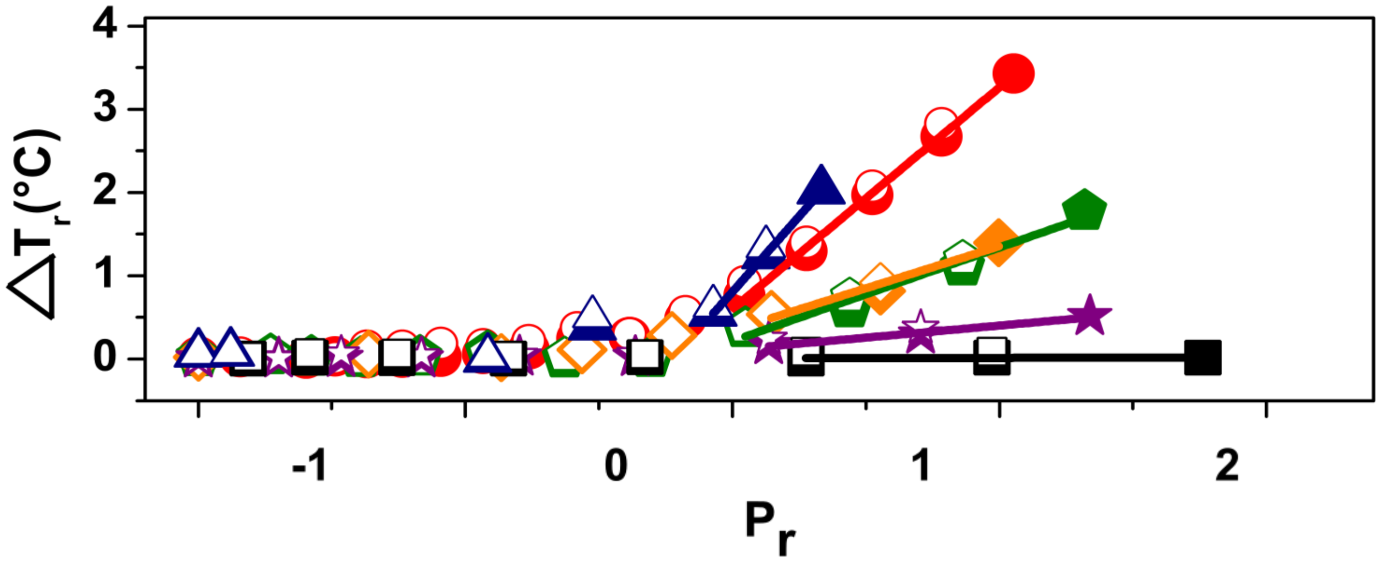


Reduced temperature difference:

$$\Delta T_r = \frac{\Delta T}{T_{lin}} - 1$$

Reduced control parameter

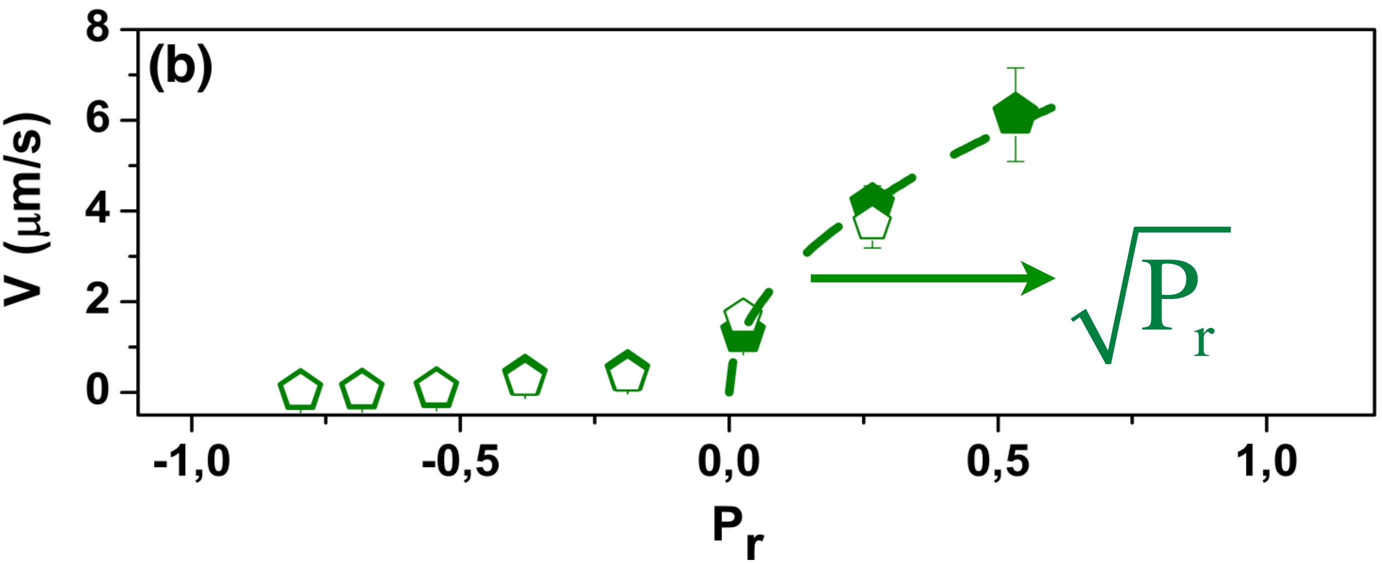
$$P_r = \frac{P}{P_c} - 1$$



(1) As in the Newtonian case, the order parameter
scales linearly with the control parameter above the onset of the transition.

(2) As in the Newtonian case, the transition is reversible upon increasing/decreasing control parameter (imperfect bifurcation).

FOCUS on LOCAL MEASUREMENTS of the CONVECTION AMPLITUDE



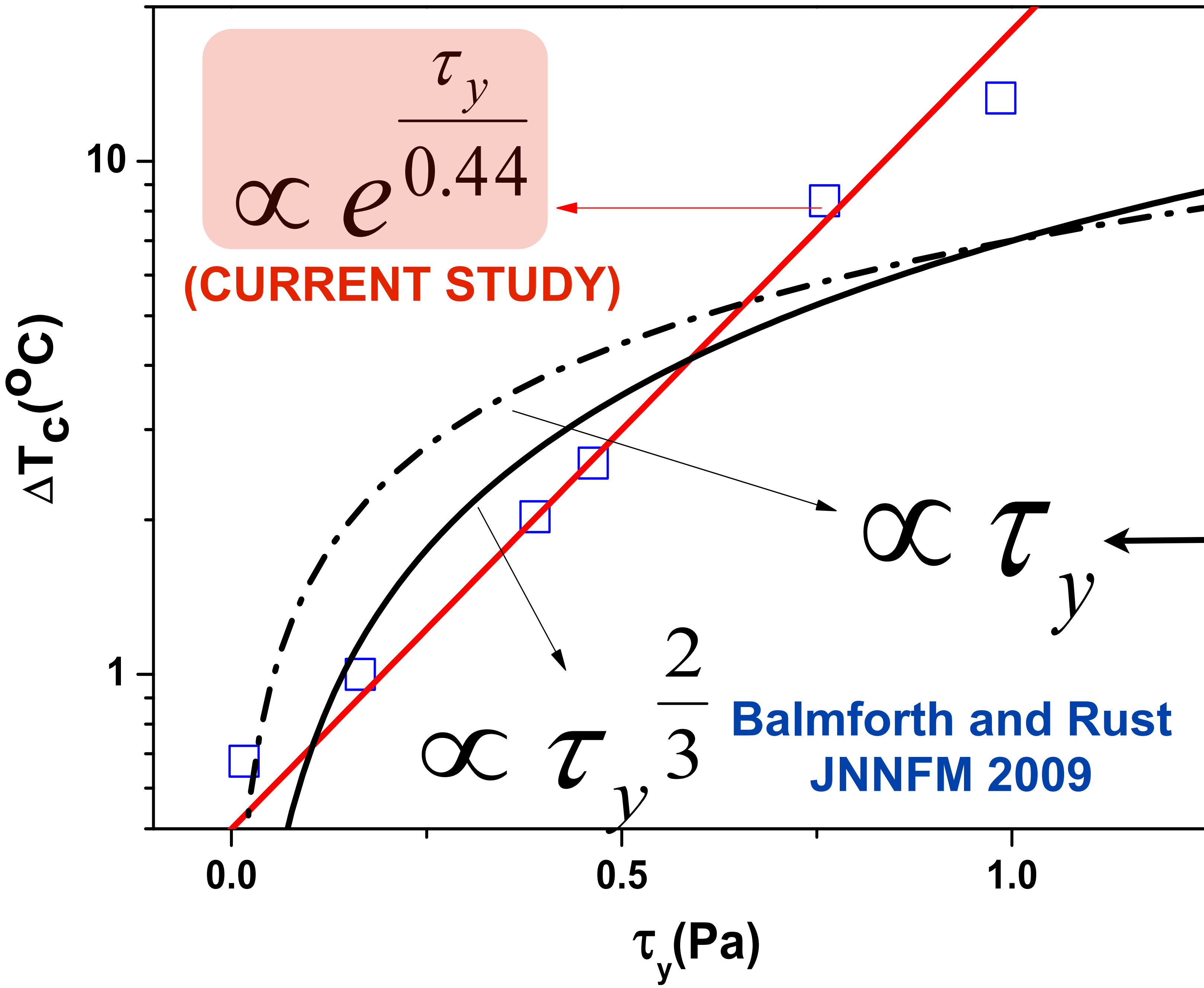
Landau Equation

$$\varepsilon = P_r = P/P_c - 1, \quad \xi = V$$

$$\varepsilon\xi - a\xi^3 + h = 0$$

As in the Newtonian case and within the entire range of Carbopol concentrations, the transition to the R-B convection in a Carbopol gel is a second order (imperfect) bifurcation that can be modelled by the Landau theory

Comparison with the litterature



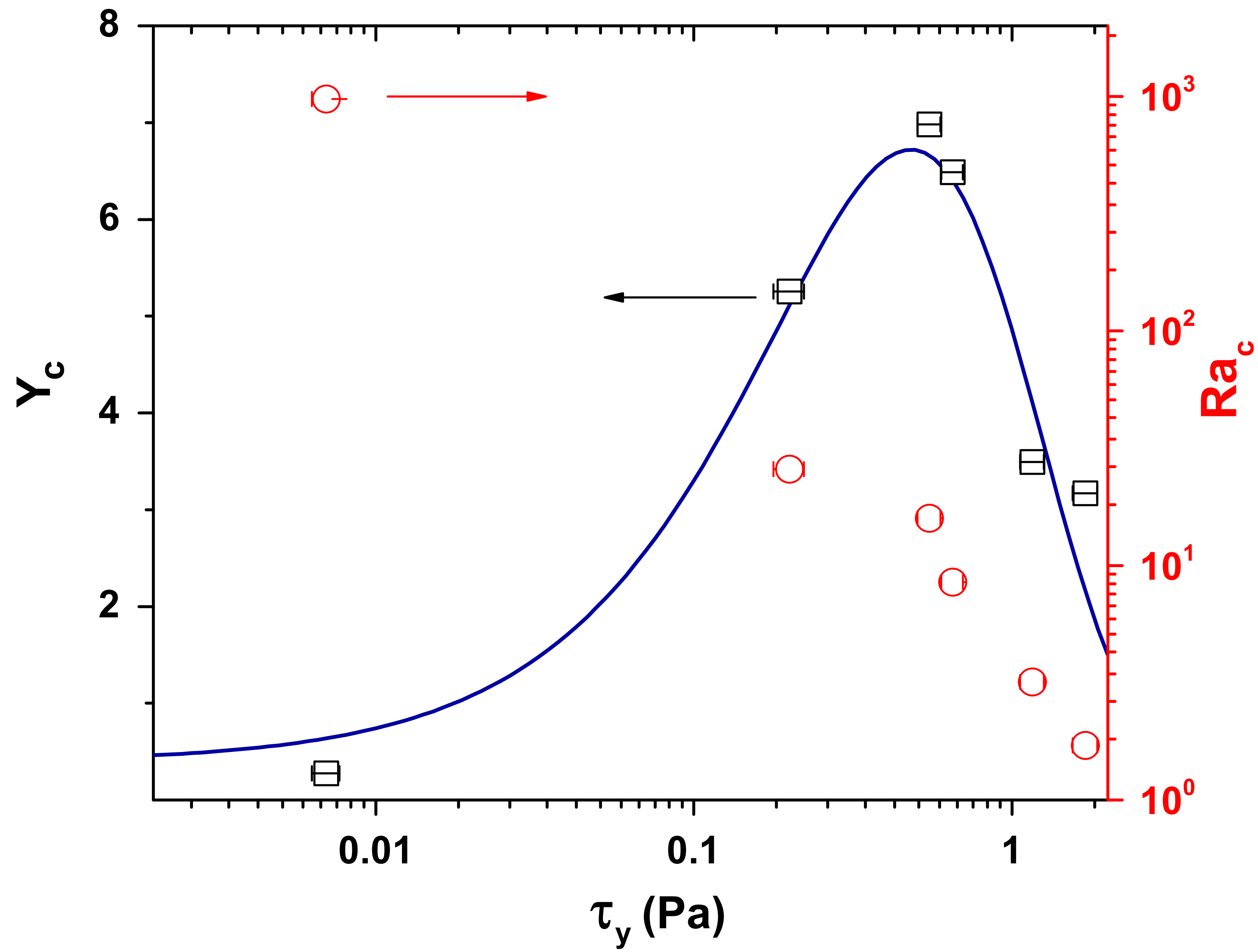
The critical temperature difference needed to trigger the R-B convection scales exponentially with the yield stress.

Experiments by Darbouli et al., Phys. Fluids 2013

Comparison with the litterature

$$Y = \frac{\tau_y}{\rho \beta g H \Delta T} \leqslant Y_c$$

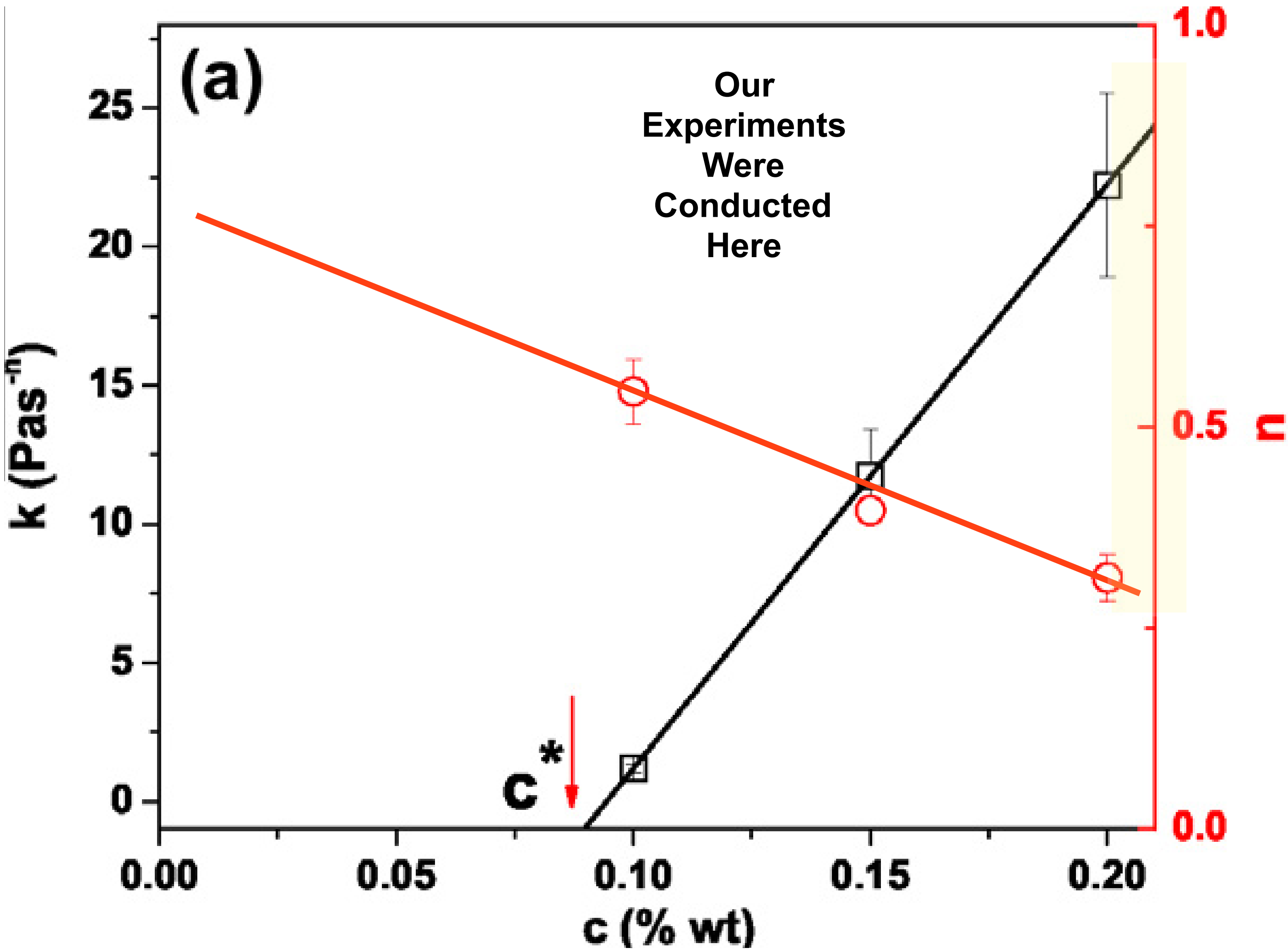
$$Ra = \frac{\rho \beta \Delta T g H}{\tau_y} \frac{t_d}{t_g} \geqslant Ra_c$$



The bifurcation towards R-B convective states is experimentally found to be supercritical in a wide range of yield stresses.

Recent theoretical developments suggest that an increase in the shear thinning behavior may turn the supercritical bifurcation into a subcritical one:
(group of Dr. Chérif Nouar in Nancy)

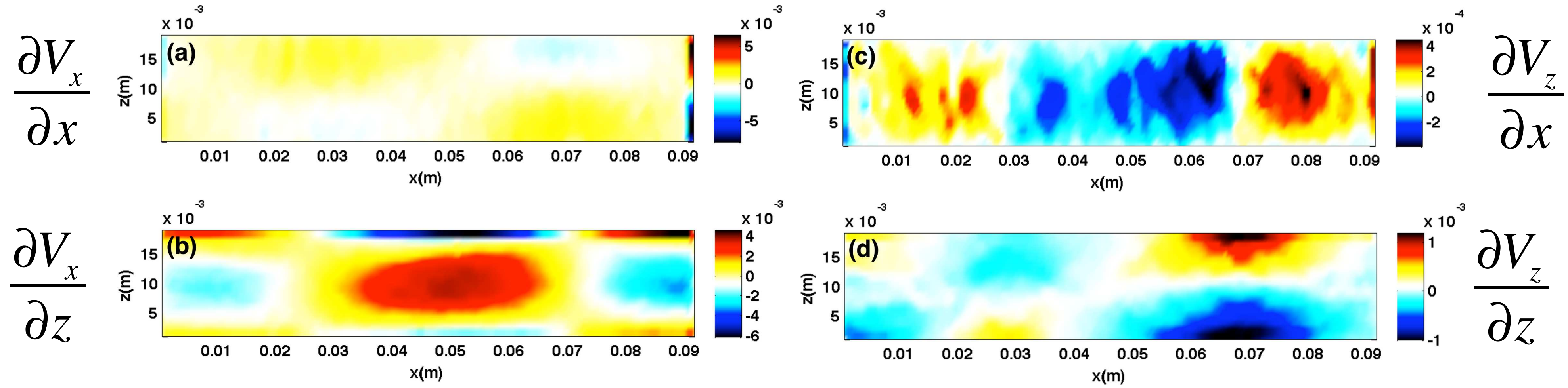
But...

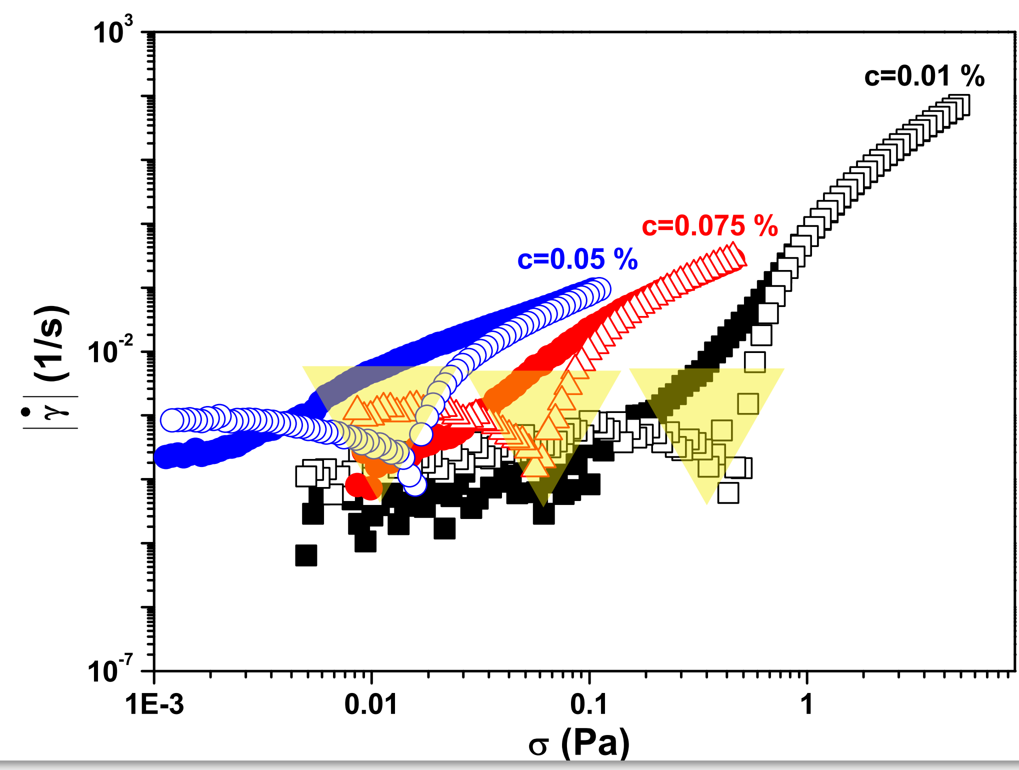


Our experiments do not confirm this theoretical prediction:

In spite of a clear shear thinning behavior, the bifurcation remains supercritical

Maybe there are some missing ingredients in the theoretical approaches?



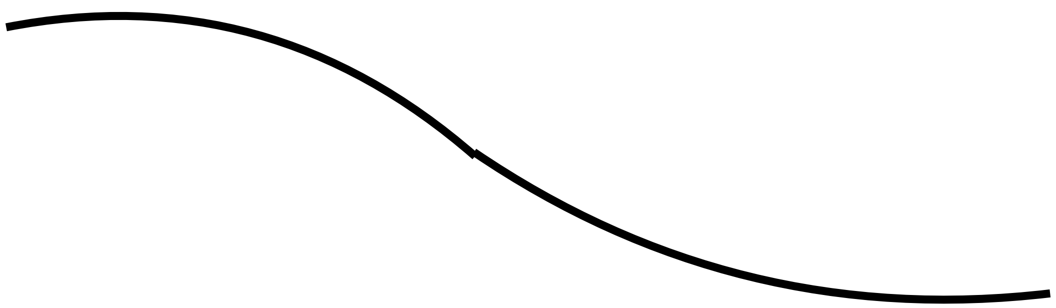


The Rayleigh-Bénard convection lives in the Bermuda triangle:

- Bingham, Herschel-Bulkley & Co. models do not apply here! (see paper by Putz&Burghelea, Rheo Acta 2009)

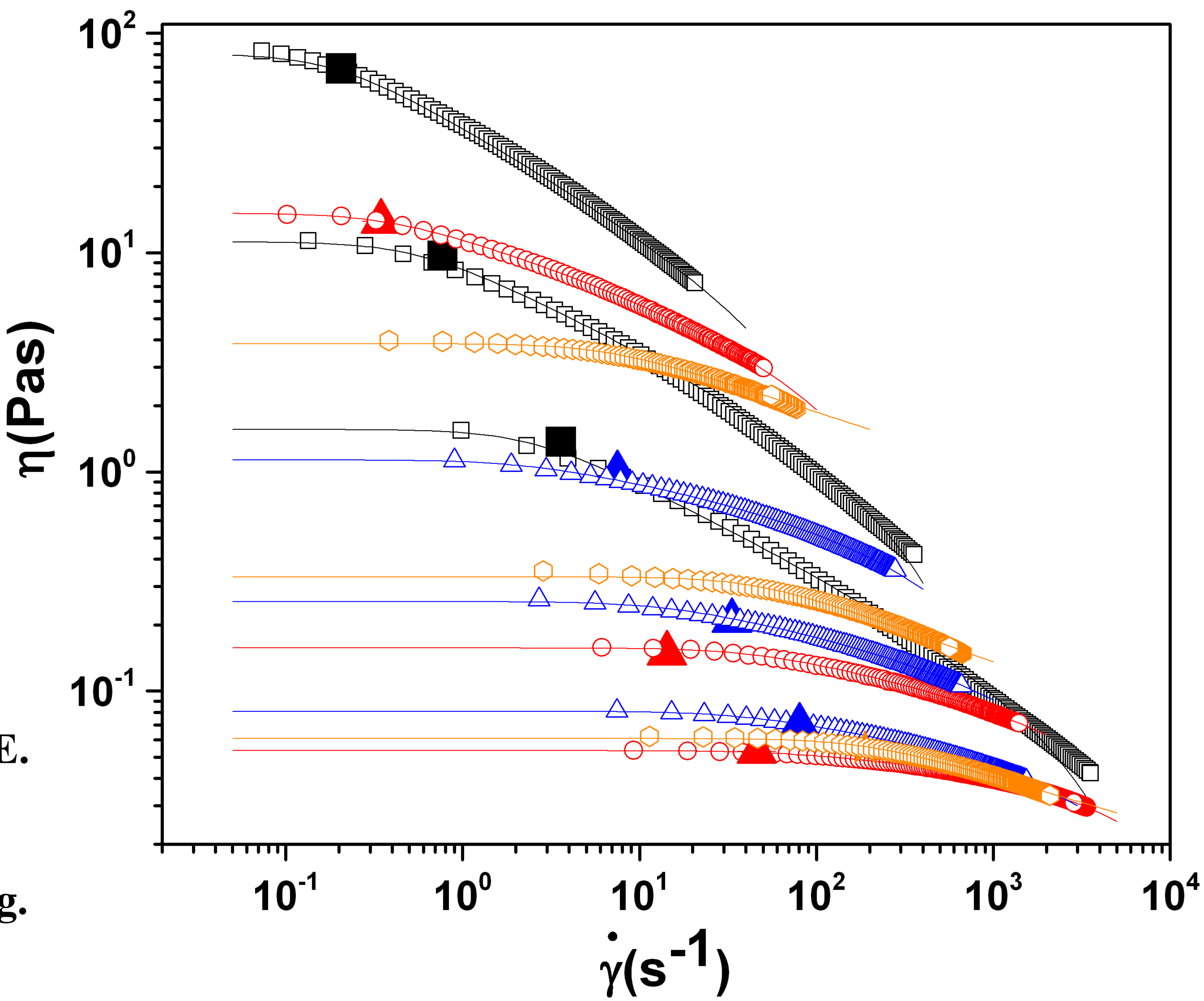
Further theoretical developments are still needed to understand the R-B convection in a Carbopol gel. A different rheological framework NEEDS to come in!

4. Experimental investigation of the Rayleigh-Bénard in a shear thinning fluid



$$\frac{\eta(T)-\eta_\infty(T)}{\eta_0(T)-\eta_\infty(T)} = [1+\dot{\gamma}^2\lambda(T)^2]^{\frac{n-1}{2}}$$

Carreau model

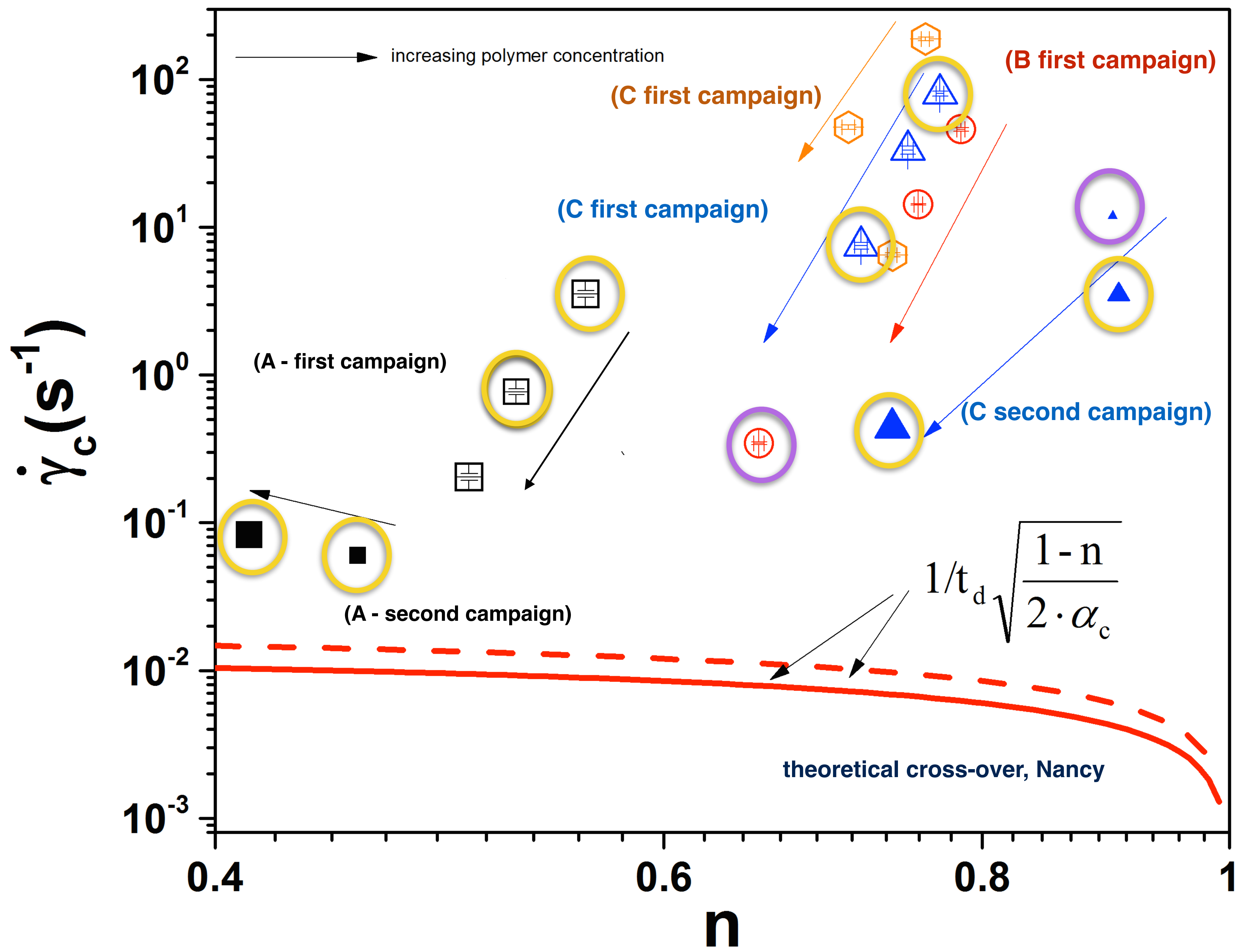


squares - A, circles - B, triangles - C, hexagons - E.

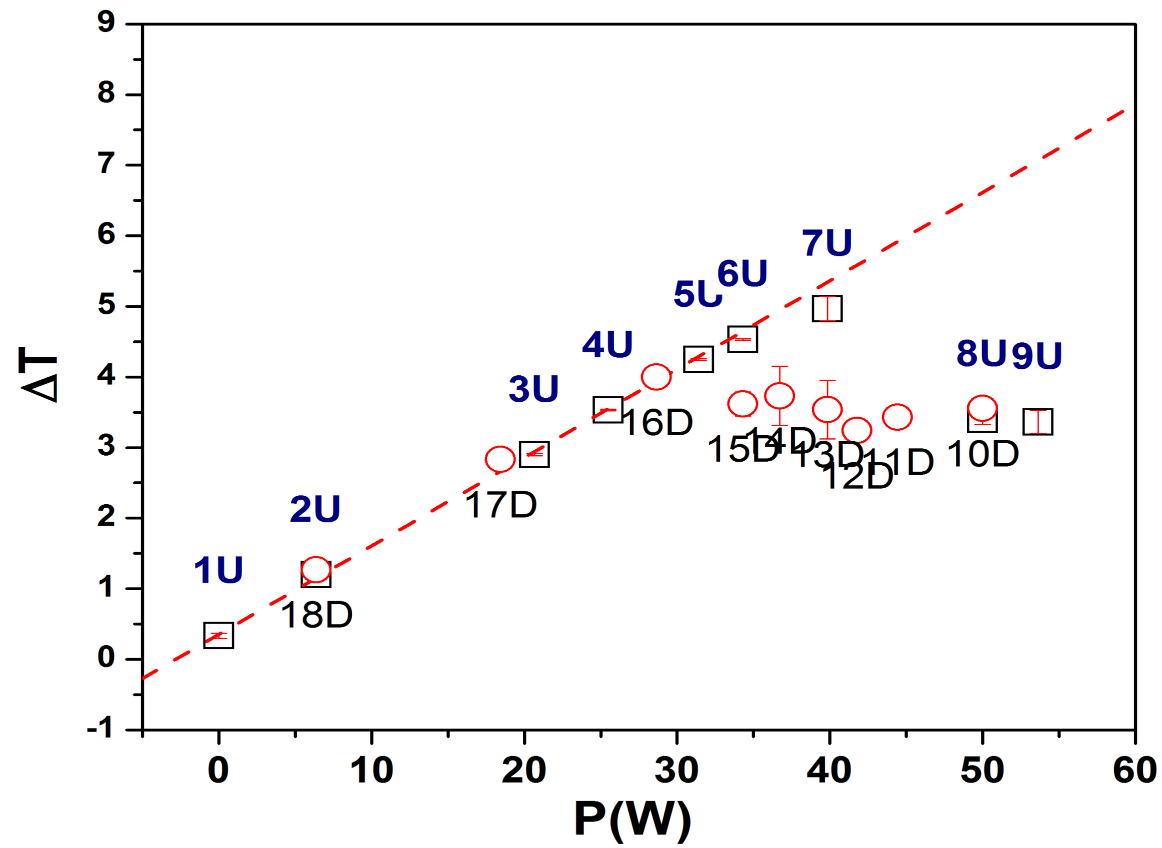
Full lines : Carreau model.

Full symbols mark the onset of the shear thinning.

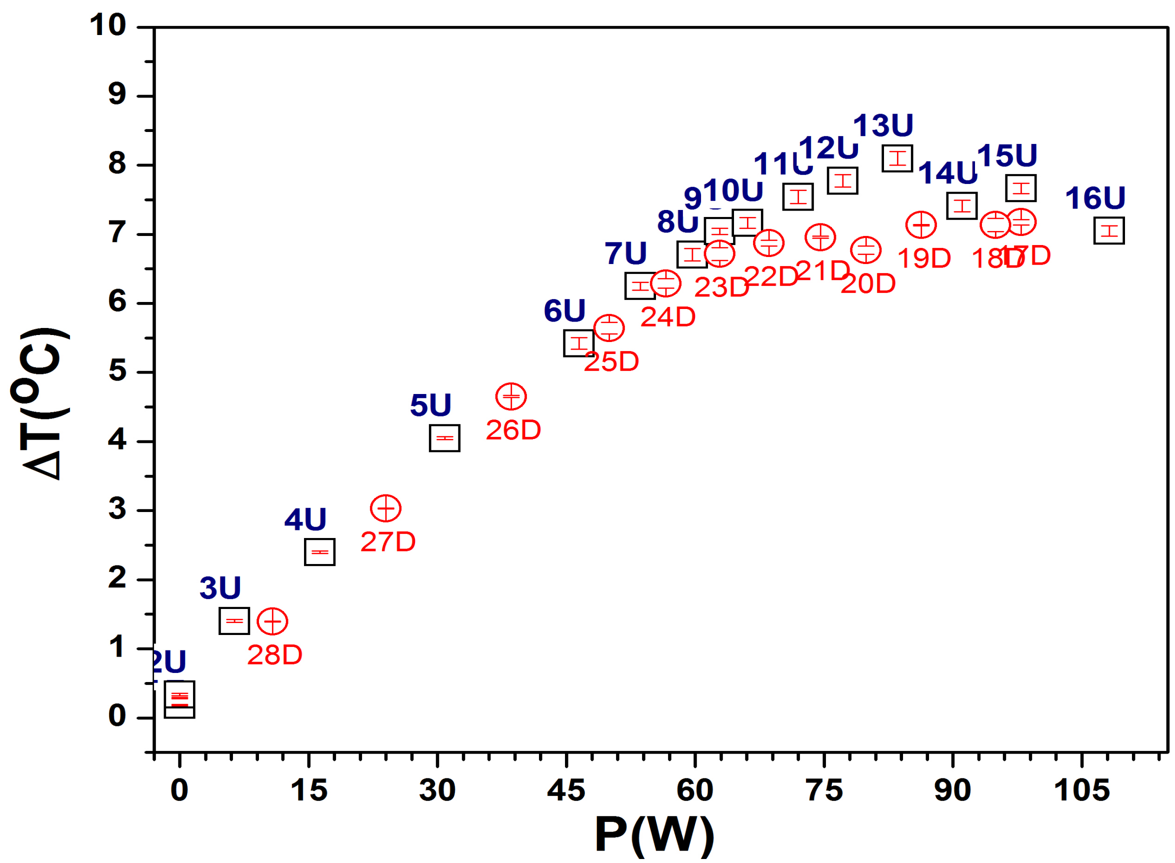
4.1. Experimental Cartography Campaign



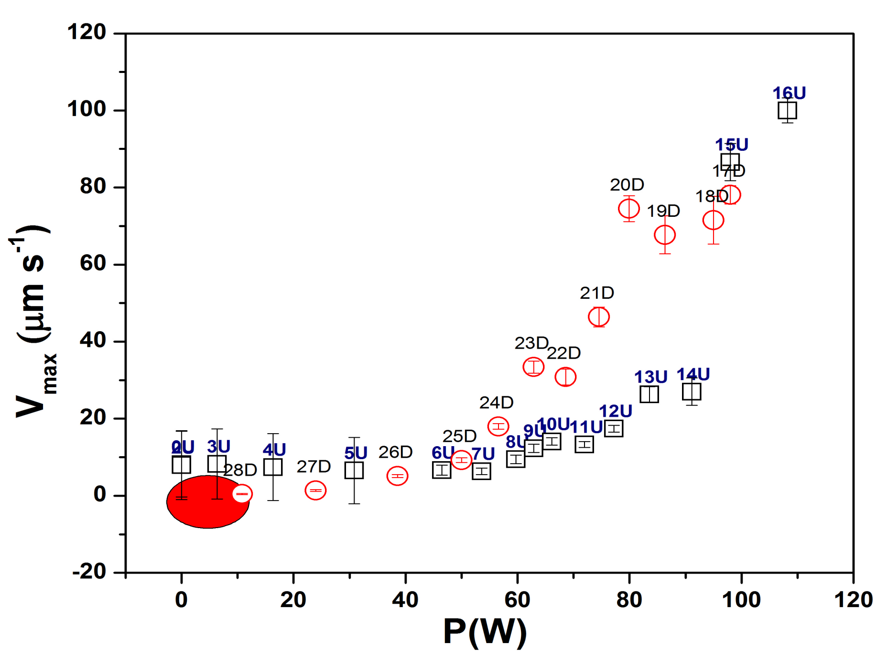
4.2. The physical nature of the R-B bifurcation in shear-thinning fluids



C solution with 2%wt



A solution with 1%wt



Our experiments confirm one part of this theoretical prediction:

the bifurcation remains subcritical

Conclusions, outlook

(1) The Rayleigh-Bénard convection **was investigated in a yield stress fluid** shear thinning fluid and **shear thinning fluid** by both **integral** measurements (T gradient between plates) and **local** ones (point-wise velocity measurements).

(2) As in the case of a Newtonian fluid, **the bifurcation towards convective states is continuous, reversible** and can be modeled by the Landau theory for yield stress fluid.

(3) In the case of a Carbopol gel, the R-B convection does not follow the Bingham, Herschel-Bulkley and Co. religion but lives in the Bermuda triangle.

(4) In the case of the shear thinning fluid, the instability is subcritical for all the experimental cases. How to find the « ideal » experimental fluid ?

Acknowledgements

Contact us:

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Thanks

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