Polarization imaging of multiply-scattered radiation based on Integral-Vector Monte Carlo Method

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Contents

Polarization imaging

Context

Size & structure characterization of complex-shaped particles in semi-transparent media

Non intrusive techniques → Electromagnetic waves (light)
Intensity + polarization (Stokes formalism, Mueller matrix)

Existing applications

Optically thin media → single scattering regime

in-between

Optically thick media → Diffusion approximation

multiple scattering effects

+

To increase information: 2D distribution = imaging
Polarization pattern = signature of the medium

Objectives

2D Optical diagnostic of semi transparent heterogeneous media analyzing polarization state with the Stokes parameter formalism

Characterization of :
- Optical / Radiative properties
- Morphology
- Size dispersion
- Volumetric fraction

Examples of a large scope of applications
- Particle suspensions, **soot**

- Biological cells

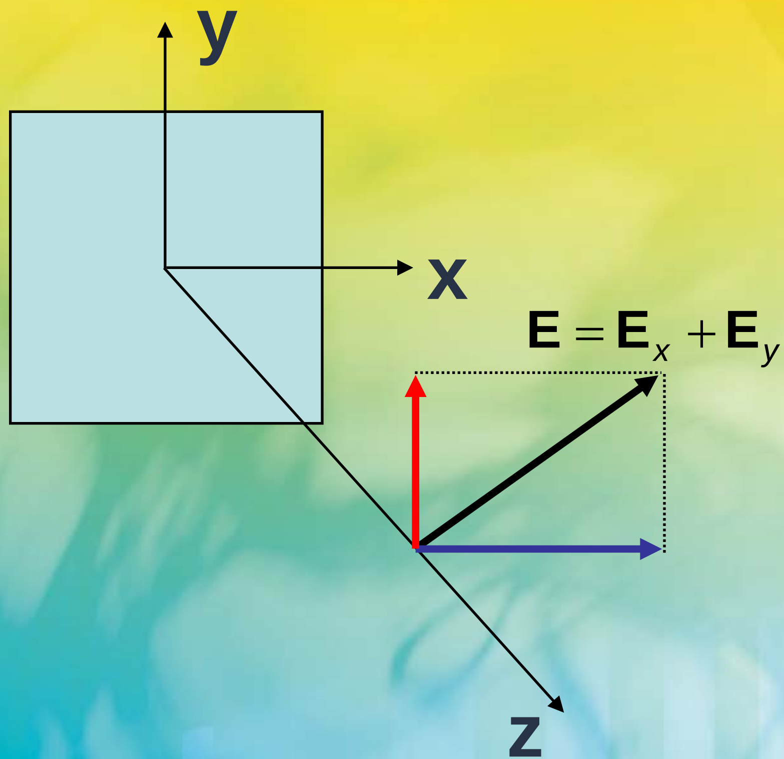
- Circumstellar disks

Polarization pattern from multiple scattering effects
→ need of comprehensive models

Modeling polarization imaging in a 3D system:
Integral-Vector Monte Carlo Method

Formalism – Polarization

Plane harmonic wave propagating in homogeneous medium



$$\mathbf{E}_x(z,t) = E_{0x} \cos(kz - \omega t)\,\mathbf{x}$$

$$\mathbf{E}_y(z,t) = E_{0y} \cos(kz - \omega t - \delta)\,\mathbf{y}$$

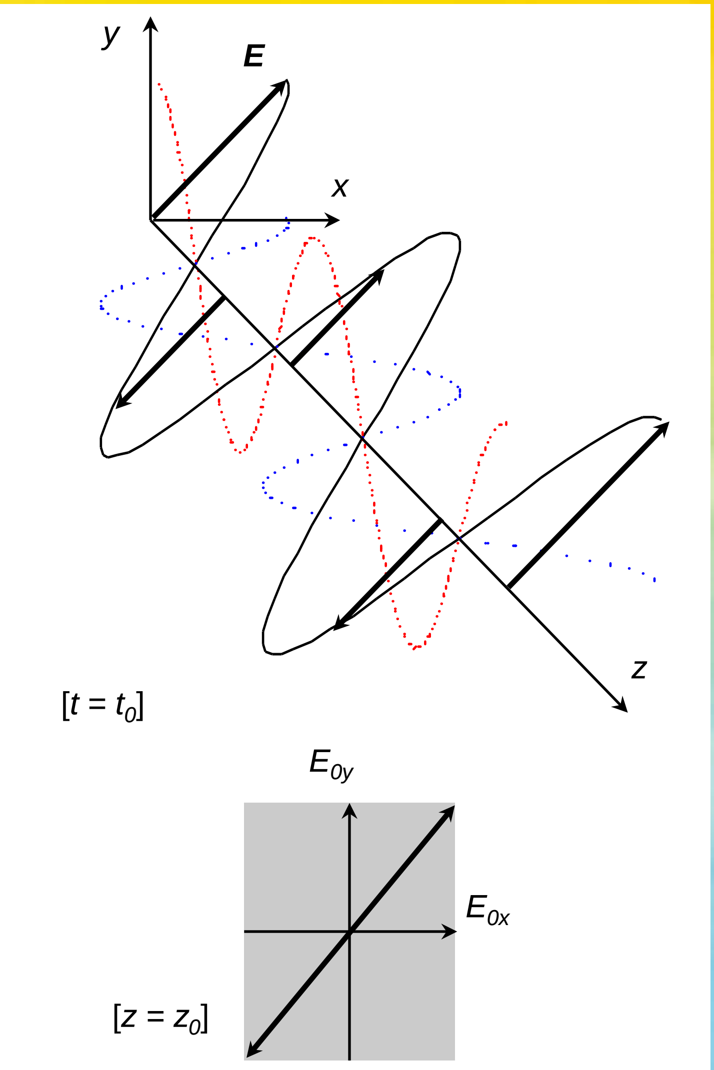
ω : angular frequency

k : wave vector

E_{0x}, E_{0y} : component amplitudes

δ : phase difference between components

Formalism – Polarization



If $\delta = 0\ [2\pi]$

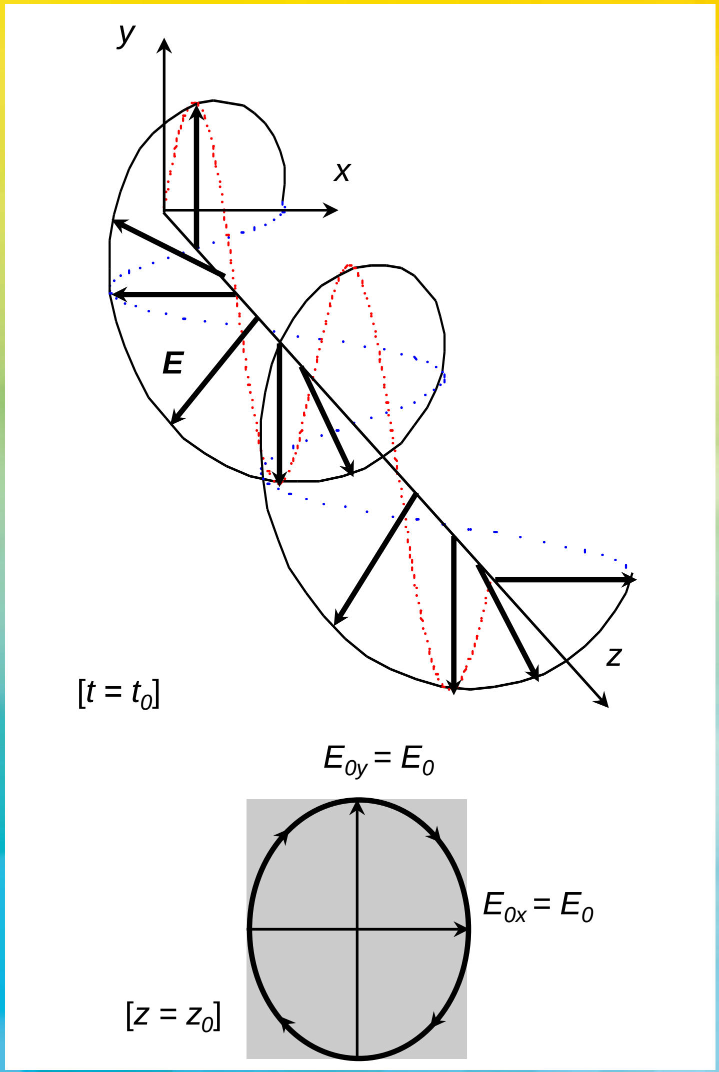
$$\mathbf{E}(z,t) = \left(E_{0x}\,\mathbf{x} + E_{0y}\,\mathbf{y}\right)\cos\left(kz - \omega t\right)$$

constant orientation of **E**

(time independent)

linear polarization

Formalism – Polarization



If $\delta = \pi/2\ [2\pi]$

$E_{0x} = E_{0y} = E_0$

$$\mathbf{E}_x(z,t) = E_0 \cos(kz - \omega t)\,\mathbf{x}$$

$$\mathbf{E}_y(z,t) = E_0 \cos(kz - \omega t - \pi/2)\,\mathbf{y}$$

$$= E_0 \sin(kz - \omega t)\,\mathbf{y}$$

$$\mathbf{E}(z,t) = E_0\left[\cos(kz - \omega t)\,\mathbf{x} + \sin(kz - \omega t)\,\mathbf{y}\right]$$

constant module of **E** (E_0)

end of **E** (in transverse plane) describes a circle

right circular polarization

Formalism – Stokes vector & Mueller matrix

- General case: Polarization state → elliptic polarization → ellipsometry
- How to describe polarization state of light with measurable quantities?
  → Stokes vector

$$\mathbf{I} = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_x\overline{E_x} + E_y\overline{E_y} \\ E_x\overline{E_x} - E_y\overline{E_y} \\ E_x\overline{E_y} + E_y\overline{E_x} \\ i(E_x\overline{E_y} - E_y\overline{E_x}) \end{pmatrix}$$

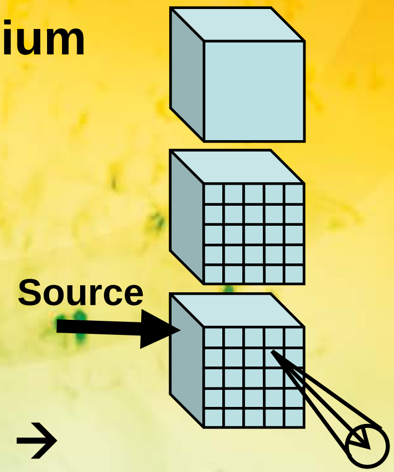
Information about
Intensity
Linear polarization (horizontal / vertical)
Linear polariaztion (oblique: +/- 45°)
Circular polarization (right / left)

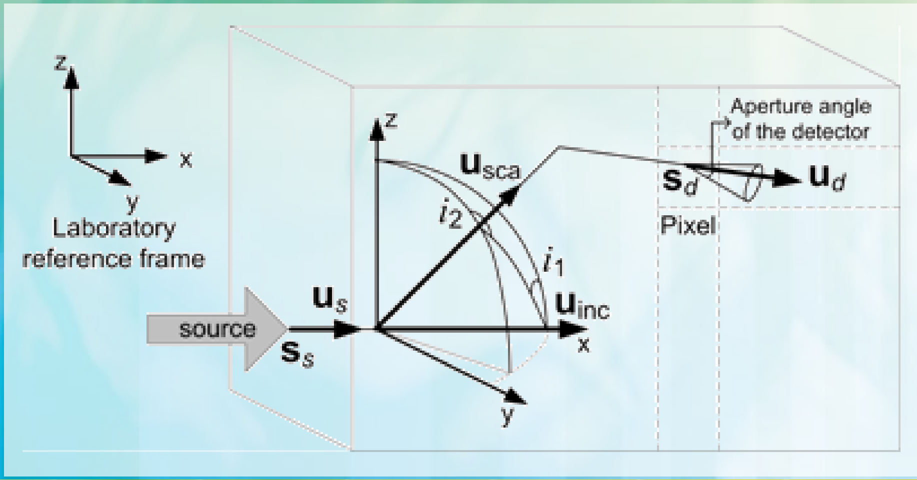
- Relation between a Stokes vector source and a detected Stokes vector
  → effective Mueller matrix

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_{\text{detected}} = \begin{pmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_{\text{source}}$$

Problem definition

- System = Scattering, absorbing, non emitting cold medium
- Data = 2D distribution of M_{ij} on a defined surface in the space surrounding the system → subdivision in pixels
- Detection within a given solid angle
- External radiation source
- Laboratory frame to keep track of the polarization state → definition of the meridian plan containing the direction of propagation under consideration





$$\mathbf{I}_{sca} = \frac{\sigma}{4\pi}\mathbf{L}(\pi - i_2)\,\mathbf{S}\,\mathbf{L}(-i_1)\,\mathbf{I}_{inc} = \frac{\sigma}{4\pi}\mathbf{S_R}\mathbf{I}_{inc}$$

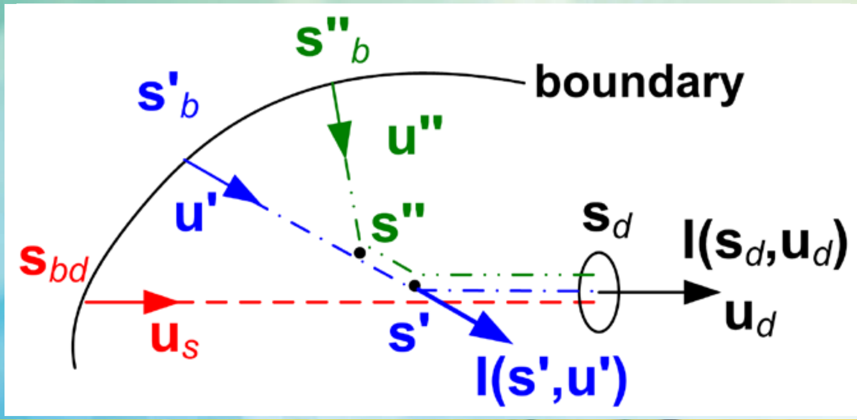
Integral formulation of the VRTE with a SOSS

The Vector Radiative Transfer Equation (VRTE)

Detected Stokes **vector**; Position; Direction; Transmittance; Scattering coefficient; Rotated scattering **matrix**

$$\mathbf{I}(\mathbf{s}_d,\mathbf{u}_d)=\mathbf{I}(\mathbf{s}_{bd},\mathbf{u}_d)t(\mathbf{s}_{bd},\mathbf{s}_d)+\int_{\mathbf{s}_{bd}}^{\mathbf{s}_d} ds' t(\mathbf{s}',\mathbf{s}_d)\frac{\sigma}{4\pi}\int_{4\pi} du' \mathbf{S}_{\mathbf{R}}(\mathbf{s}',\mathbf{u}',\mathbf{u}_d)\mathbf{I}(\mathbf{s}',\mathbf{u}')$$

$$\mathbf{I}(\mathbf{s}_d,\mathbf{u}_d)=\mathbf{I}(\mathbf{s}_{bd},\mathbf{u}_d)t(\mathbf{s}_{bd},\mathbf{s}_d)+\int_{\mathbf{s}_{bd}}^{\mathbf{s}_d} ds'\, t(\mathbf{s}',\mathbf{s}_d)\frac{\sigma}{4\pi}\int_{4\pi} du'\, \mathbf{S}_{\mathbf{R}}(\mathbf{s}',\mathbf{u}',\mathbf{u}_d)\Big\{\mathbf{I}(\mathbf{s}'_b,\mathbf{u}')t(\mathbf{s}'_b,\mathbf{s}')+$$
$$\int_{\mathbf{s}'_b}^{\mathbf{s}'} ds''\, t(\mathbf{s}'',\mathbf{s}')\frac{\sigma}{4\pi}\int_{4\pi} du''\, \mathbf{S}_{\mathbf{R}}(\mathbf{s}'',\mathbf{u}'',\mathbf{u}')\mathbf{I}(\mathbf{s}'',\mathbf{u}'')\Big\}$$



Integral formulation of the VRTE with a SOSS

The Scattering Order of Scattering Series (SOSS)

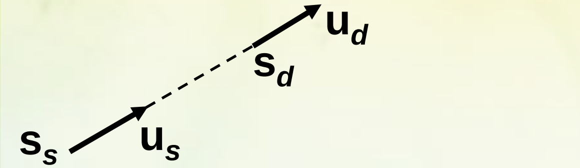
Detected Stokes **vector** → Scattering orders

$$\mathbf{I}\left(s_d, \mathbf{u}_d\right) = \sum_{k=0}^{\infty} \mathbf{I}_k$$

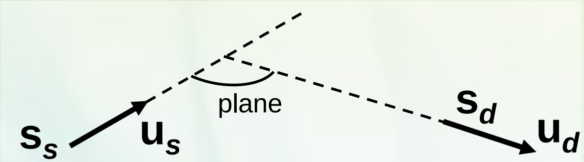
I_k is the kth scattering order Stokes vector
viz. the ensemble of contributions which are considered with k scattering modifications of propagation direction between a source and the detector

I_0 ≠ 0 only if a source is aligned with the detector

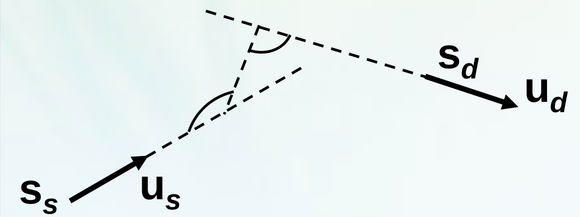
$$\mathbf{I}_0\left(\mathbf{s}_d, \mathbf{u}_d\right) = \mathbf{I}\left(\mathbf{s}_s, \mathbf{u}_s\right) t\left(\mathbf{s}_s, \mathbf{s}_d\right)$$



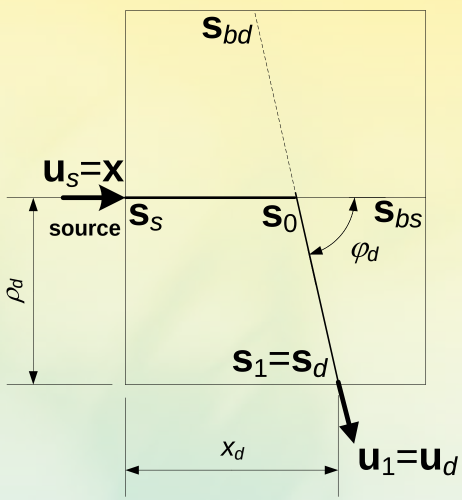
I_1 ≠ 0 only if a source and the detector directions (**u_s,u_d**) are in the same plane



General 3D case
at least two modifications of propagation direction are necessary to reach the detector from the source,
viz. **I_0** = **I_1** = 0



Integral formulation of the VRTE with a SOSS



First scattering orders in a uniform media

Albedo

Extinction coefficient

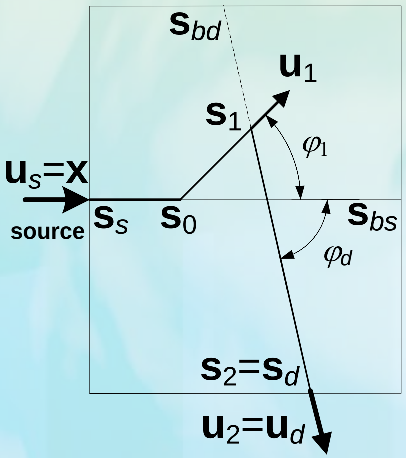
Scattering angle

Position and direction of the source

$$I_1(s_d,u_d)=\frac{\omega}{4\pi\rho_d\sin\varphi_d}\exp\left(-\beta\left(x_d+\rho_d\tan\left(\frac{\varphi_d}{2}\right)\right)\right)\mathbf{S_R}(\mathbf{u}_d,\mathbf{x})\mathbf{I}(\mathbf{s}_s,\mathbf{x})$$

distance from **x** axis

distance from the source on **x** axis

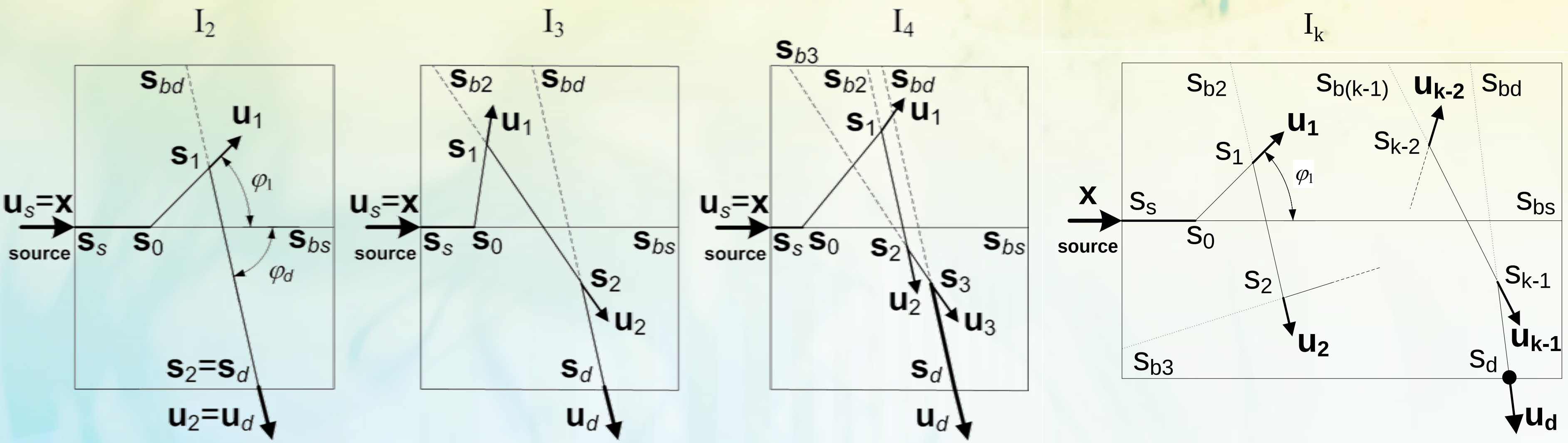


$$\mathbf{I}_2(\mathbf{s}_2,\mathbf{u}_2)=\int_{\mathbf{s}_{b2}}^{\mathbf{s}_2}\int_{\mathbf{s}_{bs}}^{\mathbf{s}_s}\left(\frac{\omega}{4\pi}\right)^2\frac{t(\mathbf{s}_s,\mathbf{s}_0)t(\mathbf{s}_0,\mathbf{s}_1)t(\mathbf{s}_1,\mathbf{s}_2)}{\left\|\mathbf{s}_1-\mathbf{s}_0\right\|^2}\mathbf{S_R}(\mathbf{u}_2,\mathbf{u}_1)\mathbf{S_R}(\mathbf{u}_1,\mathbf{x})\mathbf{I}(\mathbf{s}_s,\mathbf{x})d\mathbf{s}_0 d\mathbf{s}_1$$

Integral formulation of the VRTE with a SOSS

- **Recursive formulation for higher orders**

$$\mathbf{I}_k\left(\mathbf{s}_d,\mathbf{u}_d\right)=\frac{\omega}{4\pi}\int_{\mathbf{s}_{bd}}^{\mathbf{s}_d}\int_0^{2\pi}\int_{-1}^{1}t(\mathbf{s}_{k-1},\mathbf{s}_d)\mathbf{S_R}\left(\mathbf{u}_d,\mathbf{u}_{k\text{-}1}\right)\mathbf{I}_{k\text{-}1}\left(\mathbf{s}_{k-1},\mathbf{u}_{k-1}\right)d\eta_{k-1}d\psi_{k-1}d\mathbf{s}_{k-1}$$



- **These integrals cannot be solved analytically for complex geometries**

Monte Carlo integration

- **Principle**

Probability density function of a random variable *X*

Estimation

$$\int_D f(x)dx = \int_D pdf_X(x)\frac{f(x)}{pdf_X(x)}dx = \int_D pdf_X(x)w(x)dx = \lim_{N\to\infty} m, \quad m = \frac{1}{N}\sum_{i=1}^{N} w(x_i)$$

ROGER, M., BLANCO, S., EL HAFI, M., FOURNIER, R., Monte Carlo Estimates of Domain-Deformation Sensitivities, *Phys. Rev. Lett.*, volume 95, issue 18, pages 180601.1-4, 2005

- Judicious choices for **probability density functions (pdf)**

+

- Computation of the series is a **backward** process:
  a priori better adapted to **directional detection** than a forward process

➔Efficient reduction of variance

MC Integration → sampling integration domains thanks to appropriate chosen pdfs

Monte Carlo integration

- **Implementation**
  - **Variance and accuracy control (sample variance)**

$$\mathrm{var}_m = \frac{1}{N}\left(\frac{1}{N}\sum_{i=1}^{N} w(x_i)^2 - \left(\frac{1}{N}\sum_{i=1}^{N} w(x_i)\right)^2\right), \mathrm{var}_{sp} = \frac{N_{sp}}{N_{sp}-1}\left(\frac{1}{N_{sp}}\sum_{i=1}^{N_{sp}} m_i^2 - \left(\frac{1}{N_{sp}}\sum_{i=1}^{N_{sp}} m_i\right)^2\right)$$

  - **MC Formulation**

$$\mathbf{I}_k\left(\mathbf{s}_d,\mathbf{u}_d\right) = \frac{\omega}{4\pi}\int_{\mathbf{s}_{bd}}^{\mathbf{s}_d}\int_0^{2\pi}\int_{-1}^{1} t(\mathbf{s}_{k-1},\mathbf{s}_d)\mathbf{S_R}\left(\mathbf{u}_d,\mathbf{u}_{k\text{-}1}\right)\mathbf{I}_{k\text{-}1}\left(\mathbf{s}_{k-1},\mathbf{u}_{k-1}\right)d\eta_{k-1}d\psi_{k-1}d\mathbf{s}_{k-1}$$

    - pdf choice → cumulative density function (cdf) → variable change

$$R_{\mathbf{s}_{k-1}} = \frac{1-t(\mathbf{s}_{k-1},\mathbf{s}_d)}{1-t(\mathbf{s}_{bd},\mathbf{s}_d)},\ R_{\psi_{k-1}} = \frac{\psi_{k-1}}{2\pi},\ dR_{\eta_{k-1}} = \frac{\mathbf{S}_{\mathbf{R}11}\left(\mathbf{u}_d,\mathbf{u}_{k\text{-}1}\right)}{2}d\eta_{k-1}$$

$$\mathbf{I}_k\left(\mathbf{s}_d,\mathbf{u}_d\right) = \omega\left(1-t(\mathbf{s}_{bd},\mathbf{s}_d)\right)\int_0^1\int_0^1\int_0^1 \frac{\mathbf{S_R}\left(\mathbf{u}_d,\mathbf{u}_{k\text{-}1}\right)}{\mathbf{S}_{\mathbf{R}11}\left(\mathbf{u}_d,\mathbf{u}_{k\text{-}1}\right)}\mathbf{I}_{k\text{-}1}\left(\mathbf{s}_{k-1},\mathbf{u}_{k-1}\right)dR_{\eta_{k-1}}dR_{\psi_{k-1}}dR_{\mathbf{s}_{k-1}}$$

Backscattering configuration (validation)

i. against the semi-analytical results of Crosbie and Dougherty (1982) for **scalar backscattered intensities** in the case of a plane parallel layer of **isotropic scattering** media subjected to a Gaussian narrow beam

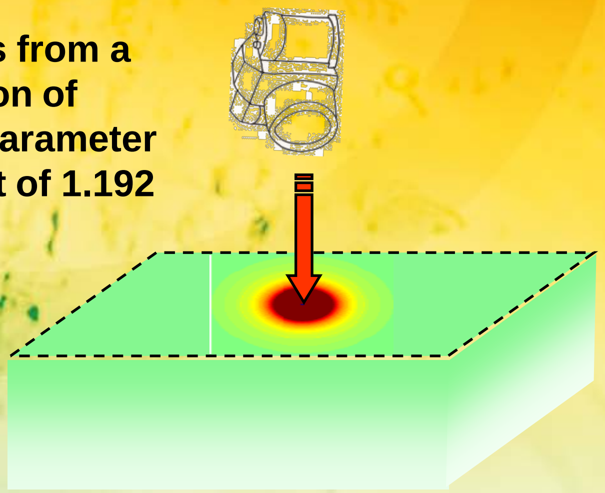
ii. against Ambirajan and Look (1997) results for the **backscattered Stokes vector** intensities calculated as a function of the distance of observation from a right circularly polarized narrow beam illuminating a plane-parallel medium laden with **spherical particles**

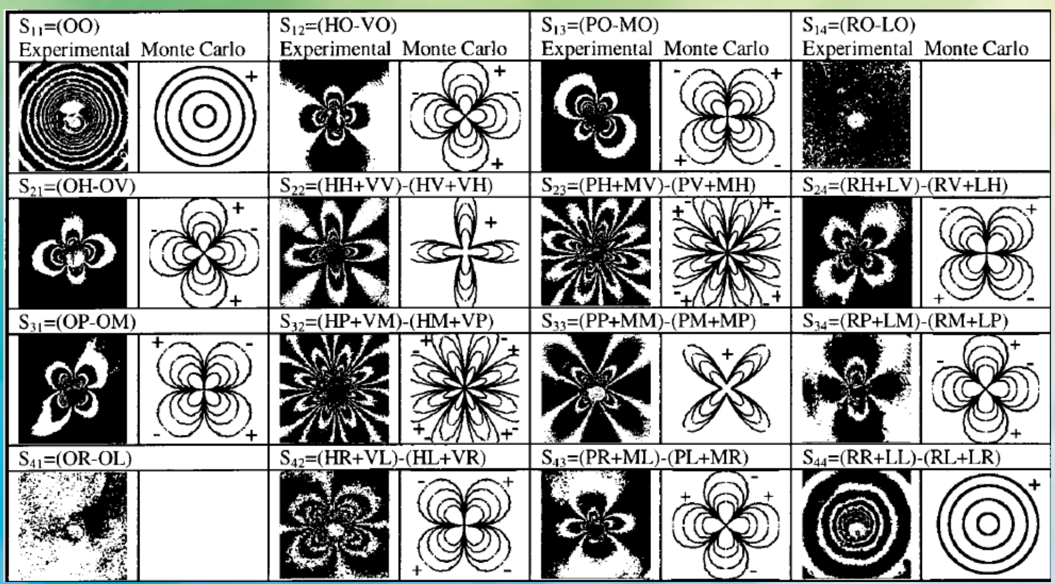
Backscattering configuration (validation)

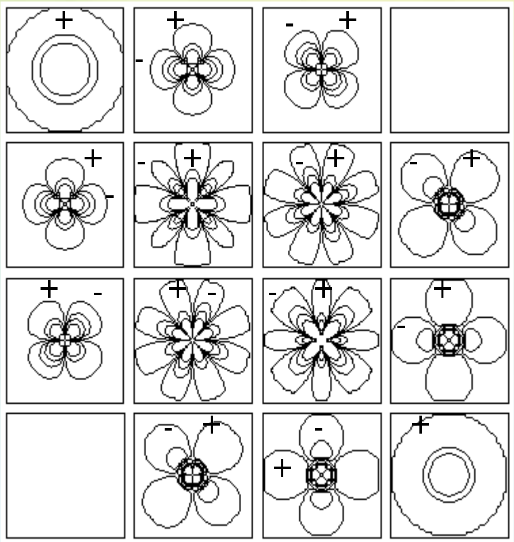
Rakovic et al (1999)

M_{11}	M_{12}	M_{13}	M_{14}
M_{21}	M_{22}	M_{23}	M_{24}
M_{31}	M_{32}	M_{33}	M_{34}
M_{41}	M_{42}	M_{43}	M_{44}

2D effective Mueller matrix contours from a half-space filled with a suspension of monodispersed spheres with a size parameter *x*=13.4 and a refractive index contrast of 1.192





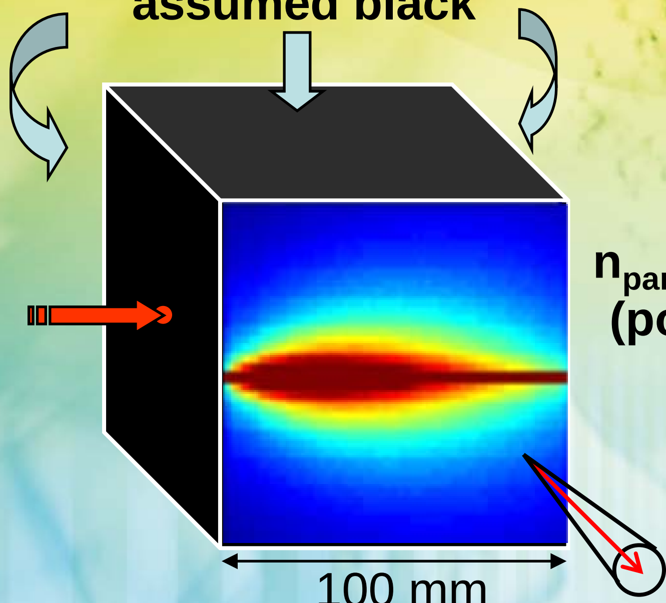


Lateral configuration

Effective Mueller matrix images of a uniform non absorbing particle-laden solution

All other faces are assumed black

Polarized monochromatic source collimated at the center
2D Gaussian shape
optical FWHM of 1.10^{-3} (narrow)
λ= 0.633 μm



The refractive index ratio
$n_{particles} / n_{surrounding\ medium}$ = 1.195
(polystyrene particles in water at λ= 0.633 μm)

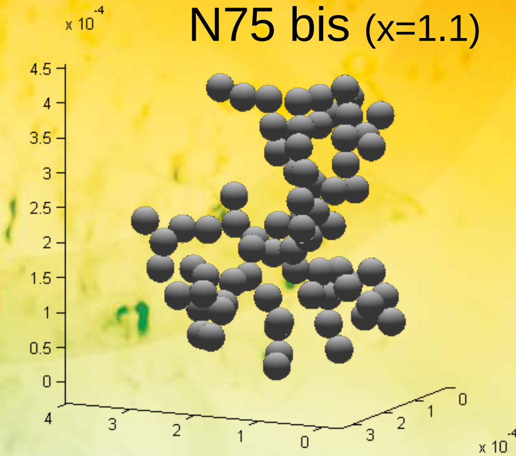
normal detection direction within a conical aperture of 2°

Sensitivity to morphology

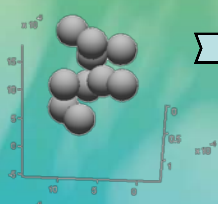
"Aggregation"

- Random generation
- Fractal dimension $D_f = 1.8$
- Prefactor $k_f = 2$
- Monomer diameter : 40 nm
- Constant volume fraction, f_v = 2.E-4

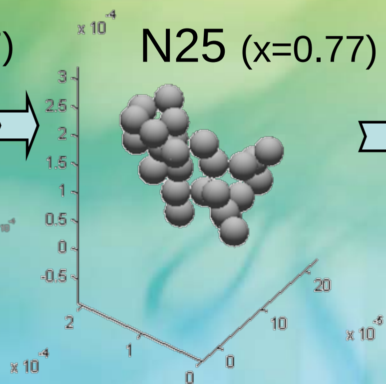
N75 bis (x=1.1)



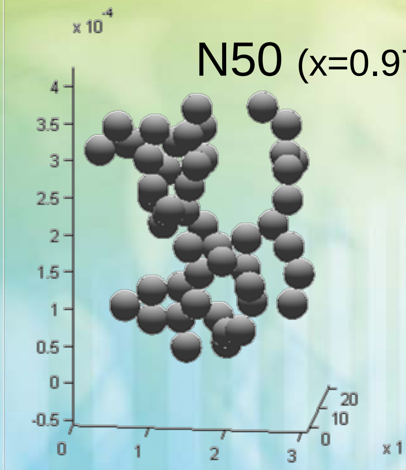
N10 (x=0.57)



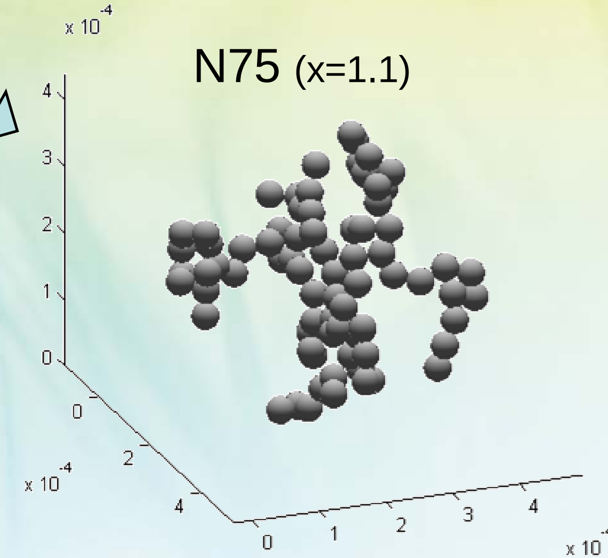
N25 (x=0.77)



N50 (x=0.97)

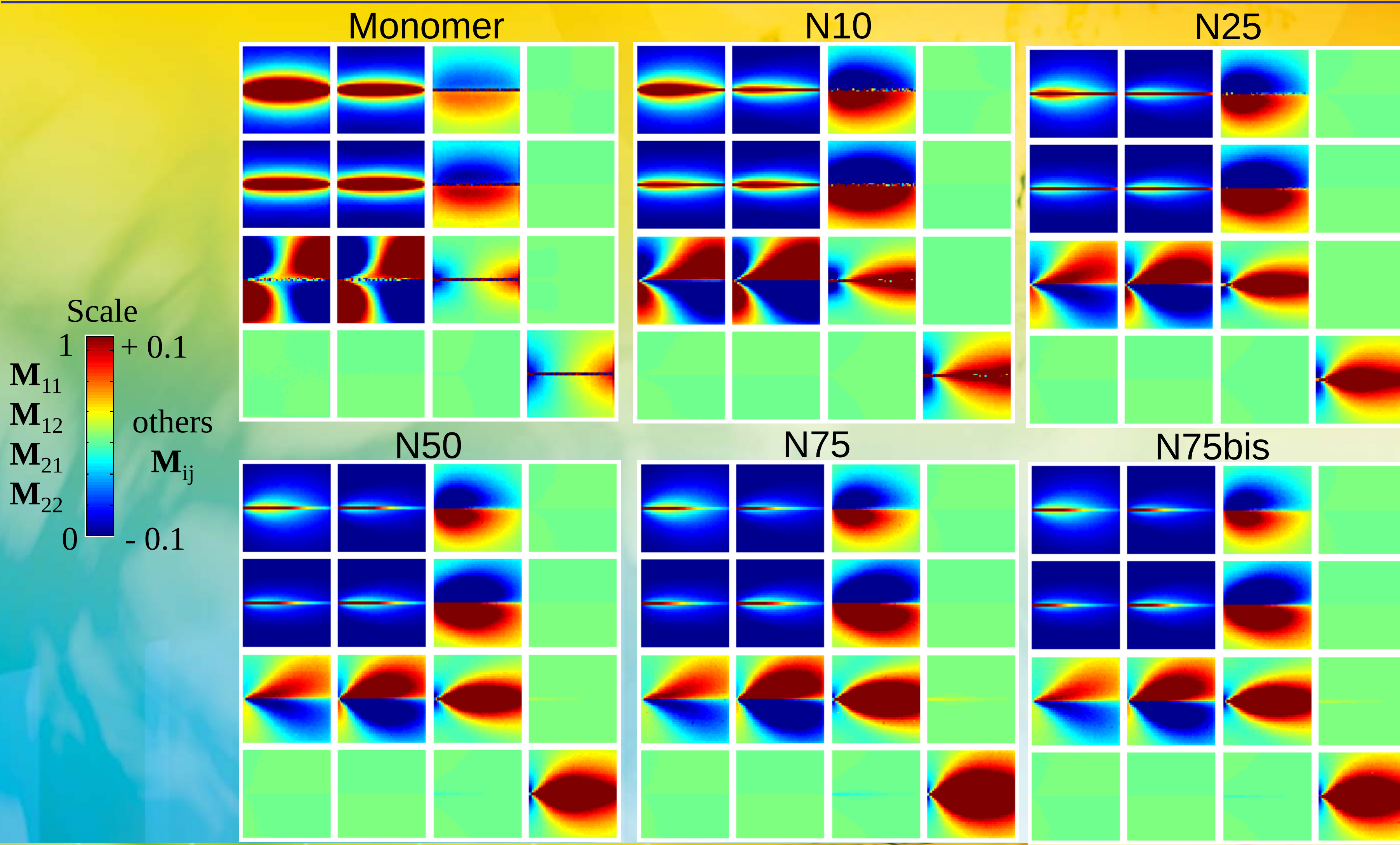


N75 (x=1.1)



Agg	Mono	N10	N25	N50	N75	N75bis
σ (m^{-1})	3.822	11.87	20.93	26.87	27.30	29.02

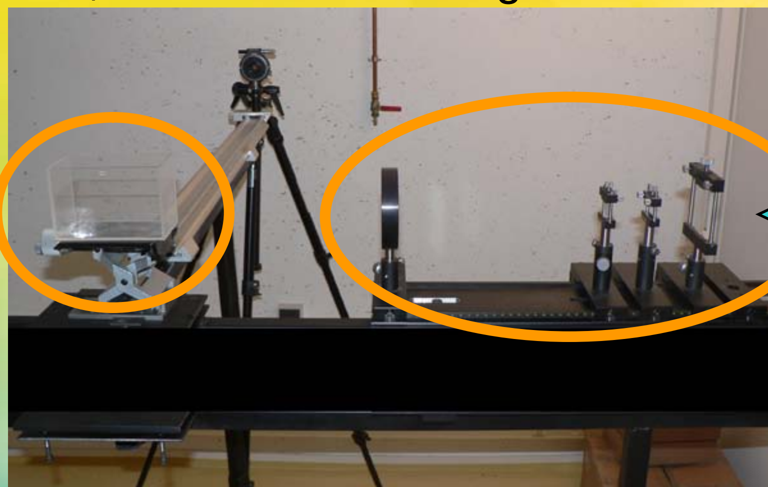
Sensitivity to morphology



Experimental setup

- Measure of Stokes parameters **I** = (I Q U V)^t, on each pixel of different pictures, at 90°, in a small solid angle

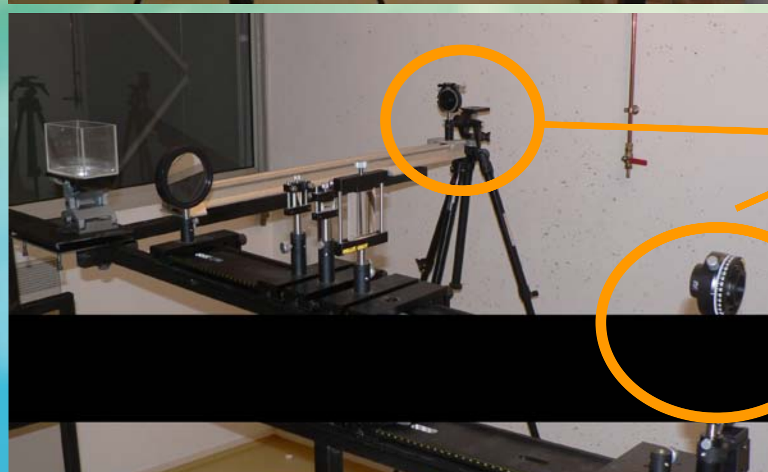
Sample



Shaping the polarized collimated laser sheet

458<λ<514 nm

1 < Power < 4 W



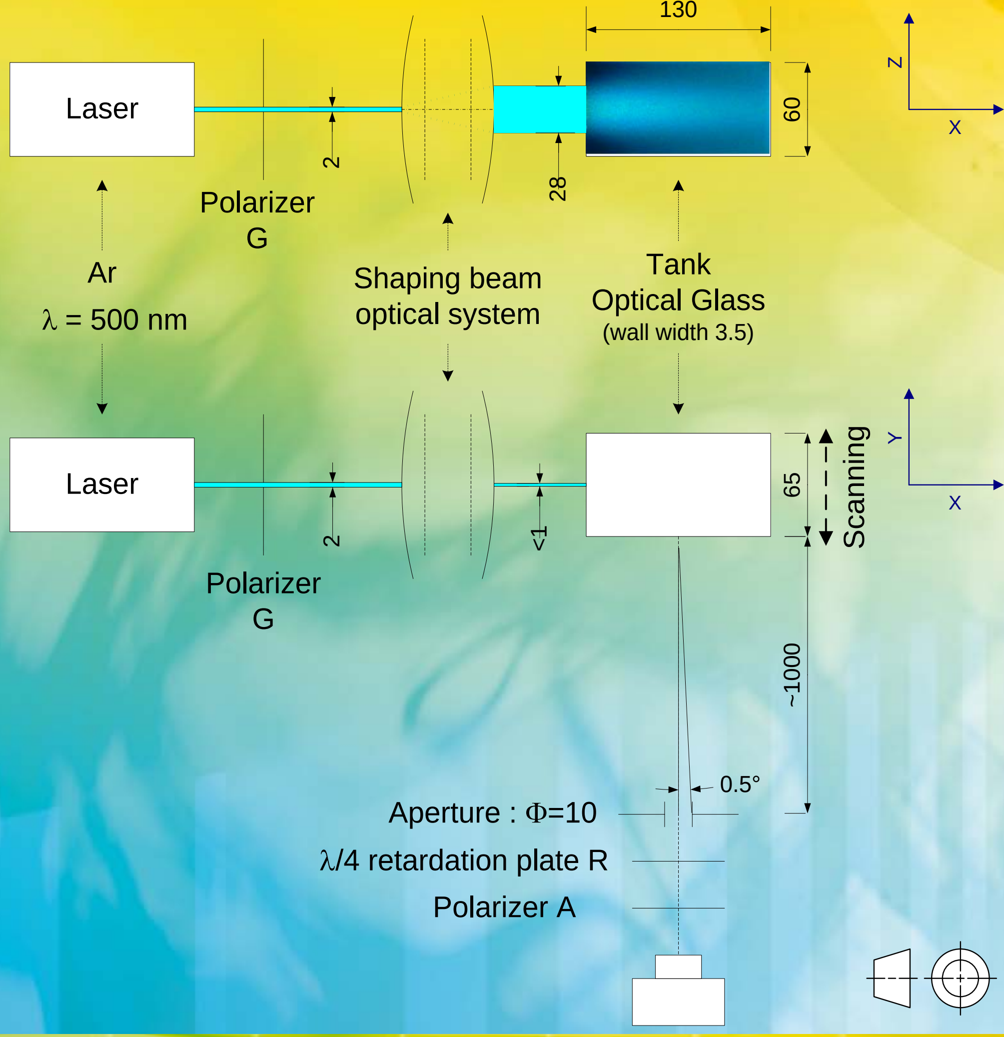
Polarizers

(generators and analyzers)

Extinction ratio < 10^{-5}

Experimental setup

- Scheme



- Principle
  - For a given source, polarized by G, (I,Q,U,V) are measured with combinations of R and A
    - $I = I_{0°} + I_{90°}$
    - $Q = I_{0°} - I_{90°}$
    - $U = I_{+45°} - I_{-45°}$
    - $V = I_{right} - I_{left}$
  - Then other quantities can be computed : Q/I, U/I
    - Polarization degree

Total	Linear	Circular
$\frac{\sqrt{Q^2+U^2+V^2}}{I}$	$\frac{\sqrt{Q^2+U^2}}{I}$	$\frac{V}{I}$

    - 2D Mueller matrix

Polarization imaging

Some experimental results

source vertically polarized $(1,1,0,0)^t$

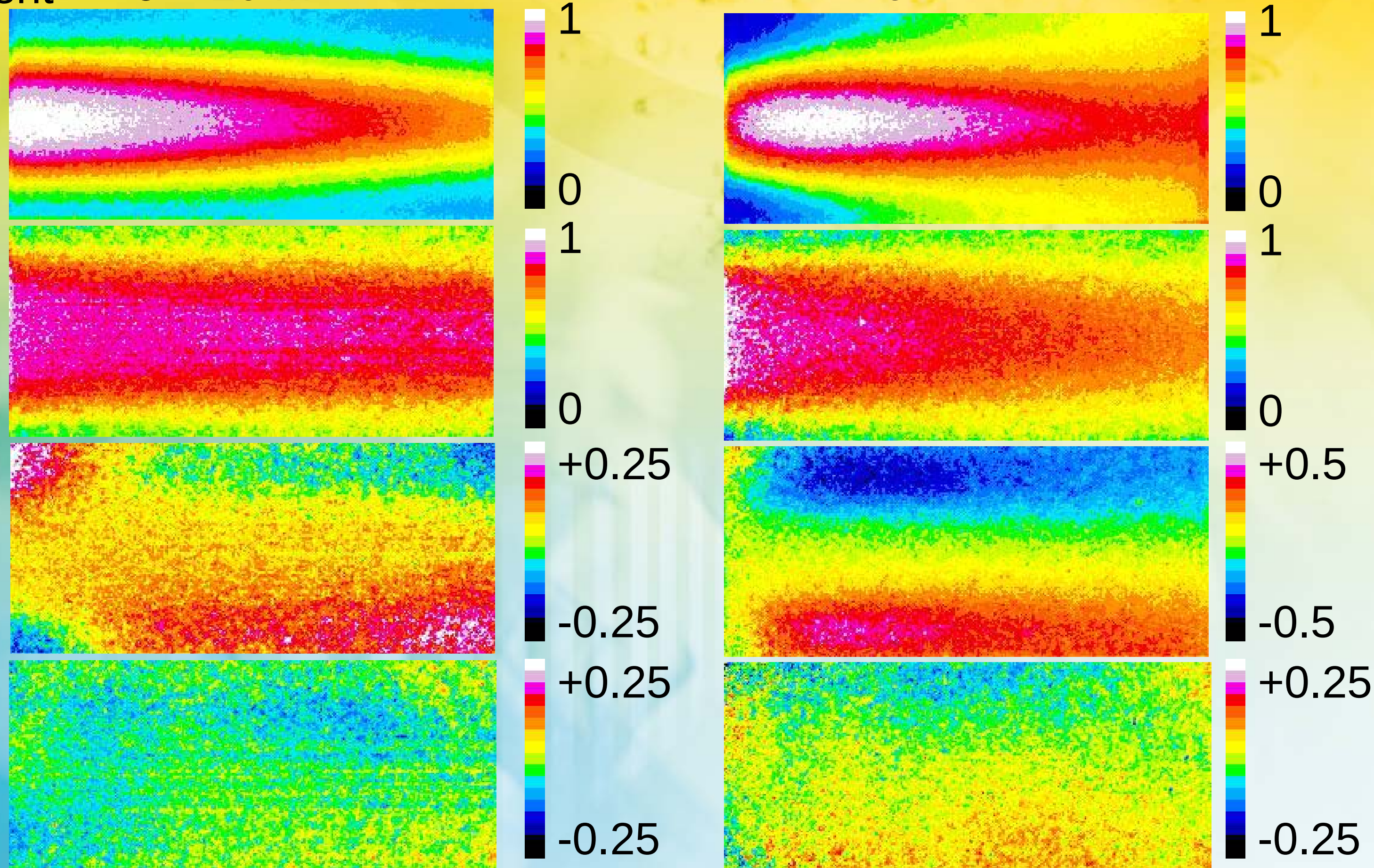
Size parameter	x = 0.8	x = 2.35
Volume fraction	f_v = 0.01 %	f_v = 0.001 %
Scattering coefficient	$\sigma = 20\ m^{-1}$	$\sigma = 17\ m^{-1}$

I/Imax

Q/I

U/I

V/I



Conclusion

- **A new model to generate polarization images in multiple scattering particle-laden media**
  - **From the integral formulation of the VRTE**
  - **With efficient statistical principles for convergence optimization**

    ➔ Integral-Vector Monte Carlo Method
- **Could potentially be applied to any 3D geometry and kind of particles** (provided that the issue of reflections at boundaries is addressed)
- Validated in the case of plane-parallel backscattering configurations
- **2D lateral Mueller matrix elements for a cubic tank filled with a uniform suspension of monodispersed particles**
  - **→ Sensitivity of different Mueller matrix elements to particle size and morphology**

Expectation

- Further work
  - Continuation of this analysis from physical and statistical points of view
  - Extension to realistic situations such as systems undergoing an aggregation process
  - Experimental investigations for comparison and application
  - Step by step development of a parameter estimation methodology

Centre de Thermique de Lyon

ACKNOWLEDGEMENTS:

Le Conseil Régional de la région Rhône-Alpes (bourse explora doc')

Le Radiative Transfer Laboratory (RTL) de l'Université du Kentucky (USA)

Merci